SEQUENCES AND SERIES REVIEW (WHITE BOARD)

Formulas to remember:

$S_n = \frac{n(t_1 + t_n)}{2}$

Arithmetic Sequence

General Term: $t_n = a + (n-1)d$

Recursive Formula: $t_n = t_{n-1} + d$

(n must be greater than 1)

Arithmetic series

$$S_n = \frac{n[2a + (n-1)d]}{2}$$

(n is a natural number)

Geometric Sequence

General Term: $t_n = ar^{n-1}$

Recursive Formula: $t_n = rt_{n-1}$

Geometric Series

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Problems that are not arithmetic or geometric: Find a pattern and write in terms of n or write a recursive formula.

Pascal's Triangle: Patterns and application of binomial expansion $(a+b)^n$

1. Determine if the following sequence is arithmetic, geometric or neither. Determine the general term for the sequence. Challenge: Write the recursive formula.

a) 29, 21, 13, ...

Arithmetic

$a = 29$

$d = 21 - 29 = -8$

$t_n = a + (n-1)d$

$= 29 + (n-1)(-8)$

$= 29 - 8n + 8$

$t_n = 37 - 8n$

i) determine t_{10}

$t_{10} = 37 - 8(10)$

$= 37 - 80$

$= -43$

b) 23, -46, 92, ...

Geometric

$t_n = ar^{n-1}$

$= 23(-2)^{n-1}$

$a = 23$

$r = \frac{-46}{23} = -2$

i) determine t_{10}

$t_{10} = 23(-2)^{10-1}$

$= -11\,776$

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2. Determine the general term for the arithmetic sequence if...

a) $t_1 = 13$ and $d = -7$

$a = 13$
$d = -7$

$t_n = a + (n-1)d$
$= 13 + (n-1)(-7)$
$= 13 - 7n + 7$
$= 20 - 7n$

b) $t_5 = 91$ and $t_7 = 57$

$d = \frac{57 - 91}{7 - 5}$
$= \frac{-34}{2}$
$= -17$

$t_n = a + (n-1)d$
$57 = a + (7-1)(-17)$
$57 = a + (6)(-17)$
$159 = a$

$t_n = a + (n-1)d$
$= 159 + (n-1)(-17)$
$= 176 - 17n$

3. Determine the general term for the geometric sequence if ...

a) the first term is 144 and the second term is 36

$t_n = ar^{n-1}$

$= 144\left(\frac{1}{4}\right)^{n-1}$

$r = \frac{36}{144} = \frac{1}{4}$ OR 0.25

$a = 144$

b) $t_5 = 45$ and $t_8 = 360$

$r^{8-5} = \frac{360}{45}$

$r^3 = 8$

$r = \sqrt[3]{8}$

$r = 2$

$t_n = ar^{n-1}$

$45 = a(2)^{5-1}$

$45 = a(16)$

$\frac{45}{16} = a$

$t_n = ar^{n-1}$

$= \frac{45}{16}(2)^{n-1}$

4. Calculate the sum of the first 10 terms in each series.

a) -103, -110, -117, ...

$d = -7$

$a = -103$

$n = 10$

$$S_n = \frac{n[2a + (n-1)d]}{2}$$

$$S_{10} = \frac{10[2(-103) + (10-1)(-7)]}{2}$$

$$= -1345$$

b) 8, -24, 72, - ...

