2 Geometry

What You'll Learn

To apply measurement and geometry concepts to composite figures and objects and to investigate optimization problems in the real world

And Why

Area, volume, and surface area calculations are important when estimating the supplies needed for painting, pouring concrete, and other construction jobs. Knowledge of surface area and volume can also help manufacturers reduce the amount of material used to package items.

Key Words

- perimeter
- area
- capacity
- volume
- surface area
- composite figure
- composite object
- optimization

CHAPTER 2

Activate Prior Knowledge

Metric and Imperial Units of Length

Prior Knowledge for 2.1

Use conversion factors to change imperial units to metric units, or vice versa.

Imperial to Metric	Metric to Imperial
1 inch = 2.54 cm	1 cm ≐ 0.3937 inch
1 foot = 30.48 cm	1 m ≐ 39.37 inches
1 foot = 0.3048 m	1 m ≐ 3.2808 feet
1 mile ≐ 1.609 km	1 km ≐ 0.6214 mile

The symbol ′ represents feet and the symbol ″ represents inches.

Example

Materials

- scientific calculator

Round each answer to the same degree of accuracy as the given measurement.

Convert each length.

a) $5\frac{1}{4}$ inches to centimetres

b) 7.3 km to miles

Solution

a) First, express $5\frac{1}{4}$ as a decimal: 5.25

Each inch is 2.54 cm.

$5.25 \times 2.54 = 13.335$

So, $5\frac{1}{4}$ inches is about 13.34 cm.

b) Each kilometre is about 0.6214 mile.

$7.3 \times 0.6214 = 4.53622$

So, 7.3 km is about 4.5 miles.

CHECK ✓

1. Convert each length.

a) A van is 17 feet long. Convert to metres.

b) A regular soccer pitch must be between 90 m and 120 m long. Convert to feet.

c) A car travels 120 km. Convert to miles.

d) A piece of paper is 21.6 cm wide. Convert to inches.

2. Room dimensions are often written using feet and inches instead of decimals.

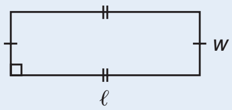
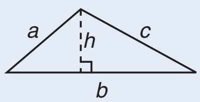
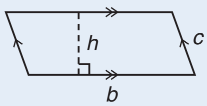
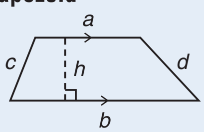
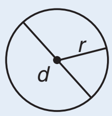
a) A room measures 3.5 m by 4.2 m. Convert to feet.

b) Express the decimal parts of your answer to part a in inches, rounded to the nearest inch. Explain your method.

1 foot = 12 inches

Perimeter and Area

The **perimeter** of a figure is the distance around it.
The **area** of a figure is the number of square units needed to cover it.

Figure	Perimeter	Area
Rectangle 	$P = 2\ell + 2w$ or $P = 2(\ell + w)$	$A = \ell w$
Triangle 	$P = a + b + c$	$A = \frac{1}{2}bh$
Parallelogram 	$P = 2b + 2c$	$A = bh$
Trapezoid 	$P = a + b + c + d$	$A = \frac{1}{2}(a + b)h$
Circle 	$C = \pi d$ or $C = 2\pi r$	$A = \pi r^2$

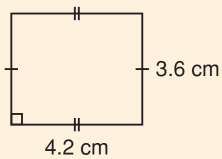
Example

Materials

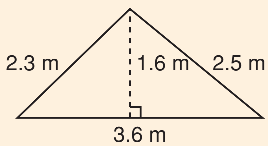
- scientific calculator

Determine the perimeter and area of each figure.

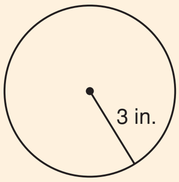
a)



b)



c)



Solution

Round the answer to the same degree of accuracy as the least accurate measurement used in the calculation.

a) $P = 2\ell + 2w$
$= 2(4.2) + 2(3.6)$
$= 15.6$

The perimeter is 15.6 cm.

$A = \ell w$
$= (4.2)(3.6)$
$= 15.12$

The area is about 15.1 cm^2.

b) $P = a + b + c$
$= 2.3 + 2.5 + 3.6$
$= 8.4$

The perimeter is 8.4 m.

$A = \frac{1}{2}bh$
$= \frac{1}{2}(3.6)(1.6)$
$= 2.88$

The area is about 2.9 m^2.

c) $C = 2\pi r$
$= 2\pi(3)$
$\doteq 18.850$

The circumference is about 19 inches.

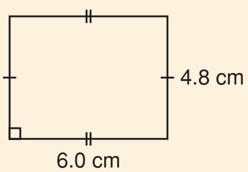
$A = \pi r^2$
$= \pi(3)^2$
$\doteq 28.274$

The area is about 28 square inches.

CHECK ☑

1. Determine the perimeter and area of each figure.

a)



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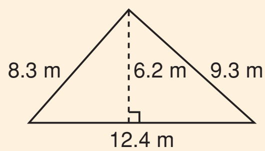
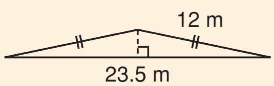


c)

2. Determine the perimeter and area of a football field that is 120 yards by 52 yards.

3. The circle at centre ice on a hockey rink has a diameter of 15 feet. Determine its circumference and area.

4. The roof of a house is made from trusses. The frame of each truss is an isosceles triangle, as shown.



a) Use the Pythagorean Theorem to determine the height of the triangle.

b) Determine the perimeter of the truss and the area it encloses.

c) Which measure from part b would be helpful when estimating how much wood will be needed for the trusses? Justify your answer.

Metric and Imperial Units of Capacity

Capacity is a measure of how much liquid a container can hold. Use conversion factors to change imperial units to metric units, or vice versa.

Imperial to Metric	Metric to Imperial
1 fluid ounce ≐ 28.413 mL	1 mL ≐ 0.0352 fluid ounce
1 pint ≐ 0.568 L	1 L ≐ 1.7598 pints
1 quart ≐ 1.1365 L	1 L ≐ 0.8799 quart
1 gallon ≐ 4.546 L	1 L ≐ 0.22 gallon

The capacity units used in the U.S. differ from those used in Canada. Unless stated otherwise, use Canadian units.

Example

Materials

- scientific calculator

Convert each measure.

a) 8.0 gallons of gas to litres

b) 170 mL of water to fluid ounces

Solution

a) Each gallon is about 4.546 L.
$8.0 \times 4.546 = 36.368$
8.0 gallons are about 36.4 L.

b) Each millilitre is about 0.0352 fluid ounce.
$170 \times 0.0352 = 5.984$
170 mL are about 6 fluid ounces.

Round each answer to the same degree of accuracy as the given measurement.

CHECK ✓

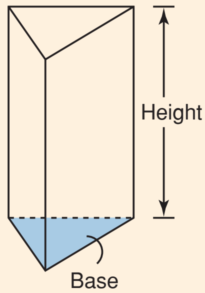
1. Convert each measure.
   a) A container holds 5 pints. Convert to litres.
   b) A gas tank can hold 80 L. Convert to gallons.
   c) A bottle contains 4.2 fluid ounces of perfume. Convert to millilitres.
   d) A large metal garbage can holds 40 gallons. Convert to litres.
   e) A can of pasta in tomato sauce contains 398 mL. Covert to fluid ounces.

2. In 2005, Canadians consumed on average 94.7 L of milk per person; Americans consumed on average 21.2 U.S. gallons per person. Each U.S. gallon is equivalent to 3.785 L. Which country had the greater milk consumption per person? Justify your answer.

Volumes of Prisms and Cylinders

Prior Knowledge for 2.2

Volume is the amount of space occupied by an object, measured in cubic units.
The volume of a cylinder or prism is the product of the base area and the height.
$V =$ base area $\times$ height
An object's height is measured perpendicular to its base.



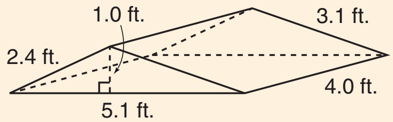
Example

Materials

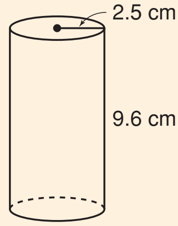
- scientific calculator

Identify the base of each object and calculate its area.
Use the base area and height to calculate the volume of the object.

a)



b)



Solution

a) The base is a triangle with base 5.1 feet and height 1 foot.
Its area is: $\frac{1}{2}bh = \frac{1}{2}(5.1)(1)$
$= 2.55$ square feet.
The height of the prism is 4 feet.
$V =$ base area $\times$ height
$= 2.55 \times 4$
$= 10.2$
The volume is 10.2 cubic feet.

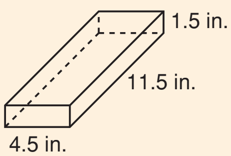
b) The base is a circle with radius 2.5 cm.
Its area is: $\pi r^2 = \pi(2.5)^2$,
or about 19.635 cm^2.
The height of the cylinder is 9.6 cm.
$V =$ base area $\times$ height
$= 19.635 \times 9.6$
$= 188.496$
The volume is about 188.5 cm^3.

Round the final answer to the same degree of accuracy as the least accurate measurement used in the calculations.

CHECK ✓

1. Identify the base of each object and calculate its area.
Use the base area and height to calculate the volume of the object.

a)



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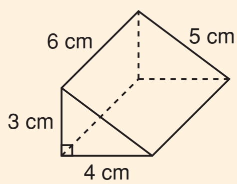

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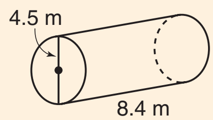


2. How do you know which face is the base of a triangular prism?

Surface Areas of Prisms and Cylinders

Surface area is the total area of the surface of an object.

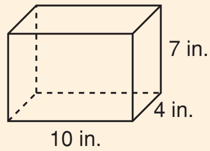
Example

Materials

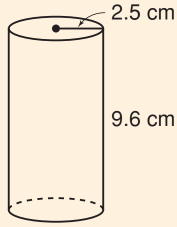
- scientific calculator

Describe the faces of each object. Then determine the surface area.

a)



b)



Solution

a)

Face	Number	Area of each	Total area (square inches)
10-in. by 4-in. rectangle	2	$10 \times 4 = 40$	$2 \times 40 = 80$
4-in. by 7-in. rectangle	2	$4 \times 7 = 28$	$2 \times 28 = 56$
10-in. by 7-in. rectangle	2	$10 \times 7 = 70$	$2 \times 70 = 140$

$SA = 80 + 56 + 140 = 276$; the surface area is 276 square inches.

b)

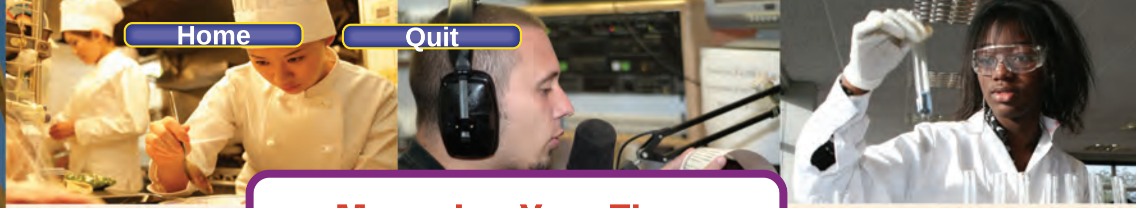
Face	Number	Area of each	Total area (cm^2)
Circle with radius 2.5 cm	2	$\pi(2.5)^2 \doteq 19.635$	$2 \times 19.635 = 39.270$
One curved surface that can be unrolled to form a rectangle with length $2\pi(2.5)$, or 5π cm, and width 9.6 cm	1	$5\pi \times 9.6 \doteq 150.796$	150.796

$SA = 39.270 + 150.796$; the surface area is about 190.1 cm^2.

CHECK ✓

1. Use the art in question 1 on page 64. For each object, describe the faces of the object and determine its surface area.

