3.2 Properties of Quadratic Functions

Recall: 3 Forms of Quadratics

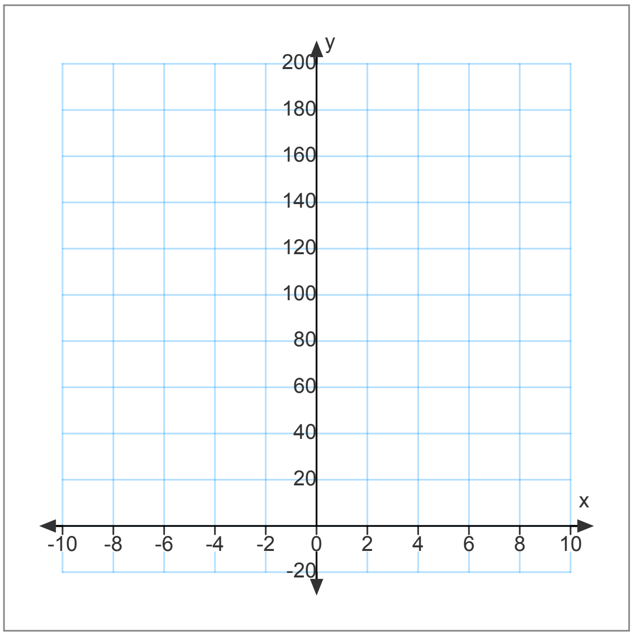
Quadratic Form	In General	What does it tell us?
Standard Form	$y = ax^2 + bx + c$	Direction of opening and the y-intercept
Factored Form	$y = a(x-r)(x-s)$	Direction of the opening and the zeros
Vertex Form	$y = a(x-h)^2 + k$	Direction of the opening, the vertex (max/min) and the axis of symmetry

Recall other important facts
-When the second differences are equal, the relation is quadratic
-If a is less than 0, the quadratic is reflected over the x-axis (opens down)
-The equation of the axis of symmetry is the x-value of the vertex
-The max/min value is the y-value of the vertex.

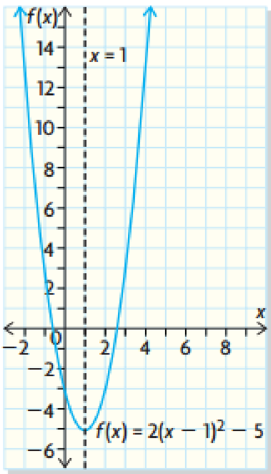
$$y = -2(x-3)^2 + 20$$

$$y = 3(x-4)(x+2)$$

$$y = x^2 - 2x - 8$$



For each of the following determine the vertex, the equation of the axis of symmetry, the max or min value and the zeros. (x, y)

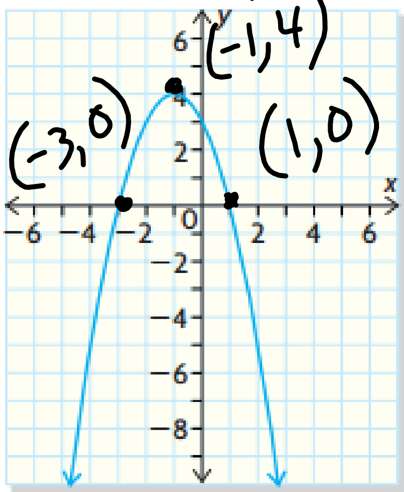


$f(x) = 2(x-1)^2 - 5$

Vertex $(1, -5)$

$x = 1$

min at $y = -5$



Determine the equation for the second graph

$(-1, 4)$

$y = a(x - r)(x - s)$

$y = a(x+3)(x-1)$

$4 = a(-1+3)(-1-1)$

$4 = a(2)(-2)$

$\frac{4}{-4} = \frac{a(-4)}{-4}$

$-1 = a$

$y = -(x+3)(x-1)$

Remember

"Solve" means find the roots which means find the x-intercepts which means set y=0 and solve in factored form.

To find the vertex you can either

1) Complete the square
2) Find the midpoint of the zeros (axis of symmetry)
   Sub that value into the equation to find the y-value (max/min)
   State the vertex (x, y)

Eg.1) Using BOTH methods, determine the max/min value of the function $f(x)=3x^2-16x-12$

1) Complete the square

$$y=3x^2-16x-12$$

$\frac{1}{2}\times\frac{-16}{3}=\frac{-16}{6}$

$$=3\left(x^2-\frac{16}{3}x\right)-12$$

$\left(\frac{16}{6}\right)^2=\frac{256}{36}=\frac{64}{9}$

$$=3\left(\underbrace{x^2-\frac{16}{3}x+\frac{64}{9}}-\frac{64}{9}\right)-12$$

$$=3\left(x-\frac{16}{6}\right)^2-\frac{192}{9}-12$$

$$=3\left(x-\frac{16}{6}\right)^2-\frac{192}{9}-\frac{108}{9}$$

$$=3\left(x-\frac{16}{6}\right)^2-\frac{300}{9}$$

min at $-\frac{300}{9}=-\frac{100}{3}$ when $x=\frac{16}{6}$

$$y=2x^2-60x+4$$

$\frac{1}{2}\times -30 = -15$

$$=2(x^2-30x)+4$$

$(-15)^2=225$

$$=2\left(x^2-30x+225-225\right)+4$$

$$=2(x-15)^2-450+4$$

$$=2(x-15)^2-446$$

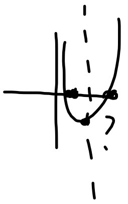
min at -446 when $x=15$

Find the vertex using the x-intercepts

$$y=x^2-8x+12$$

$$0=(x-2)(x-6)$$

$\therefore x-2=0$, $x=2$ AND $x-6=0$, $x=6$



To find x value of vertex

$$x=\frac{2+6}{2}$$

$$=\frac{8}{2}$$

$$x=4$$

sub $\rightarrow$ $y=x^2-8x+12$

$$=4^2-8(4)+12$$

$$=16-32+12$$

$$=-4$$

Vertex $(4,-4)$

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Eg. 2 The function that models the height of a golf ball is

$$h(t) = -5t^2 + 40t + 100$$

where *h(t)* is the height in metres after *t* seconds.

a) Determine how long it takes after it has been hit to touch the ground.

$$0 = -5t^2 + 40t + 100$$
$$0 = -5(t^2 - 8t - 20)$$
$$0 = -5(t+2)(t-10)$$

$\therefore t + 2 = 0$ AND $t - 10 = 0$

$t = -2$ $\quad t = 10$

-2 is impermissible because we can't have negative time.

$\therefore$ It takes 10 seconds for the golf ball to hit the ground.

b) Determine the maximum height the ball will reach during it's flight.

x-value of vertex

$$x = \frac{-2+10}{2} = \frac{8}{2} = 4$$

sub $\rightarrow y = -5t^2 + 40t + 100$

$= -5(4)^2 + 40(4) + 100$

$= 180$

$\therefore$ The maximum height is 180 meters.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

To find zeros when you can't factor

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