EXAM REVIEW – Chapter 6

CHAPTER 6: Quadratic Equations

61. ANS:

$$\begin{aligned} y &= (x^2+6x+9-9)+4 \\ &= (x^2+6x+9)+4-9 \\ &= (x+3)^2-5 \\ \therefore vertex &= (-3,-5) \end{aligned}$$

62. ANS:

$$\begin{aligned} y &= (-4x^2+24x \quad)-13 \\ &= -4(x^2-6x \quad)-13 \\ &= -4(x^2-6x+9-9)-13 \\ &= -4(x^2-6x+9)-13+36 \\ &= -4(x-3)^2+23 \\ \therefore \max &= 23 \end{aligned}$$

63. ANS: (a)

$$\begin{aligned} x^2+x-6&=0 \\ (x+3)(x-2)&=0 \\ \therefore x=-3 \; &and \; x=2 \end{aligned}$$

(b)

$$\begin{aligned} &3p^2+15p=0 \\ &3p(p+5)=0 \\ &\therefore 3p=0 \quad and \quad p+5=0 \\ &\therefore \; p=0 \quad and \quad p=-5 \end{aligned}$$

(c)

$$\begin{aligned} &10x^2+21x-10=0 \\ &\frac{(10x+25)(10x-4)}{10}=0 \\ &\frac{5(2x+5)2(5x-2)}{10}=0 \\ &(2x+5)(5x-2)=0 \\ &\therefore x=-\frac{5}{2} \quad and \quad x=\frac{2}{5} \end{aligned}$$

64. ANS: (a)

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-9\pm\sqrt{(9)^2-4(2)(3)}}{2(2)} \\ &= \frac{-9\pm\sqrt{57}}{4} \\ \therefore x &= \frac{-9+\sqrt{57}}{4} \quad or \quad x=\frac{-9-\sqrt{57}}{4} \\ x &= -0.36 \quad or \quad x=-4.14 \end{aligned}$$

(b)

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{12\pm\sqrt{(-12)^2-4(4)(9)}}{2(4)} \\ &= \frac{12\pm\sqrt{144-144}}{8} \\ \therefore x &= \frac{12\pm\sqrt{0}}{8} \\ x &= \frac{12}{8} = 1.5 \end{aligned}$$

65.

$$(3x-4)^2+(x+5)(x-3)=0$$
$$(3x-4)(3x-4)+(x+5)(x-3)=0$$
$$9x^2-12x-12x+16+(x^2-3x+5x-15)=0$$
$$9x^2-24x+16+(x^2+2x-15)=0$$
$$10x^2-22x+1=0$$

$Let\ \ a=10,\ b=-22,\ c=1$

$$\therefore \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{22\pm\sqrt{(-22)^2-4(10)(1)}}{2(10)}$$
$$x=\frac{22\pm\sqrt{444}}{20}$$
$$\therefore\ x=\frac{22+\sqrt{444}}{20}\quad or\quad x=\frac{22-\sqrt{444}}{20}$$
$$x=2.15\quad or\quad x=0.05$$

66. Let x be one of the integers. Let y be the other integer.

$$x-y=31 \qquad ...(1)$$
$$x^2+y^2=485 \quad ...(2)$$
$$x=31+y \quad ...(1b)$$
$sub\ (1b)\ in\ (2)$
$$(31+y)^2+y^2=485$$
$$961+62y+y^2+y^2=485$$
$$2y^2+62y+961-485=0$$
$$2y^2+62y+476=0$$

$Let\ \ a=2,\ b=62,\ c=476$

$Let\ \ a=2,\ b=62,\ c=476$

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-62\pm\sqrt{(62)^2-4(2)(476)}}{2(2)}$$
$$=\frac{-62\pm\sqrt{36}}{4}$$
$$\therefore\ y=\frac{-62+\sqrt{36}}{4}\quad or\quad y=\frac{-62-\sqrt{36}}{4}$$
$$=-14 \qquad\qquad =-17$$

There are 2 solutions: $x=17\ and\ y=-14$ or $x=14\ and\ y=-17$

67. Let b represent the base of the triangle. Therefore, $h=b+3$.

$$h=b+3 \quad \ldots(1)$$
$$\frac{bh}{2}=18 \quad \ldots(2)$$

Sub (1) into (2)

$$\frac{b(b+3)}{2}=18$$
$$b(b+3)=36$$
$$b^2+3b-36=0$$

$Let\ \ a=1,\ b=3,\ c=-36$

$$b=\frac{-3\pm\sqrt{(3)^2-4(1)(-36)}}{2(1)}$$
$$=\frac{-3\pm\sqrt{9+144}}{2}$$
$$b=4.68cm\quad or\quad b=-7.68$$

Since b must be positive, then the base is $4.68\ cm$ long.

68. Let w be the width of the flower bed. Therefore, the length is $2w+2$

$$A = l \times w$$
$$6 = (2w+2)\times(w)$$
$$6 = 2w^2 + 2w$$
$$0 = 2w^2 + 2w - 6$$
$$0 = 2(w^2 + w - 3)$$

$Let\ a = 1,\ b = 1,\ c = -3$

$$w = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{1+12}}{2}$$
$$= \frac{-1 \pm \sqrt{13}}{2}$$
$$\therefore w = \frac{-1+\sqrt{13}}{2} \quad or \quad w = \frac{-1-\sqrt{13}}{2}$$
$$= 1.30\,m \qquad\qquad = -2.30\ (inadmissible)$$

Therefore the dimensions of the flower bed are $1.3m\ \ by\ \ 4.6m$.

69. (a) The initial velocity is the "b" value in the formula $h = at^2 + bt + c$.
So the initial velocity is $51\ m/s$.

(b) This is the value "c" in the formula $h = at^2 + bt + c$.
So the initial height of the rocket is $1.3\ m$.

(c) Factor the equation $h = -4.9t^2 + 51t + 1.3$ using the quadratic formula.

$$h = -4.9t^2 + 51t + 1.3$$
$$a = -4.9,\ b = 51,\ c = 1.3$$
$$t = \frac{-51 \pm \sqrt{(51)^2 - 4(-4.9)(1.3)}}{2(-4.9)}$$
$$t = \frac{-51 \pm \sqrt{(51)^2 - 4(-4.9)(1.3)}}{2(-4.9)}$$
$$= \frac{-51 \pm \sqrt{2626.48}}{2(-4.9)}$$
$$t = \frac{-51+51.2492}{-9.8} \quad or \quad t = \frac{-51-51.2492}{-9.8}$$
$$= -0.025\text{ s} \qquad\qquad = 10.43\ s$$

The rocket hits the ground after 10.4 seconds.

(d) Add the zeros and divide by 2. Sub this value into the given equation.

$$r = -0.025 \quad and \quad s = 10.43$$
$$x = \frac{r+s}{2} = \frac{-.025+10.43}{2} = 5.20$$

Sub $x = 5.20$ into $h = -4.9t^2 + 51t + 1.3$ to get $h = -4.9(5.2)^2 + 51(5.2) + 1.3 = 134.004$
Therefore, the rocket DOES NOT reach a height of 135 metres.