2. How many faces does a rectangular prism have? A triangular prism?



Managing Your Time

In college, or in a job or apprenticeship, you will be expected to manage your workload in the time designated for the task and to meet deadlines. You will need to set priorities, create a realistic plan for completing them, and follow your plan, making adjustments when necessary.

- **Set priorities**
  - Rank the importance of the tasks on your to-do list.
  - Decide which tasks must be done as soon as possible. Plan when the others must or should be done.
- **Create and follow a schedule**
  - Prioritize your tasks to help you decide what to do in a day, week, month, or year.
  - Be realistic. Allow time for socializing, travelling to and from work or school, eating, sleeping, and completing other responsibilities and activities.
  - Get an early start on longer tasks and work regularly on them so you have enough time to adapt your schedule to any unexpected delays.

• Complete questions 2 to 4 when it is appropriate as you work through this chapter.

1. What is another strategy you would suggest for managing your time?
2. Apply strategies for managing your time as you prepare for the chapter test or work on the chapter problem.
3. Compare the schedule you have now with a schedule you may have in college or an apprenticeship. Use the information below, or research information about courses or apprenticeships that interest you.
   - Some college students spend about 15 h per week in class and, for each hour in class, about 2 h on independent work and study.
   - Required workplace and classroom hours for apprenticeship programs vary. Assume you will spend 30 h on the job and 5 h in the classroom each week.
4. Describe a situation in a future apprenticeship or job in which you might use time-management strategies.

2.1 Area Applications

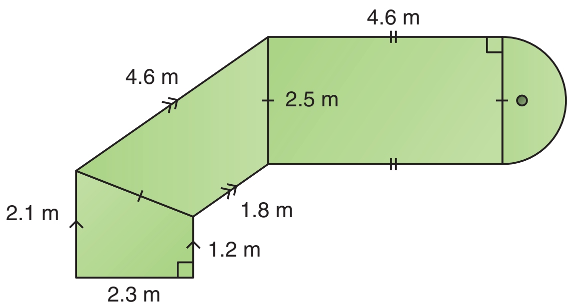
Shantel has a summer job working at a miniature golf course. Before the course can open in the spring, the old carpeting at each hole is inspected. If the carpeting is too worn, Shantel calculates how much carpeting is needed and replaces it.

Investigate

Determining the Area of a Composite Figure

Materials
- scientific calculator

The design for one hole of a miniature golf course is shown. The curve is a semicircle.



- Determine the area of this miniature golf hole.
- Trade solutions with a classmate. Check your classmate’s solution. Did you solve the problem the same way?
- Compare areas. If you have different answers, find out why.

Reflect

- Suppose the length of the rectangular part of the hole were only 3.6 m. How would this affect the area? Justify your answer.
- Suppose this hole needs new carpeting. How would knowing the area help Shantel decide how much carpet to buy? What other information does she need to know about how the carpeting is sold?

Connect the Ideas

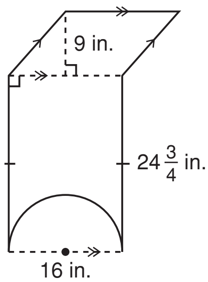
Composite figures

A figure that is made up from other simpler figures is called a **composite figure**.

Example 1

Describing a Composite Figure

Describe the figures that make up this composite figure.



Solution

The composite figure is made up of these figures:

- A rectangle with width 16 inches and height $24\frac{3}{4}$ inches.
- A parallelogram with height 9 inches and base 16 inches, on top of the rectangle.
- A semicircle with diameter 16 inches, removed from the base of the rectangle.

Area of a composite figure

To determine the area of a composite figure:

- Break it into simpler figures for which you know how to calculate the area.
- Calculate the area of each part.
- Add the areas.
- Subtract the areas of any parts removed from the figure.

Example 2

Materials
- scientific calculator

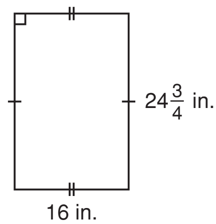
Calculating the Area of a Composite Figure

Determine the area of the composite figure in *Example 1*.

Solution

Determine the area of each simple figure.

- The rectangle has dimensions 16 inches by $24\frac{3}{4}$ inches.



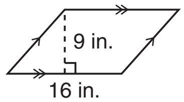
Express $24\frac{3}{4}$ as a decimal: 24.75

> Use subscripts to organize calculations of similar properties. For example, $A_{\text{rectangle}}$ refers to the area of the rectangle and A_{figure} refers to the area of the composite figure.

$$\begin{aligned} A_{\text{rectangle}} &= \ell w \qquad \text{Substitute: } \ell = 16 \text{ and } w = 24.75 \\ &= 16 \times 24.75, \text{ or } 396 \end{aligned}$$

The area of the rectangle is 396 square inches.

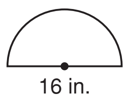
- The parallelogram has base length 16 inches and height 9 inches.



$$\begin{aligned} A_{\text{parallelogram}} &= bh \qquad \text{Substitute: } b = 16 \text{ and } h = 9 \\ &= 16 \times 9, \text{ or } 144 \end{aligned}$$

The area of the parallelogram is 144 square inches.

- The radius of the semicircle is half the diameter:



16 inches ÷ 2 = 8 inches

The area of the semicircle is half the area of a circle with the same radius.

$$\begin{aligned} A_{\text{semicircle}} &= \frac{1}{2}\pi r^2 \qquad \text{Substitute: } r = 8 \\ &= \frac{1}{2}\pi(8)^2 \\ &\doteq 100.531 \end{aligned}$$

Press: 1 [÷] 2 [×] [π] [×] 8 [x^2] [ENTER]

> Round the final answer to the same degree of accuracy as the least accurate measurement used in the calculations.

The area of the semicircle is about 100.531 square inches.

$$\begin{aligned} A_{\text{figure}} &= A_{\text{rectangle}} + A_{\text{parallelogram}} - A_{\text{semicircle}} \\ &= 396 \text{ square inches} + 144 \text{ square inches} - 100.531 \text{ square inches} \\ &= 439.469 \text{ square inches} \end{aligned}$$

The area of the composite figure is about 439 square inches.

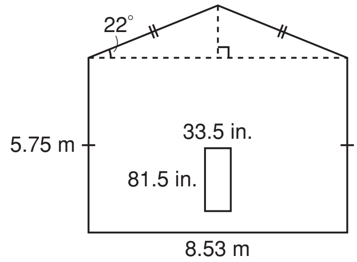
Finding missing lengths

Sometimes you need to use trigonometric ratios to determine missing lengths before you can calculate the area of a composite figure.

Example 3 Using Trigonometry to Determine an Unknown Length

Materials
- scientific calculator

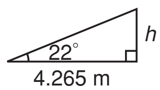
Carpenters have constructed the frame for a house and will nail pressboard over the frame. Determine the area of pressboard they need for the back wall of the house.



Solution

This composite figure is made up of a large rectangle, with an isosceles triangle on top and a rectangular door cut out.
Determine the area of each simple figure.

- $A_{\text{rectangle}} = \ell w$ Substitute: $\ell = 8.53$ and $w = 5.75$
  $= 8.53 \times 5.75$, or 49.0475
  The area of the rectangle is about 49.05 m.
- The isosceles triangle is made up of two congruent right triangles, each with one 22° angle and the same height as the isosceles triangle. The base of each right triangle is half the width of the wall:
  $8.53 \text{ m} \div 2 = 4.265 \text{ m}$



$\tan A = \dfrac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$

$\tan 22° = \dfrac{h}{4.265}$

$h = 4.265 \times \tan 22°$, or about 1.72

The height of the triangle is about 1.72 m.

$A_{\text{triangle}} = \frac{1}{2}bh$ Substitute: $b = 8.53$ and $h = 1.72$
$= 0.5 \times 8.53 \times 1.72$, or 7.3358

The area of the triangle is about 7.34 m.

- The dimensions of the door are in inches.
  $A_{\text{door}} = \ell w$ Substitute: $\ell = 33.5$ and $w = 81.5$
  $= 33.5 \times 81.5$, or 2730.25
  Convert the dimensions to square metres.
  1 inch = 2.54 cm, or 0.0254 m
  So, $(1 \text{ inch})^2 = (0.0254 \text{ m})^2$, or about $0.000\,645\,2 \text{ m}^2$.
  $2730.25 \times 0.000\,645\,2 \doteq 1.7616$
  The area of the rectangular door is about 1.76 m.

$A_{\text{total}} = A_{\text{rectangle}} + A_{\text{triangle}} - A_{\text{door}}$
$= 49.0475 \text{ m} + 7.3358 \text{ m} - 1.7616 \text{ m}$, or 54.6217 m

They need about 54.6 m² of pressboard for this wall of the house.

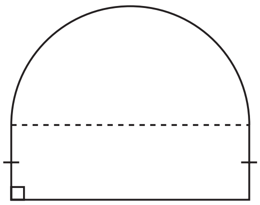
Practice

A

■ For help with questions 1 to 3, see Example 1.

1. Describe the figures that make up each composite figure. The curve is a semicircle.

a)



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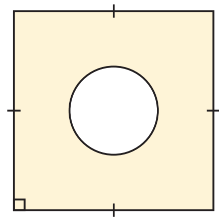

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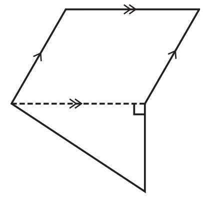
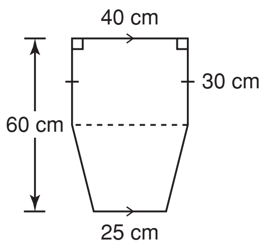


2. Describe how you would determine the area of each composite figure in question 1.

3. Sketch the figures that make up each composite figure. Include measurements in your sketches. All curves are circles or semicircles.

a)



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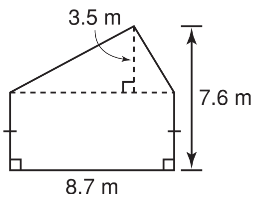
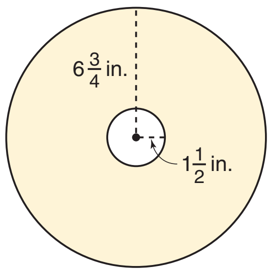
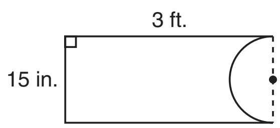


1 foot = 12 inches

c)



d)

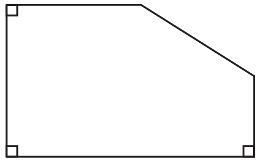


B

■ For help with question 4, see Example 2.

4. Determine the area of each composite figure in question 3.

5. Two students are calculating the area of this figure.



Tasmin's method
This is a rectangle with a trapezoid on top. I will add the area of the rectangle to the area of the trapezoid.

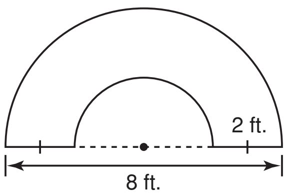
Jeffrey's method
This is a rectangle with a triangle removed from the corner. I will subtract the area of the triangle from the area of the rectangle.

Who is correct? Justify your answer.
Include diagrams in your explanation.

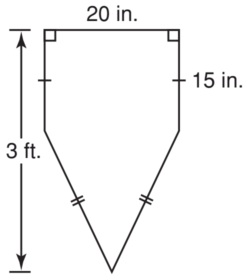
1 foot = 12 inches

6. Describe the figures that make up each composite figure.
Determine the area of each composite figure. All curves are semicircles.

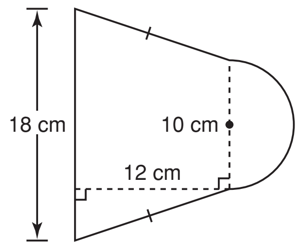
a)



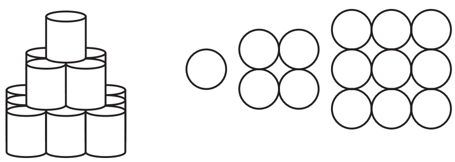
b)



c)

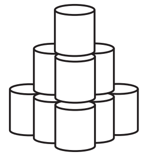


7. A grocery store display is built using cans of juice, stacked in layers. The display has 10 layers. The top three layers and an overhead view of each of these layers are shown.



a) Describe the pattern in the number of cans in each layer.
b) What shape is each layer of the display?
c) How many cans are in the tenth layer?
d) Each can has diameter 12.5 cm. Determine the amount of floor space needed for the display. What assumption are you making?