Geometric

$$r = \frac{-24}{8} = -3$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{8((-3)^{10} - 1)}{-3 - 1}$$

$$= -118096$$

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5. Determine the sum of the first 7 terms of the geometric series if ...

a) the third term is 18 and the terms increase by a factor of 3

$r = 3$

a

$n = 7$

$t_n = ar^{n-1}$

$18 = a(3)^{3-1}$

$18 = a(9)$

$2 = a$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{2(3^7 - 1)}{3 - 1}$

$= 2186$

b) $t_5 = 5$ and $t_8 = -40$

$r^{8-5} = \frac{-40}{5}$

$r^3 = -8$

$r = \sqrt[3]{-8}$

$r = -2$

$t_n = ar^{n-1}$

$5 = a(-2)^{5-1}$

$5 = a(-2)^4$

$5 = a(16)$

$\frac{5}{16} = a$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{\frac{5}{16}((-2)^7 - 1)}{-2 - 1}$

$\doteq 13.4$

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6. Determine the sum of the first 7 terms of the arithemetic series if ...

a) $t_1 = 31$ and $t_{20} = 109$ $S_7 \doteq 303$

$d \doteq 4.1$

$a = 31$

b) $t_7 = 43$ and $t_{13} = 109$ $S_7 = 70$

7. Determine the number of terms in the sequence

-63, -57, -51, ... +63

$d = 6$

$a = -63$

$t_n = a + (n-1)d$

$63 = -63 + (n-1)6$

$63 = -63 + 6n - 6$

$22 = n$

8. Determine the sum of the geometric series.

17 - 51 + 153 - ... -334 611

$r = \frac{-51}{17} = -3$

$a = 17$

$n = ?$

$t_n = ar^{n-1}$

sub to solve

$n = 10$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{10} = \frac{17((-3)^{10} - 1)}{-3 - 1}$

$= -250\,954$

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9. During a skydiving lesson, Chandra jumps out of a plane and falls 4.9 m during the first second. For each second afterward, she continues to fall 9.8 m more than the previous second. After 15 s, she opens her parachute. How far did Chandra fall before she opened her parachute?

4.9, 14.7, . . .

$t_n = a + (n-1)d$

$a = 4.9$

$d = 9.8$

$n = 15$

$t_n = 4.9 + (15-1)(9.8)$

$= 142.1$ m

$\therefore$

10. At a fish hatchery, fish hatch at different times even though the eggs were all fertilized at the same time. The number of fish that hatched on each of the first four days after fertilization was 2, 10, 50, and 250, respectively. If the pattern continues, calculate the total number of fish hatched during the first 10 days.

2, 10, 50, . . .

$r = \frac{10}{2} = 5$

$a = 2$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{2(5^{10} - 1)}{5 - 1}$$

$$= 4\,882\,812$$

11. a) Determine the 7th term of the sequence 3, 2, 5, 7, 12, ...?
Explain your reasoning.

19, 31

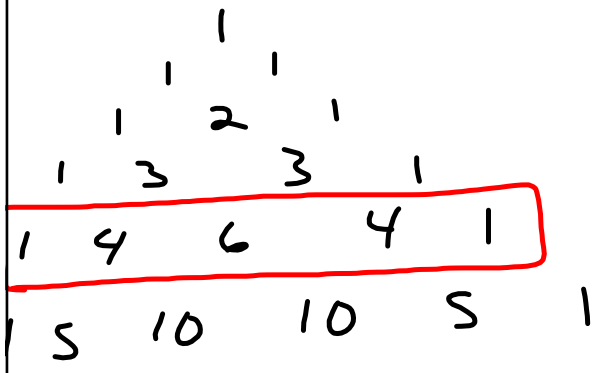
each term is the sum of the previous 2 terms

b) Write the recursive formula for the sequence in part (a).

$t_n = t_{n-1} + t_{n-2}$

12. Write the first 5 rows of Pascals triangle.

a) Describe three patterns you see in the triangle.