8. The display in question 7 is changed to have a triangular base. The top three layers are shown. The cans are still stacked 10 layers high.



a) Draw and describe the pattern in the number of cans in each layer.

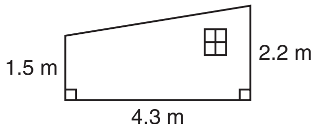
b) How many cans are in the bottom layer?

c) Determine the amount of floor space needed for the display. What assumption are you making?

d) Will the display hold more or fewer cans than the display in question 7? Justify your answer.

9. A decorator is painting this wall of an attic room. The window measures 0.6 m by 0.5 m.

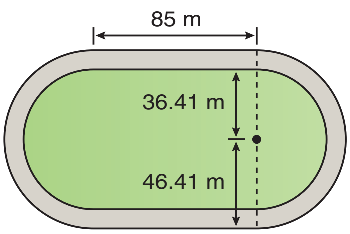
1 m ≐ 3.2808 feet



a) What is the area of the wall in square metres and square feet?

b) The paint is sold in 1-pint containers. Each container should cover between 50 square feet and 60 square feet. How many cans of paint should the decorator buy?

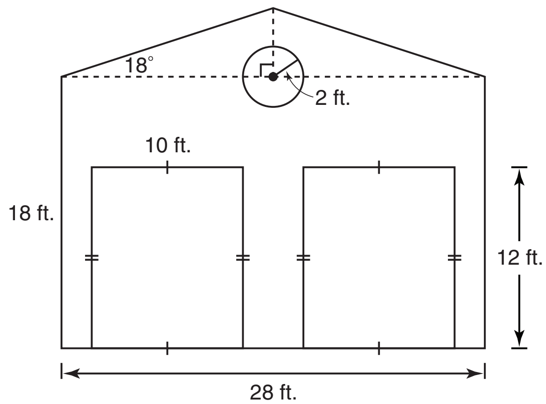
10. **Assessment Focus** The running track in this diagram consists of two parallel sections with semicircular sections at each end. Determine the area of the track.



11. **Literacy in Math** Write a step-by-step description of how to determine the area of a composite figure. Include an example.

For help with question 12, see Example 3.

12. An outdoor garage is being built on a farm to house vehicles and equipment. The front has two congruent garage door entrances and a round window at the top.



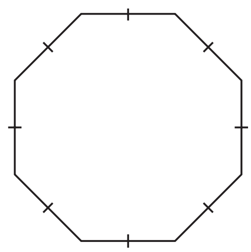
a) The front wall will be covered in sheet metal. Determine the amount of sheet metal needed, to the nearest square foot.

b) Suppose the sheet metal is priced by the square metre. How many square metres will be needed for this project?

1 foot = 0.3048 m

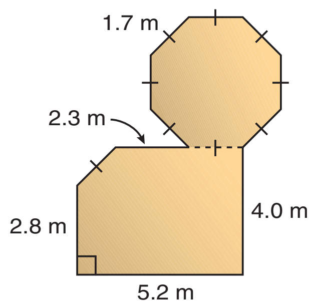
The short lines on each side mean that the sides have equal length.

13. a) Describe how to use composite figures to determine the area of a regular octagon. Use your technique to determine the area of a regular octagon with side length 20 cm.



b) Compare your work in part a with another student. If your answers are different, try to determine why. If you used different methods, which do you think is easier?

14. The design for a backyard deck is shown. It will be built using plastic lumber made from recycled materials.



a) Determine the area of the deck.

b) A circular hot tub with diameter 2 m is to be installed in the octagonal portion of the deck. How much wood needs to be cut out to make room for the hot tub?

c) The backyard is a rectangle measuring 65 feet by 45 feet. What is the area of the backyard not covered by the deck?

1 m ≐ 3.2808 feet

15. Each year, car manufacturers create concept cars to try out new technology and design ideas. For an auto show, a manufacturer wishes to display three electric concept cars on a raised platform.

- a four-door, four-passenger car with length 4856 mm and width 1915 mm
- a four-passenger sport wagon with length 4390 mm and width 1743 mm
- a two-seater sport utility vehicle with length 3885 mm and width 1598 mm

a) Choose a figure for the platform. Sketch how the cars should be arranged on the platform. Explain the decisions you made about the figure and the arrangement of the cars.

b) Determine reasonable dimensions for the platform in part a. Include an explanation of how you decided how much space to leave between the cars.

c) How much carpeting should be ordered to cover the top of the display platform you have designed?

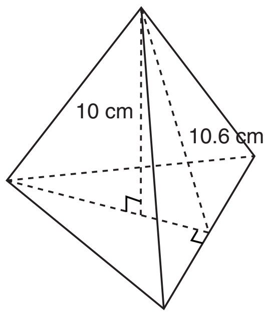
C

A net is a drawing that can be cut out and folded to create a three-dimensional object.

16. Monique wants to make a regular equilateral pyramid with a vertical height of 10 cm.

a) Draw a net for this object.

b) Determine the area of the net.



In Your Own Words

Suppose you are hired to paint the walls of your classroom. Develop a plan describing how you would measure and calculate the area to be painted. What other information would you need to include in your plan?

Home Quit

2.2 Working with Composite Objects

Antiques, works of art, and other valuable objects are carefully packaged for shipping. A cardboard box or wooden crate is built to fit the object. Dimensions must be accurately measured to ensure that the container is large enough to hold the object and protective packing material.

Investigate

Building a Model

Materials
- scientific calculator

Work with a partner.
Choose an object in the classroom that can be viewed as a combination of two or more rectangular prisms, triangular prisms, or cylinders.
Alternatively, build your own object.

- Sketch your object. Describe the rectangular prisms, triangular prisms, or cylinders that make up the object.
- Calculate the volume and surface area of your object.

Reflect

- What strategies did you use to calculate the volume and surface area of your object?
- Did your strategies produce reasonable results? If not, how could they be improved?
- Suppose you had to build a package to hold the object. Which of the data you collected or calculated would be most useful? Why?

Connect the Ideas

Volume and surface area

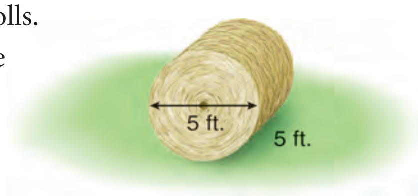
The volume of a cylinder or prism is the product of the base area and the height: $V =$ base area $\times$ height. The surface area of a cylinder or prism is the sum of the areas of the faces.

Example 1

Finding the Volume and Surface Area of a Cylinder

Materials
- scientific calculator

A machine bales hay in cylindrical rolls. For storage, a shrink-wrap protective cover is placed on the bale.



a) Determine the volume of hay in a bale and the area of the shrink-wrap covering it.

b) How much shrink-wrap is needed if the wrap does not cover the ends of the roll?

c) Suppose the shrink-wrap is priced by the square metre. To the nearest square metre, how many square metres of wrap are needed to cover the bale as described in part b?

Solution

The radius of the base is half the diameter, or 2.5 feet.

a) The base area of the cylinder is: $\pi(2.5 \text{ feet})^2 \doteq 19.635$ cubic feet
The height is 5 feet.
So, the volume of the cylinder is:
19.635 square feet $\times$ 5 feet $=$ 98.175 cubic feet
The volume of hay is about 98 cubic feet.
The surface area is the sum of the areas of the faces.

Face	Shape	Number	Area of each face (cm^2)
Curved surface	Unrolls to form a rectangle	1	$2 \times \pi \times 2.5 \times 5 \doteq 78.54$
Ends	Circles	2	$2 \times \pi \times 2.5^2 \doteq 39.27$

$78.54 + 39.27 = 117.81$
About 118 square feet of shrink-wrap are needed to cover the bale.

If the ends of the roll are not covered, the surface area is just the area of the curved surface.

b) From the table, about 79 square feet of shrink-wrap are needed.

c) 1 foot $=$ 0.3048 m, so $(1 \text{ foot})^2 = (0.3048 \text{ m})^2$, or about 0.092 903 m^2
$78.54 \times 0.092\,903 \doteq 7.2966$
So, about 7 m^2 of shrink-wrap are needed.

Composite objects

When a structure or object is made up from several simple objects, it is called a **composite object**.

The house in the photograph can be thought of as a rectangular prism with a triangular prism on top and another rectangular prism for the chimney.

Other objects, such as this sewer pipe, can be thought of as simple three-dimensional objects with a piece removed.

Determining the volume

To determine the volume of a composite object:

- Calculate the volume of each part of the object.
- Add the volumes.
- Subtract the volume of any parts that were removed.

Sometimes, the composite object can be viewed as a prism whose base is a composite object. In these cases, you can use the formula:

$V =$ base area $\times$ height

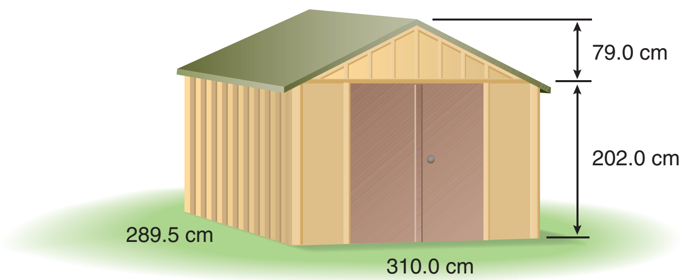
Example 2

Finding the Volume of a Composite Object

Materials

- scientific calculator

Determine the volume of this shed in cubic metres.



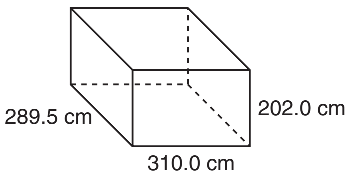
Solution

Method 1

Think of the shed as a triangular prism on top of a rectangular prism.

Calculate the volume of each part:

- The base area of the rectangular prism is:

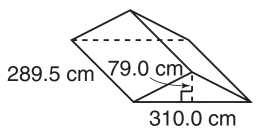


$310.0 \text{ cm} \times 289.5 \text{ cm}$
$= 89\ 745 \text{ cm}^2$

The height is 202.0 cm.

So, $V_{\text{rectangular prism}}$
$= 89\ 745 \text{ cm}^2 \times 202.0 \text{ cm}$
$= 18\ 128\ 490.0 \text{ cm}^3$

- The base area of the triangular prism is:



$\frac{1}{2} \times 310.0 \text{ cm} \times 79.0 \text{ cm}$
$= 12\ 245 \text{ cm}^2$

The height is 289.5 cm.

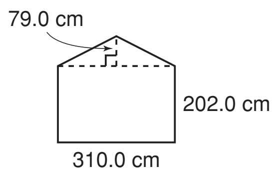
So, $V_{\text{triangular prism}}$
$= 12\ 245 \text{ cm}^2 \times 289.5 \text{ cm}$
$= 3\ 544\ 927.5 \text{ cm}^3$

$V_{\text{shed}} = V_{\text{rectangular prism}}$
$+ V_{\text{triangular prism}}$
$= 18\ 128\ 490.0 \text{ cm}^3$
$+ 3\ 544\ 927.5 \text{ cm}^3$
$= 21\ 673\ 417.5 \text{ cm}^3$

The volume is $21\ 673\ 417.5 \text{ cm}^3$.

Method 2

Think of the shed as a prism with this 5-sided base:



Calculate the area of each part of the base:

- $A_{\text{rectangle}}$
  $= 310.0 \text{ cm} \times 202.0 \text{ cm}$
  $= 62\ 620 \text{ cm}^2$
- A_{triangle}
  $= \frac{1}{2} \times 310.0 \text{ cm} \times 79.0 \text{ cm}$
  $= 12\ 245 \text{ cm}^2$

So, the base area of the prism is:
$62\ 620 \text{ cm}^2 + 12\ 245 \text{ cm}^2$
$= 74\ 865 \text{ cm}^2$

The height of the prism is 289.5 cm.

The volume of the prism is:
base area × height
$= 74\ 865 \text{ cm}^2 \times 289.5 \text{ cm}$
$= 21\ 673\ 417.5 \text{ cm}^3$

The volume is $21\ 673\ 417.5 \text{ cm}^3$.

$21\ 673\ 417.5 \div 1\ 000\ 000 \doteq 21.67$

The volume of the shed is approximately 21.67 m^3.

1 m = 100 cm
So, $(1 \text{ m})^3 = (100 \text{ cm})^3$,
or $1\ 000\ 000 \text{ cm}^3$

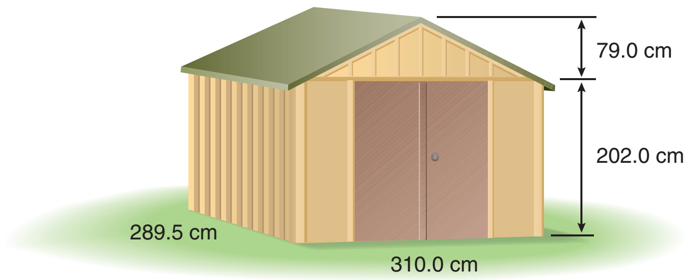
Determining the surface area

When you determine the surface area of a composite object, include only those faces that are faces of the composite object. That is, the faces that are part of the surface of the object.

Example 3 Finding the Surface Area of a Composite Object

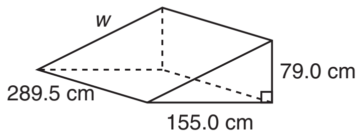
Materials
- scientific calculator

Determine the surface area of the shed in *Example 2* in square metres. Assume the shed has a floor that you should include in your calculations.



Solution

Before you can determine the area of the roof panels, you have to determine their width.
Draw a sketch showing a roof panel and half of the triangular panel from the front of the shed.
Use the Pythagorean Theorem.



$155.0^2 + 79.0^2 = w^2$
$30\ 266 = w^2$
$w \doteq 173.97$

The width is about 173.97 cm.
Use a table to keep track of the faces included in the surface area.

Face	Shape	Number	Area of each face (cm^2)
Roof panels	Rectangle	2	$289.5 \times 173.97 \doteq 50\ 364.3$
Front and back triangular panels	Triangle	2	$\frac{1}{2}(310.0 \times 79.0) = 12\ 245$
Front and back	Rectangle	2	$310.0 \times 202.0 = 62\ 620$
Sides	Rectangle	2	$289.5 \times 202.0 = 58\ 479$
Floor	Rectangle	1	$310.0 \times 289.5 = 89\ 745$

$$SA_{total} = 2(50\ 364.3 + 12\ 245 + 62\ 620 + 58\ 479) + 89\ 745$$
$$= 367\ 416.6 + 89\ 745$$
$$= 457\ 161.6$$

1 m = 100 cm
So, $(1\text{ m})^2 = (100\text{ cm})^2$, or 10 000 cm^2
Divide by 10 000 to convert the area to square metres.
The surface area of the shed is about 45.72 m^2.

Practice

A

For help with questions 1 or 2, see Example 1.

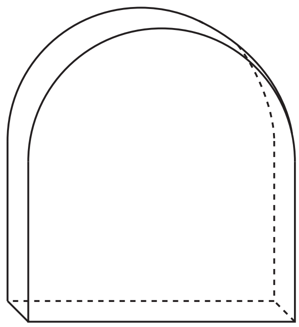
1. Describe the simple three-dimensional objects that make up each cake.

a)

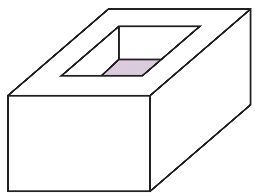
b)

2. Describe the simple three-dimensional objects that make up each object.

a)

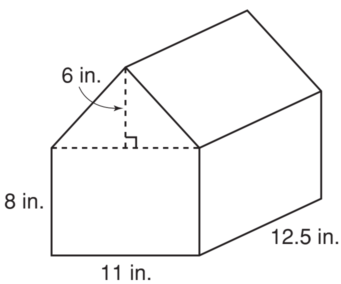


b)



3. Describe the simple three-dimensional objects that make up each object.

a)



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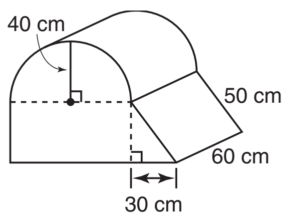


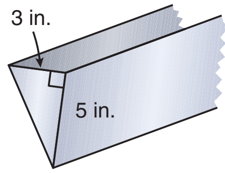
4. A fruit drink is sold in a box that contains 10 drink pouches. The dimensions of the box are shown.

a) Determine the surface area of the cardboard used for the box.

b) Each drink pouch uses about 340 cm^2 of material. How much is used for the 10 pouches?

c) Each drink pouch contains 200 mL. What is the total amount of drink in the package?

5. A section of water trough for a poultry farm is shown on the right. The triangular face is a right triangle with a base of 3 inches and height of 5 inches. The trough runs the length of the barn, which is 120 feet long.



1 foot = 12 inches

a) Determine the amount of sheet metal required to build the trough.

b) Determine the volume of the trough in cubic inches.

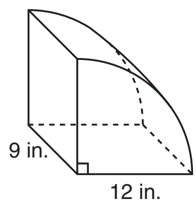
1 gallon ≐ 4.546 L

c) A cubic inch is about 0.004 33 gallons. About how many litres of water can the trough hold?

1 cubic foot ≐ 6.23 gallons

6. In many places, regulations state that all the milk a dairy farmer has in a holding tank must be picked up in one trip. A milk truck has a cylindrical tank with radius 9 feet and length 14 feet. Is the tank large enough to pick up all the milk from a full 6000-gallon holding tank? Justify your answer.

7. Joshua is calculating the volume and surface area of this quarter cylinder.



a) How would its volume compare to the volume of a cylinder with the same radius and height?

b) Calculate the object's volume.

c) Complete the calculations in the table Joshua has created. Then add areas to determine the total surface area.

Face	Shape	Number	Area of each face (square inches)
Front and back	Quarter circle	2	$\frac{1}{4}(\pi \times 12^2) =$
Bottom and side	Rectangle	2	$9 \times 12 =$
Curved surface	Unrolls to form rectangle	1	$\frac{1}{4}(2\pi \times 12 \times 9) =$

d) Explain how Joshua developed the expressions for the area of each face.

B

8. The bottom layer of the cake in part a of question 1 has diameter 39 cm and height 10 cm. The top layer has diameter 26 cm and height 10 cm. Assume that the entire top of the bottom layer is iced, but only the portion that can be seen is decorated.

a) Determine the volume of each layer and the total volume of the two layers.

b) Determine the area that is iced.

c) Determine the area that is decorated.

9. The bottom layer of the cake in part b of question 1 has length and width 14 inches and height 4 inches. The top layer has length and width 10 inches and height 4 inches. Assume that the entire top of the bottom layer is iced, but only the portion that can be seen is decorated.

a) Determine the volume of each layer and the total volume of the two layers.

b) Determine the surface area that is iced.

c) Determine the area that is decorated.

1 inch = 2.54 cm

10. Suppose you could buy either of the cakes shown in question 1 for the same price. Use your answers to questions 8 and 9 to decide which cake is the better deal. Justify your answer.

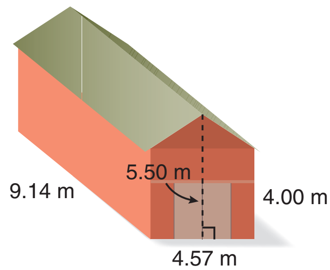
For help with question 11, see Example 2.

1 m ≐ 3.2808 feet

11. The front and back faces of the roof of this barn are isosceles triangles.

a) Determine the volume of the barn.

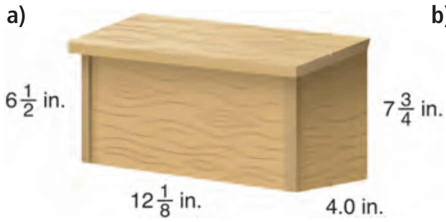
b) Would the barn be large enough to store 5000 cubic feet of hay? Justify your answer.



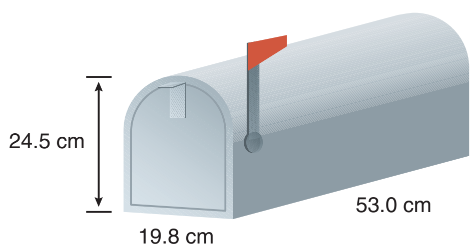
1 inch = 2.54 cm

12. Two different mailboxes are shown, one wooden, one made out of sheet metal. Which mailbox has the greater volume? Justify your answer.

a)



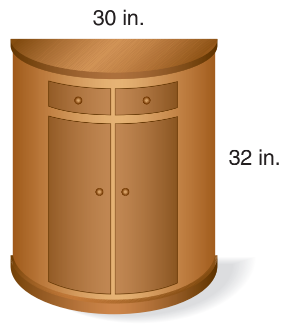
b)



Home Quit

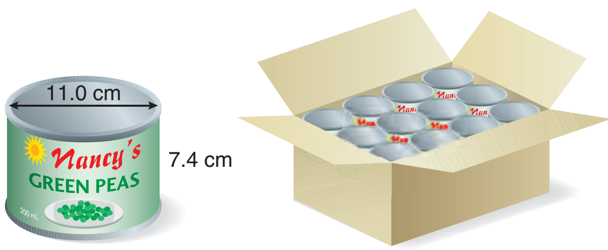
13. **Literacy in Math** Create a flow chart that outlines the steps to follow when calculating the volume of a composite object.

14. Olivia owns a furniture store. She has sold a half-cylinder console table like the one shown here. Olivia needs to build a crate to ship the table to the customer.



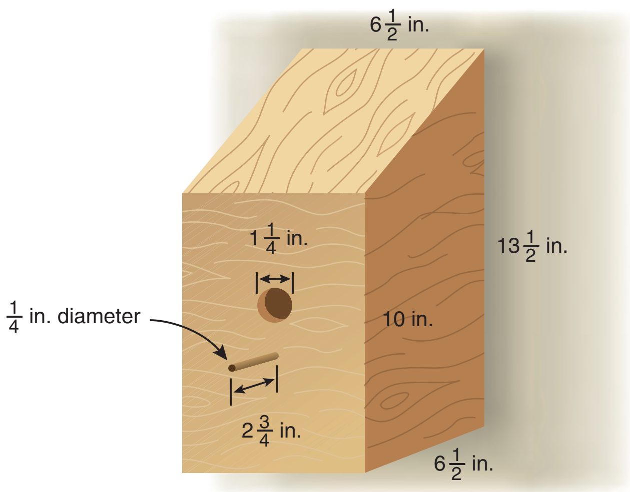
a) What dimensions would you recommend for the shipping crate? Justify your answer.
b) What will be the volume of the shipping crate?
c) How much empty space will there be around the table for protective packing material?

15. A can of peas has a diameter of 11.0 cm and a height of 7.4 cm. The cans are packed for shipping in a box. They are arranged in 2 layers of 3 rows by 4. The box is constructed to fit the cans snugly. Determine the amount of empty space in the box.

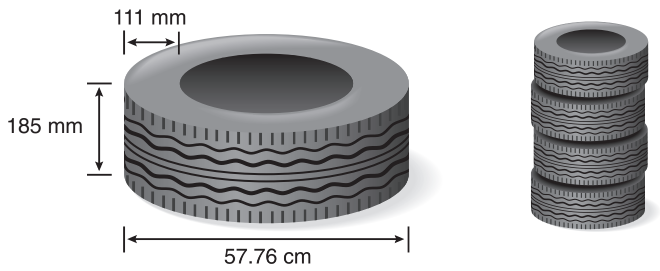


■ For help with question 16, see Example 3.