b) Use pascal's triangle to expand $(3x-2y)^4$

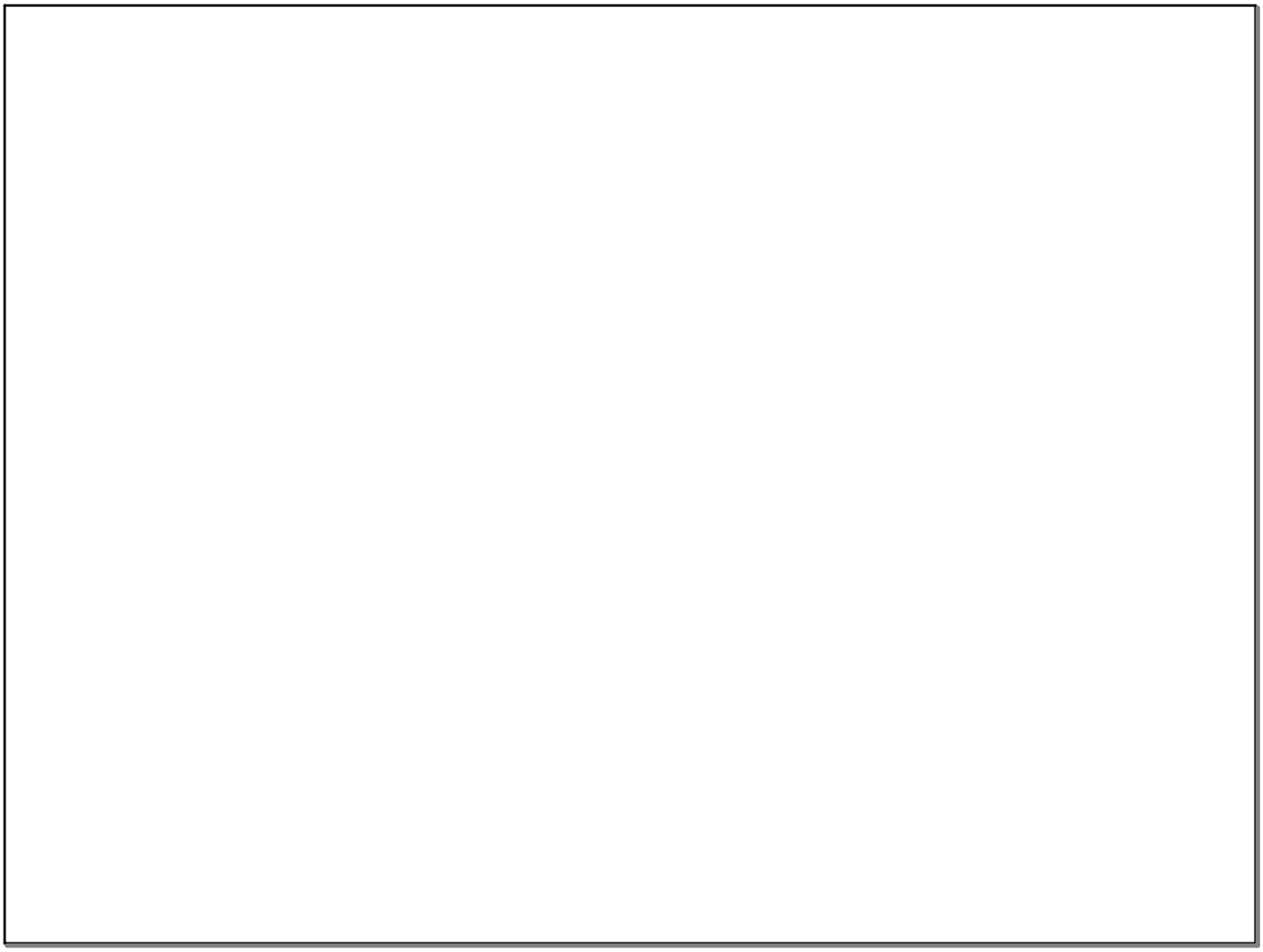
$(x+y)^4$ (simple)

$= 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$

$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$= 1(3)^4x^4(-2)^0y^0 + 4(3)^3x^3(-2)^1y^1 + 6(3)^2x^2(-2)^2y^2$
$+ 4(3)^1x^1(-2)^3y^3 + 1(3)^0x^0(-2)^4y^4$

$= 81x^4 - 216x^3y + 216x^2y^2 - 96xy^3 + 16y^4$



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