16. Amutha has built a birdhouse, and decides to paint it. Determine the surface area that requires painting.



17. A tire manufacturer sells tires in packages of four. The tires are shrink-wrapped together, without covering the hole in the centre. Determine the amount of shrink-wrap required for each package, in square centimetres.



1 inch = 2.54 cm

18. Two different mailboxes are shown in question 12. Which mailbox has the lesser surface area? What is the difference in surface areas?

19. **Assessment Focus**

a) Suppose the objects in question 3 are to be moulded using concrete. Determine the volume of concrete required to make each object.

b) Which object has the greater volume?

c) Suppose the objects in question 3 are to be made out of sheet metal. Determine the area of metal required to build each object.

d) Which object has the greater surface area?

C

20. A manufacturing company uses sheet metal and a press to cut out washers. The sheet metal is $\frac{1}{16}$-inch thick. The washers have outer diameter 1 inch and inner diameter $\frac{1}{4}$ inch. The sheet has length 8 yards and width 3 yards.

1 yard = 3 feet
1 foot = 12 inches

a) Determine how many washers can be cut from one sheet.

b) Calculate the volume of material not used. Include the material cut from the centre of each washer.

c) The washers are to be sprayed with a protective coating. Determine the surface area of the washers from one sheet.

In Your Own Words

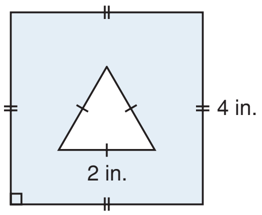
Suppose you are given an object and asked to design a shipping carton. Describe the steps you would follow. Include an example with diagrams in your explanation.

Mid-Chapter Review

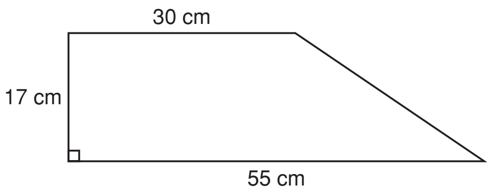
2.1

1. Describe the figures that make up each composite figure. Then determine the area of each composite figure.

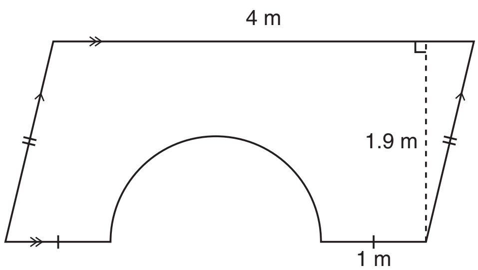
a)



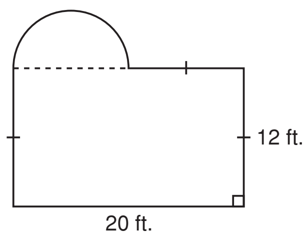
b)



c)



2. A winter cover will be made for this swimming pool. The cover must extend 1 foot beyond the edges along the perimeter.



a) What will be the area of the cover in square feet?

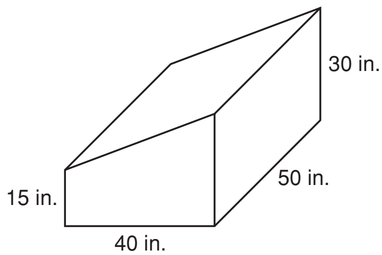
1 m ≐ 39.37 inches

b) The material for the cover sells for $2.50 per square metre. How many square metres are needed and how much will they cost?

2.2

3. Quarters are packaged in rolls of 40. Each quarter has a diameter of 2.4 cm, and is 1.5 mm thick. Determine the volume and surface area of a roll of quarters.

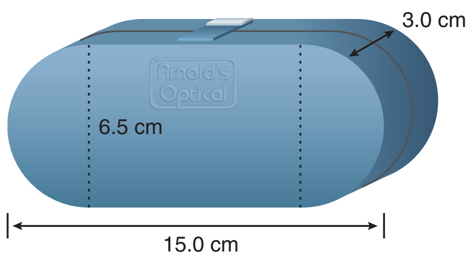
4. Describe the simple objects that make up this object. Then determine its volume and surface area.



5. Max needs to make 3 copies of the object shown in question 4 from cement. He has 2 m^3 of cement. Will this be enough? Justify your answer.

1 m ≐ 39.37 inches

6. Determine the volume and surface area of this sunglasses case.



Home Quit

2.3 Optimizing Areas and Perimeters

A gardener wants to determine the greatest rectangular area that can be enclosed by a given length of edging. A dog breeder wants to find the rectangle with a given area that has the least perimeter. Both are ***optimization*** *problems.*

Investigate

Finding Maximum Areas and Minimum Perimeters

Materials
- grid paper
- 24 toothpicks
- 36 square tiles

or
- TI-83 or TI-84 graphing calculator

Choose *Using Manipulatives* or *Using a Graphing Calculator*.
Work with a partner.

Using Manipulatives

Part A: Investigating Optimal Areas
Suppose you have twenty-four 1-m sections of edging to enclose a rectangular garden. What is the maximum area that you can enclose?

- Each toothpick represents a section of edging. Construct as many rectangles as you can using all 24 toothpicks. For each rectangle, record the dimensions, area, and perimeter in a table like this.

Use only whole toothpicks for the sides.

Length (m)	Width (m)	Area (m^2)	Perimeter (m)

- Create a graph of area versus length. Draw a smooth curve through the points.
- What are the dimensions of the garden with the maximum area? Did you use the table or the graph?

Part B: Investigating Optimal Perimeters

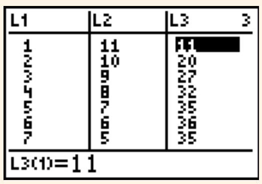
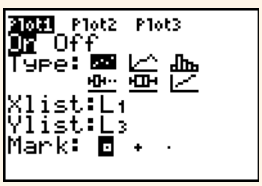
Suppose you want to build a rectangular patio with thirty-six 1-m^2 patio tiles. What is the minimum perimeter the patio can have?

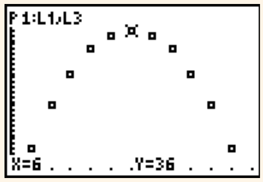
- Each square tile represents a patio tile. Construct as many rectangles as you can using all 36 square tiles. For each rectangle, record the dimensions, area, and perimeter in a table.
- Create a graph of perimeter versus length. Draw a smooth curve through the points.
- What are the dimensions of the patio with the minimum perimeter?

Using a Graphing Calculator

Part A: Investigating Optimal Areas

Suppose you have twenty-four 1-m sections of edging to enclose a rectangular garden.
What is the maximum area that you can enclose?

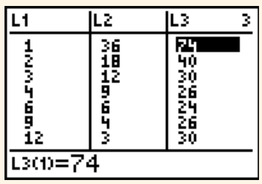
Steps	Display	Notes
Press [STAT] 1. • In L1, list possible lengths for the rectangle. Enter whole numbers from 1 to 11. • Press [▶] [▲] to move onto the list name L2. Press: 12 [−] [2nd] 1 [ENTER] • Move onto the list name L3. Press: [2nd] 1 [×] [2nd] 2 [ENTER]		If necessary, press [2nd] [+] 4 [ENTER] to clear the lists in the list editor. The formula **12 − L1** calculates the corresponding width for each length in L1. The formula **L1 × L2** calculates the area of each rectangle.
Create a graph of length and area. Press [2nd] [Y=] 1 and change the settings as shown.		Length will be plotted on the horizontal axis and area on the vertical axis.

Press $\boxed{\text{ZOOM}}$ 9. Press $\boxed{\text{TRACE}}$ and use the arrow keys to see the coordinates of each point.		

■ What are the dimensions of the garden with the maximum area?

Part B: Investigating Optimal Perimeters

Suppose you want to build a rectangular patio with thirty-six 1-m^2 patio tiles. What is the minimum perimeter the patio can have?

Steps	Display	Notes
Clear the list editor. Press: $\boxed{\text{2nd}}$ $\boxed{+}$ 4 $\boxed{\text{ENTER}}$ Press: $\boxed{\text{STAT}}$ 1 • In L1, list possible lengths for the rectangle. Enter the whole number factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36. • Move onto the list name L2. Press: 36 $\boxed{\div}$ $\boxed{\text{2nd}}$ 1 $\boxed{\text{ENTER}}$ • Move onto the list name L3. Press: 2 $\boxed{\times}$ $\boxed{(}$ $\boxed{\text{2nd}}$ 1 $\boxed{+}$ $\boxed{\text{2nd}}$ 2 $\boxed{)}$ $\boxed{\text{ENTER}}$		The formula **36 ÷ L1** calculates the corresponding width for each length in L1. The formula **2 × (L1 + L2)** calculates the perimeter of each rectangle.

■ Create a graph of length and perimeter.

■ What are the dimensions of the patio with the minimum perimeter?

Restrictions on the possible dimensions or shape of an object are called *constraints*.

Reflect

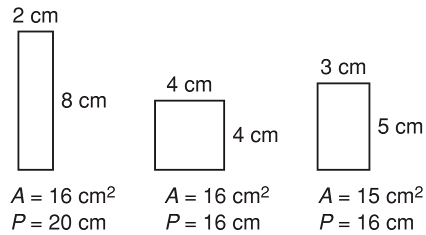
■ What constraints limited the number of rectangles you could construct in each situation?

■ Did you use a table of values or a graph to determine the maximum area or the minimum perimeter? Explain your choices.

Connect the Ideas

Optimizing the dimensions of a rectangle

Among all rectangles with a given perimeter, a square has the maximum area. Among all rectangles with a given area, a square has the minimum perimeter.



Example 1 — Finding Dimensions of Optimal Rectangles

Materials
- scientific calculator

a) What are the dimensions of a rectangle with perimeter 20 m and the maximum area? What is the maximum area?

b) What are the dimensions of a rectangle with area 45 m^2 and the minimum perimeter? What is the minimum perimeter?

Solution

a) The maximum area occurs when the rectangle is a square.
Determine the side length, s, of a square with perimeter 20 m.

$$\begin{aligned} s &= P \div 4 \\ &= 20 \div 4, \text{ or } 5 \\ A &= 5 \times 5 \\ &= 25 \end{aligned}$$

The rectangle is a square with side length 5 m.
Its area is 25 m^2.

b) The minimum perimeter occurs when the rectangle is a square.
Determine the side length, s, of a square with area 45 m^2.

$$\begin{aligned} s &= \sqrt{A} \\ &= \sqrt{45}, \text{ or about } 6.71 \\ P &= 4 \times 6.71 \\ &= 26.84 \end{aligned}$$

The rectangle is a square with side length about 6.7 feet.
Its perimeter is about 26.8 feet.

Optimizing with restrictions

There may be restrictions on the rectangle you are optimizing:
- The length and width may have to be whole numbers; or
- The length and width may have to be multiples of a given number.

In these cases, it may not be possible to form a square. The maximum area or minimum perimeter occurs when the length and width are closest in value.

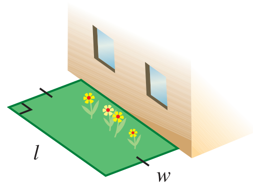
Sometimes one or more sides of the area to be enclosed are bordered by a wall or other physical barrier. In these cases, the optimal rectangle will not be a square. You can use diagrams or a table and graph to find the dimensions of the optimal rectangle.

Example 2 Optimizing with Constraints

Materials

- TI-83 or TI-84 graphing calculator

A rectangular garden is to be fenced using the wall of a house as one of side of the garden. The garden should have an area of 40 m^2. Determine the minimum perimeter and dimensions of the garden in each case:



a) The dimensions must be whole numbers of metres.

b) The dimensions can be decimals.

Solution

One length of the garden is along the wall.

Only three sides of the garden need to be fenced. So, $P = \ell + 2w$

The area is 40 m^2.

$A = \ell w$ Substitute $A = 40$

$40 = \ell w$ Isolate w.

$w = 40 \div \ell$

Use a table or a graph to determine the minimum perimeter and dimensions of the garden.

a) Substitute values into the equations above. Record possible dimensions for the garden in a table. The minimum perimeter for whole number dimensions occurs twice: when (length, width) is (10 m, 4 m) or (8 m, 5 m). The perimeter in both cases is 18 m.

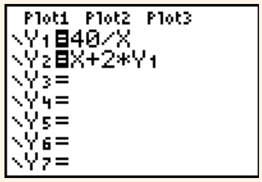
Since the dimensions are whole numbers, ℓ is a factor of 40.

Area 40 m^2		
Length (m)	**Width (m)**	**Perimeter (m)**
40	1	42
20	2	24
10	4	18
8	5	18
5	8	21
4	10	24
2	20	42
1	40	81

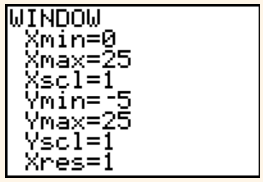
We use a different procedure from that in *Investigate* since the dimensions can be decimal lengths.

b) Use a graphing calculator to create a graph of length and perimeter.

Press [Y=].
- Use X to represent the length.
- Use Y1 to represent the width. Press: 40 [÷] [X,T,Θ,*n*] [ENTER]
- Use Y2 to represent the perimeter. Press: [X,T,Θ,*n*] [+] 2 [×] [VARS] [▸] 1 1 [ENTER]

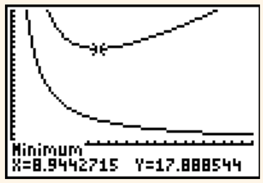


Press [WINDOW].
Change the window settings as shown.



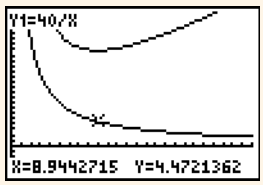
You could also use a scientific calculator to guess and check as in part a.

Press [GRAPH]. Press [2nd] [TRACE] 3 to determine the minimum of the graph.
Use the arrow keys to place the cursor on the graph of Y2. Then move the cursor to the left of the minimum and press [ENTER], move to the right of the minimum and press [ENTER], and close to the minimum and press [ENTER].
The Y-value is the minimum perimeter.



Press [2nd] [GRAPH] to view the data in a table. To change the starting value of the table, or the increment, press [2nd] [WINDOW] and adjust TblStart and ΔTbl.

Press [▾].
This Y-value is the width.



The minimum perimeter is about 17.9 m and occurs when the length along the house wall is about 8.9 m and the width is about 4.5 m.

Example 3 Enclosing Non-Rectangular Areas

Materials
- scientific calculator

A hobby farmer is creating a fenced exercise yard for her horses. She has 900 m of flexible fencing and wishes to maximize the area. She is going to fence a rectangular or a circular area. Determine which figure encloses the greater area.

Solution

Rectangular Area

The rectangle with perimeter 900 m and greatest area is a square.

Substitute $P = 900$ in the formula for the perimeter of a square.

$$P = 4s$$
$$900 = 4s \qquad \text{Solve for } s.$$
$$s = \frac{900}{4}$$
$$= 225$$

The side length of the square is 225 m.

Substitute $s = 225$ in the formula for the area of a square.

$$A = s^2$$
$$= 225^2, \text{ or } 50\,625$$

The greatest rectangular area that can be enclosed is 50 625 m^2.

Circular Area

Substitute $P = 900$ in the formula for the circumference of a circle.

$$C = 2\pi r$$
$$900 = 2\pi r \qquad \text{Solve for } r.$$
$$r = 900 \div 2\pi$$
$$\doteq 143.24$$

The radius of the circle is about 143.2 m.

Substitute $r = 143.24$ in the formula for the area of a circle.

$$A = \pi r^2$$
$$\doteq \pi \times 143.24^2$$
$$\doteq 64\,458.25$$

The greatest circular area that can be enclosed is about 64 458 m^2.

The circular pen encloses the greater area.

Practice

A

For help with questions 1 to 3, see Example 1.

1. For each perimeter, what are the dimensions of the rectangle with the maximum area? What is the area?

a) 40 cm **b)** 110 feet **c)** 25 m **d)** 87 inches

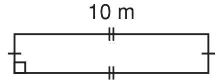
2. For each area, what are the dimensions of the rectangle with the minimum perimeter? What is the perimeter?

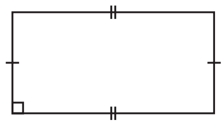
a) 25 square feet **b)** 81 m^2 **c)** 144 cm^2 **d)** 169 square inches

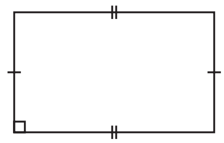
3. For each area, what are the dimensions of the rectangle with the minimum perimeter? What is the perimeter? Round your answers to one decimal place.

a) 30 square feet **b)** 65 m^2 **c)** 124 cm^2 **d)** 250 square inches

4. A gardener uses 24 m of fencing to enclose a rectangular vegetable garden. Some possible rectangles are shown. Determine the missing dimension for each diagram.

a)


b)


c)


5. Calculate the area of each garden shown in question 4.

6. A farmer has 400 feet of fencing. Determine the greatest rectangular area that he can enclose with the fencing.

B

7. At an outdoor festival, 2-m sections of fencing are used to enclose an area for food sales. There are 100 sections of fencing available.

a) How many metres of fencing are available altogether?

b) Determine the maximum rectangular area that could be enclosed. How does the fact that the fencing is in sections affect your answer?

8. Lindsay has 20 pipe cleaners, each measuring 8 inches. She attaches them end to end to build a frame. Determine the greatest area that can be enclosed by the frame. Describe any assumptions you make.

9. A rectangular patio is to be constructed from 100 congruent square tiles.

a) What arrangement of tiles would give the minimum perimeter?

b) Suppose each tile has side length 50 cm. What would be the minimum perimeter? What would be the area of the patio?

1 acre = 43 560 square feet

10. A rectangular patio is to be constructed from 80 congruent square tiles.

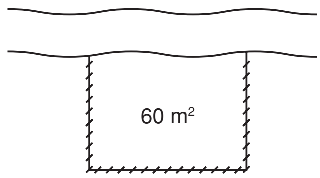
a) What arrangement of tiles would give the minimum perimeter? Justify your answer.

b) Suppose each tile has side length 50 cm. What would be the minimum perimeter? What would be the area of the patio?

11. A farmer has 650 feet of fencing. Does she have enough fencing to enclose a rectangular area of half an acre? Justify your answer.

For help with question 12, see Example 2.

12. A rectangular section of a field is to be fenced. Because one side of the field is bordered by a creek, only 3 sides need to be fenced. The fenced section should have an area of 60 m^2. Determine the minimum perimeter and dimensions of the fenced area in each case:



a) The dimensions must be whole numbers of metres.

b) The dimensions can be decimal lengths.

1 m ≐ 3.2808 feet

13. A lifeguard is roping off a rectangular swimming area using the beach as one side. She has 200 m of rope.

a) Determine the greatest area she can rope off and its dimensions.

b) Is the area in part a greater or less than 50 000 square feet? Justify your answer.

■ For help with question 14, see Example 3.

14. John buys 20 m of fencing to create a dog pen. How much more area will the dog have if John builds a circular pen rather than a square one?

15. Sasha is planning to create a garden with area 30 m^2. She could use a rectangular, triangular, or a circular design. Sasha decides to use the design that requires the least edging material. Which design should she use? How much edging will it require?

16. **Assessment Focus** The Tengs are adding a sunroom to their house. The perimeter of the sunroom will be 45 feet, not including the wall that is part of the house.

a) One design is for a rectangular sunroom. Determine the maximum possible area of the room and the dimensions that give this area.

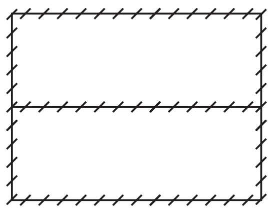
b) Another design is in the shape of a semicircle, where the straight edge is attached to the house. Determine the diameter and area of the room.

c) Which design has the greater area? How much greater is it?

17. Most of the heat loss for outdoor swimming pools is due to surface evaporation. So, the greater the area of the surface of the pool, the greater the heat loss. For a given perimeter, which surface shape would be more efficient at retaining heat: a circle or a rectangle? Justify your answer.

C

18. A farmer has 1800 m of fencing. He needs to create two congruent rectangular fields, as shown. Determine the maximum possible area of each field.



19. Twelve 2-m sections of metal fencing will be used to enclose an area. The area can have any shape, including triangle, hexagon, rectangle, and so on. The pieces do not bend, so they cannot form a circle. Determine the shape that maximizes the area. Justify your answer.

In Your Own Words

A friend has missed math class. Use an example to demonstrate that a square is the rectangle with the maximum area for a given perimeter and the minimum perimeter for a given area.

2.4 Optimizing Area and Perimeter Using a Spreadsheet

A contractor is designing a rectangular deck. One side of the deck will be against a house wall, but the other three sides will require a railing. The homeowners want the deck to have an area of 200 square feet. They also want to minimize the length of the railing.

Inquire

Materials

- *Microsoft Excel*
- areaopt.xls
- peropt.xls

Area and Perimeter Problems

Part A: Maximum Area for a Given Perimeter

A park worker has 32 m of fencing to build a rectangular pen for rabbits. What is the maximum area that she can provide for the rabbits?

- Open the file *areaopt.xls*, or start a new spreadsheet file and enter the data and formulas shown here.

	A	B	C	D
1	**Maximum Area of Pen with Perimeter 32 m**			
2				
3	**Length (m)**	**Width (m)**	**Area (m^2)**	**Perimeter (m)**
4	0.5	=16-A4	=A4*B4	=2*(A4+B4)
5	=A4+0.5			

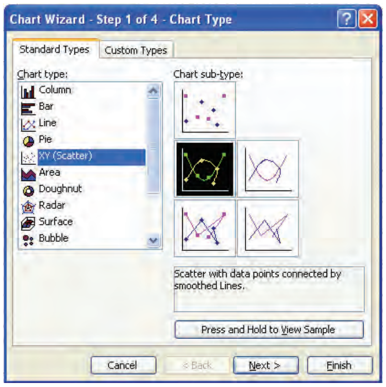
1. **a)** What is the initial length of one side of the pen?

b) By what increment does the side length increase?

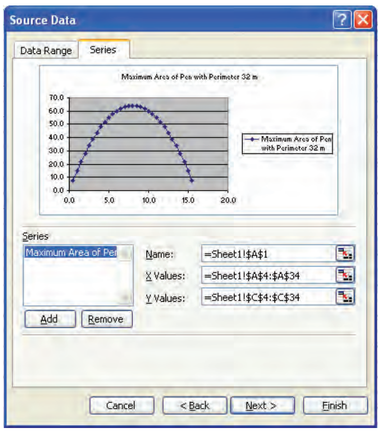
- Select cells A5 to A34. **Fill Down** to show lengths up to 15.5 m. Select cells B4 to D34. **Fill Down** to calculate the corresponding widths, areas, and perimeters.

2. Suppose the fencing comes in 0.5-m lengths. What are the dimensions of the rectangular pen with the maximum area? Justify your answer.

- Use the **Chart** feature to graph length and area.
  - From the **Insert** menu, select **Chart**.
  - In the **Chart type**: box, click on **XY**(**Scatter**). Select chart sub-type *Scatter with data points connected by smoothed lines*.



  - Click **Next** >, and select the **Series** tab. Click **Add**. Place the cursor in the **Name**: box, then click cell A1. Place the cursor in the **X Values** box, then select cells A4 to A34. Highlight ={1} in the **Y Values** box, then select cells C4 to C34.



  - Click **Next** >, and enter appropriate titles for the axes. Click **Finish**.

3. **a)** Describe the shape of the graph.

b) Suppose the fencing can be cut to decimal lengths. How can you determine the dimensions of the rectangular pen with the greatest area using the graph?

c) How could you change the initial length and increment to zoom in on the region around the maximum?

- Use the **Convert** function to express the dimensions in the spreadsheet in feet and square feet.
  - In cell E4, type: = CONVERT(A4,"m","ft")
  - In cell F4, type: = CONVERT (B4,"m","ft")
  - In cell G4, type: = E4*F4
  - In cell H4, type: = 2*(E4+F4)
  - Copy these formulas down to row 34.

4. **a)** How many feet of fencing does the park worker have?

b) What is the area of the greatest rectangle she can enclose with this fencing? What are its dimensions?

Part B: Minimum Perimeter for a Given Area

A park worker is to build a rectangular pen for rabbits with an area of 24 m^2. What is the minimum length of fencing he needs for this project?

- Open the file *peropt.xls*, or start a new spreadsheet file and enter the data and formulas shown here.

	A	B	C	D
1	Minimum Perimeter of Pen with Area 24 m^2			
2				
3	Length (m)	Width (m)	Area (m^2)	Perimeter (m)
4	0.5	=24/A4	=A4*B4	=2*(A4+B4)
5	=A4+0.5			

5. a) What is the initial length of one side of the pen?
b) By what increment does the side length increase?

- Select cells A5 to A99. **Fill Down** to show lengths up to 48.0 m. Select cells B4 to D99. **Fill Down** to calculate the corresponding widths, areas, and perimeters.

6. Suppose the fencing comes in 0.5-m lengths. What are the dimensions of the rectangular pen with the minimum perimeter? Justify your answer.

7. Use the **Chart** feature to graph length and perimeter.
a) Describe the shape of the graph.
b) Suppose the fencing can be cut to decimal lengths. How can you determine the dimensions of the rectangular pen with the least perimeter using the graph?
c) How could you change the initial length and increment to zoom in on the region around the minimum?

8. Express the dimensions in the spreadsheet in feet and square feet.
a) How many square feet must the park worker enclose?
b) What is the minimum length of fencing he will need to enclose the rectangular area? What are the dimensions of the rectangle?

Practice

A

1. For each change to the situation in Part A of *Inquire*, describe the changes you would make to the spreadsheet.

a) The park worker has 20 m of fencing.

b) The fencing comes in 1-m lengths.

c) The park worker has 38 m of fencing.

d) The fencing comes in 0.25-m lengths.

B

2. Cy plans to create a flower garden using 0.25-m edging. He has 28 pieces of edging, and he wants to use all of it without any overlap.

a) How many metres of edging does Cy have in total?

b) By what increment should the side length increase? Justify your answer.

c) Use *areaopt.xls* to determine the maximum area of the garden. What changes do you have to make to the spreadsheet?

3. You have 40 m of fencing to build a rectangular dog pen. Use *areaopt.xls* to determine the maximum area of the dog pen.

4. Asma is planning a vegetable garden with area 144 square feet. She will fence the garden to keep out rabbits, and she wants to minimize the length of fencing she must buy. Use *peropt.xls* to determine the dimensions of the rectangle with the minimum perimeter. What changes do you have to make to the spreadsheet?

5. A rectangular patio is to be constructed from 48 congruent square tiles.

a) Use *peropt.xls* to determine the minimum perimeter of a patio built using all of the tiles.

b) Explain why the patio with minimum area is not a square.

6. Refer to the deck design problem in the lesson opener on page 97. Change *peropt.xls* to investigate this situation. If you were the contractor, what dimensions would you use for the deck? What changes do you have to make to the spreadsheet?

7. Jace has 12 m of rope to enclose a rectangular display. Use *areaopt.xls* to determine the maximum area in each situation. Justify the changes you make to the spreadsheet in each case.
   a) The rope is used for all four sides of the display.
   b) A wall is on one side, so the rope is only used for three sides.
   c) The display is in a corner of the room, so the rope is only used for two sides.

8. Tyresse uses rope to enclose a 400-m^2 rectangular swimming area on a lake. Use *peropt.xls* to determine the minimum length of rope needed in each situation. Justify the changes you make to the spreadsheet in each case.
   a) The rope is used for all four sides of the swimming area.
   b) One side is along the beach, so the rope is only used for three sides.
   c) One side is along the beach and an adjacent side is along a dock, so the rope is only used for two sides.

Reflect

- Why is a spreadsheet a good tool for solving perimeter and area optimization problems?
- How did you decide what value to use for the length in cell A4 and how far down to copy the formulas in each spreadsheet?

2.5 Dynamic Investigations of Optimal Measurements

Dynamic training software can be used to help train pilots and drivers. It simulates what may happen in various situations. Dynamic geometry software can help you visualize what happens to objects as their dimensions change.

Inquire

Materials

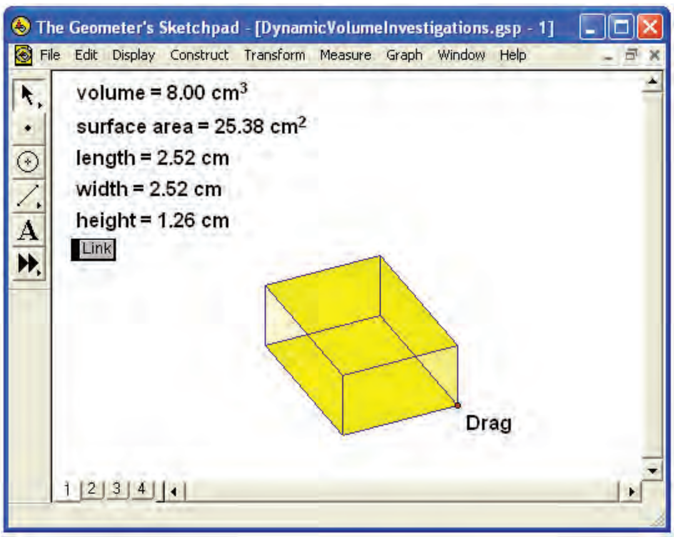
- *The Geometer's Sketchpad*
- DynamicVolumeInvestigations.gsp

Exploring Optimal Measures

Work with a partner.

Open the file: *DynamicVolumeInvestigations.gsp*

Move to any page by clicking on the tabs in the lower left corner or move to the next page by clicking the **Link** button.



1. Move to page 1. Drag the labelled point and observe the changes in the object and the measured values. Answer these questions.

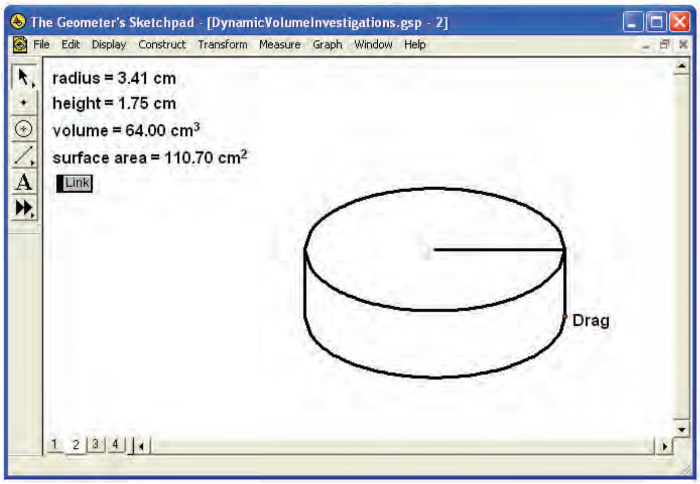
a) What three-dimensional object is shown on the page? What measurements are given?

b) As you drag the point, what measurements change? Which measurement remains the same?

c) Make some predictions about the object with the least possible surface area or the greatest possible volume.

d) Drag the point to create the object with the least possible surface area or the greatest possible volume. Record the optimal value of the surface area or volume and the dimensions of the object.

2. Move to page 2. Drag the labelled point and observe the changes in the object and the measured values. Repeat question 1.



3. Move to page 3. Drag the labelled point and observe the changes in the object and the measured values. Repeat question 1.

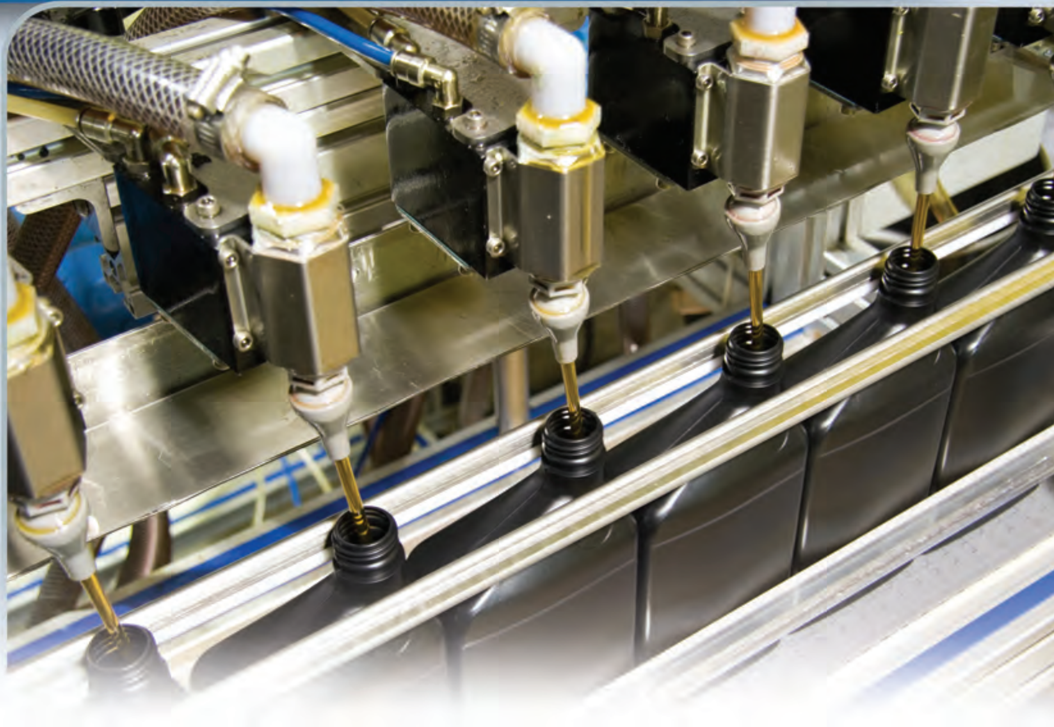
4. Move to page 4. Drag the labelled point and observe the changes in the object and the measured values. Repeat question 1.

Reflect

- Describe a rectangular prism that is optimized for volume or surface area. How does this compare to your prediction?
- Describe a cylinder that is optimized for volume or surface area. How does this compare to your prediction?
- How did you determine the optimal measurements?

2.6 Optimizing Volume and Surface Area

A container's shape affects both the volume it can hold and the amount of material used to make it. To reduce waste and costs, a packaging designer may create a container that holds the desired volume using the least material. Other factors will also affect the design.



Investigate

Minimizing the Surface Area of a Rectangular Prism

Materials
- 1-cm grid paper or light cardboard
- scissors
- tape

Work with a partner.

Design and build three boxes with volume 1000 cm^3.
All the boxes must be rectangular prisms.
- Calculate the surface area of each box.
- Which box has the least surface area? What are its dimensions?

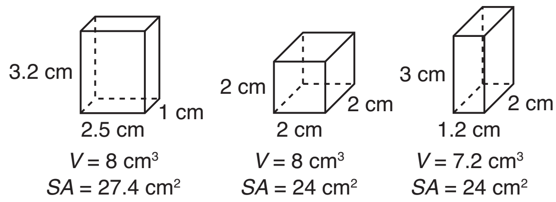
Reflect

- How did you determine the dimensions of each box?
- Compare boxes with classmates. What are the dimensions of the box with the least surface area? Describe the box.
- How does this compare with the two-dimensional problem of minimizing the perimeter of a rectangle with a given area?

Connect the Ideas

Optimizing the dimensions of a rectangular prism

Among all rectangular prisms with a given surface area, a cube has the maximum volume. Among all rectangular prisms with a given volume, a cube has the minimum surface area.



Example 1 Optimizing Rectangular Prisms

Materials

- scientific calculator

a) Rosa constructs a rectangular prism using exactly 384 square inches of cardboard. It has the greatest volume possible. What are the dimensions of the prism? What is its volume?

b) Liam constructs a rectangular prism with a volume of exactly 1331 m^3. It has the least surface area possible. What are the dimensions of the prism? What is its surface area?

Solution

For a cube with side length s:
$SA = 2s^2 + 2s^2 + 2s^2$
$= 6s^2$
$V = s \times s \times s$
$= s^3$

a) The prism with maximum volume is a cube. Determine the edge length, s, of a cube with surface area 384 square inches.

$SA = 6s^2$ Substitute: $SA = 384$

$384 = 6s^2$ Divide each side by 6 to isolate s^2.

$64 = s^2$ Take the square root of each side to isolate s.

$s = \sqrt{64}$, or 8

$V = s^3$ Substitute: $s = 8$

$= 8^3$, or 512

The rectangular prism is a cube with edge length 8 inches.
Its volume is 512 cubic inches.

b) The prism with the least surface area is a cube. Determine the edge length, s, of a cube with volume 1331 m^3.

$V = s^3$ Substitute: $V = 1331$

$1331 = s^3$ Take the cube root of each side to isolate s.

$s = \sqrt[3]{1331}$, or 11

$SA = 6s^2$ Substitute: $s = 11$

$= 6(11)^2$, or 726

The rectangular prism is a cube with edge length 11 m.
Its surface area is 726 m^2.

To determine $\sqrt[3]{1331}$:
On a TI-30XII, press:
3 [2nd] [^] 1331 [ENTER]
On a TI-83 or TI-84, press: [MATH] 4 1331 [)] [ENTER]

Optimizing with constraints

There may be constraints on the prism you are optimizing:

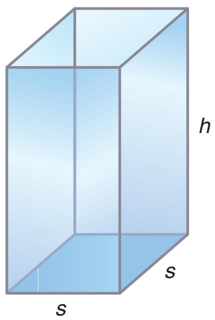
- The dimensions may have to be whole numbers; or
- The dimensions may have to be multiples of a given number.

In these cases, it may not be possible to form a cube. The maximum volume or minimum surface area occurs when the dimensions are closest in value.

Sometimes one or more sides of the object are missing or bordered by a wall or other physical barrier. In these cases, the optimal rectangular prism will not be a cube. You can use diagrams or a table and graph to find the dimensions of the optimal rectangular prism.

Example 2 Optimizing with Constraints

Yael is designing a glass candle holder. It will be a rectangular prism with outer surface area 225 cm^2, a square base, and no top.



a) Determine the maximum volume of the candle holder.

b) What are the dimensions of the candle holder with the maximum volume?

Solution

a) The base of the candle holder has area: $A = s^2$

The volume of the candle holder is: $V = s^2h$

The surface area of the candle holder is the sum of the areas of the faces: the base and four identical sides.

$SA = s^2 + 4(sh)$ Substitute: $SA = 225$

$225 = s^2 + 4sh$ Isolate h.

$225 - s^2 = 4sh$

$\frac{225 - s^2}{4s} = h$

Substitute values for s and determine the corresponding lengths, heights, and volumes.

If technology is available, use a spreadsheet or graphing calculator to create the table and graph.

Base side length (cm)	Height of prism (cm)	Volume (cm^3)	Surface area (cm^2)
1	56	56	225
2	27.625	110.5	225
3	18	162	225
4	13.0625	209	225
5	10	250	225
6	7.875	283.5	225
7	6.285714	308	225
8	5.03125	322	225
9	4	324	225
10	3.125	312.5	225
11	2.363636	286	225
12	1.6875	243	225
13	1.076923	182	225
14	0.517857	101.5	225

It looks like the maximum volume of the candle holder occurs when the side length of the base is about 8.6 cm and the volume is about 325 cm^3. Create a table with smaller increments to verify this prediction.

Base side length (cm)	Height of prism (cm)	Volume (cm^3)	Surface area (cm^2)
8.5	4.492647	324.5938	225
8.6	4.390698	324.736	225
8.7	4.290517	324.7493	225
8.8	4.192045	324.632	225

The maximum volume of the candle holder occurs when the base side length is about 8.7 cm. The volume is about 325 cm^3.

b) From the table, the candle holder with maximum volume has base side length about 8.7 cm and height about 4.3 cm.

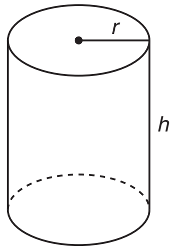
Example 3 Optimizing Other Objects

Materials

- TI-83 or TI-84 graphing calculator

1 mL = 1 cm^3

Naveed is designing a can with volume 350 mL. What is the minimum surface area of the can? Determine the dimensions of a can with the minimum surface area.



Solution

Since 1 mL = 1 cm^3, the volume will be 350 cm^3.

The surface area of the can is: $SA = 2\pi r^2 + 2\pi rh$

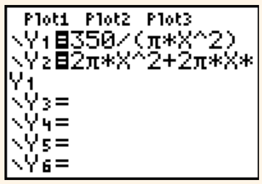
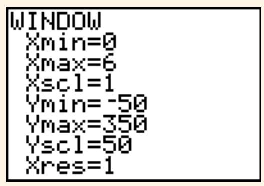
The volume of the can is:

$V = \pi r^2 h$ Divide both sides by πr^2 to isolate h.

$\frac{V}{\pi r^2} = h$ Substitute: $V = 350$

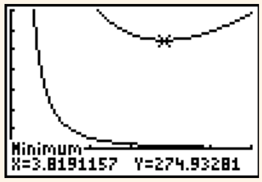
$\frac{350}{\pi r^2} = h$

Use a graphing calculator to create a graph of radius and surface area.

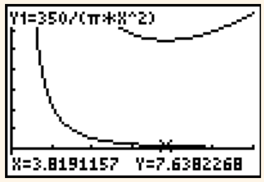
Press [Y=]. • Use X to represent the radius. • Use Y1 to represent the height. Press: 350 [÷] [(] [2nd] [^] [×] [X,T,Θ,*n*] [^] 2 [)] [ENTER] • Use Y2 to represent the surface area. Press: 2 [2nd] [^] [×] [X,T,Θ,*n*] [^] 2 [+] 2 [2nd] [^] [×] [X,T,Θ,*n*] [×] [VARS] [▸] 1 1 [ENTER]	
Press [WINDOW]. Change the window settings as shown.	

Press [2nd] [GRAPH] to view the data in a table. To change the starting value of the table, or the increment, press [2nd] [WINDOW] and adjust TblStart and ΔTbl.

Press [GRAPH].
Press [2nd] [TRACE] 3, move the cursor onto the graph of Y2, and answer the prompts to determine the minimum surface area of the can.



Press [▼]. Read the coordinates to determine the dimensions of the can with the minimum surface area.



The minimum surface area of the can is about 275 cm^2. It occurs when the radius is about 3.8 cm and the height is about 7.6 cm.

Practice

A

■ For help with questions 1 or 2, see Example 1.

1. Yasmin is constructing a rectangular prism using exactly 96 cm^2 of cardboard. The prism will have the greatest possible volume.

a) Describe the prism. What will be its dimensions?

b) What will be its volume?

2. Mathew is constructing a rectangular prism with volume exactly 729 cubic inches. It will have the least possible surface area.

a) Describe the prism. What will be its dimensions?

b) What will be its surface area?

3. The dimensions of two rectangular prisms with volume 240 cm^3 are given. Sketch each prism and predict which will have less surface area. Check your prediction.

a) 10 cm by 6 cm by 4 cm

b) 12 cm by 10 cm by 2 cm

4. Krikor has to design and build a box with the greatest volume possible. The box is a rectangular prism. For each surface area, what will be the dimensions of the box?

a) 600 square inches **b)** 1350 cm^2 **c)** 2400 square inches

5. Tanya is designing a storage box. It will be a rectangular prism with the least possible surface area. For each volume, what will be the dimensions of the box?

a) 1 m^3 **b)** 125 000 cm^3 **c)** 8 cubic feet

B

6. Jude is designing a plush activity toy for a baby. The toy will be a rectangular prism with surface area 864 cm^2.

a) Determine the maximum volume of the toy.

b) What are the dimensions of the toy with maximum volume?

1 gallon ≐ 277.42 cubic inches

7. Camping supply stores sell collapsible plastic containers for storing water. The containers are often rectangular prisms with rounded corners. Reducing the amount of plastic helps the container fold as small as possible.

a) Convert the capacity of a 5-gallon container to cubic inches.

b) Determine the minimum surface area of a container holding 5 gallons of water. What would be the dimensions of the container?

1 foot = 30.48 cm

8. An electrical transformer box is a rectangular prism constructed from sheet metal. It must have volume at least 274 625 cm^3 to hold all the necessary equipment.

a) What dimensions for the box require the least area of sheet metal?

b) What area of sheet metal is needed to build the box?

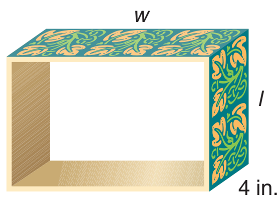
c) Tony has 20 square feet of sheet metal. Will this be enough to construct the box? Justify your answer.

For help with question 9, see Example 2.

9. Tori is designing a hanging shelf. It has volume 400 cubic inches and depth 4 inches. She will paint a design that will cover the four outside faces.

a) Determine the minimum area she will paint.

b) What are the dimensions of the shelf with the minimum area to paint?



10. A company packages sugar cubes in cardboard boxes containing 144 cubes. The cubes are arranged in 2 layers, with 12 rows of 6 cubes in each layer. The company wishes to design a box that uses less packaging, but holds the same number of cubes. How could you arrange the cubes so the least amount of cardboard is used?

11. **Literacy in Math** A company considers packaging cereal in cubic boxes.
   a) What advantages does a cube have over the tall, narrow boxes currently in use?
   b) What problems might this new shape create for each of these people?
      i) A consumer
      ii) A grocery store owner
      iii) A designer planning the package labelling

■ For help with question 12, see Example 3.

12. Filip is designing a can for a new vegetable product. The can should hold 750 mL of vegetables. To reduce waste, he wants the surface area of the can to be as small as possible.
   a) What dimensions should Filip use?
   b) What will the surface area be?

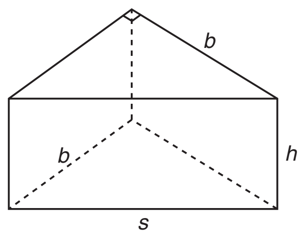
13. A cylindrical storage tank holds 1800 cubic feet of gasoline. Determine the minimum amount of material needed to build this tank.

14. Look at the dimensions of the optimal cylinders in *Example 3* and questions 12 and 13. How do the diameter and height appear to be related?

1 mL = 1 cm^3

15. **Assessment Focus** A beverage company is investigating containers that can hold 512 mL of juice. They are debating whether to use a rectangular prism or a cylinder. Which object would require less material? Justify your answer.

16. Courtney is designing a gift box. It will be a triangular prism with surface area 220 cm^2. She decides the box should have a right isosceles triangular base to make it easy to package.



   a) Determine the length of the hypotenuse of the base, *s*.
   b) Determine the maximum possible volume of the box.
   c) What are the dimensions of the box with the maximum volume?

17. Kimmia and Jan thought triangular prisms might be a good shape for juice boxes. They learned from research that to minimize surface area, the base should be an equilateral triangle. They created this spreadsheet to explore the dimensions of a prism that could hold 250 mL of juice.

	A	B	C	D	E	F
1	Minimum Surface Area of a Triangular Prism with Volume 250 cm^3					
2	Base: Equilateral triangle					
3	Side length of base (cm)	Base and top area (cm^2)	Area of sides (cm^2)	Height of prism (cm)	Volume (cm^3)	Surface area (cm^2)
4	9.4	76.5	184.3	6.5	250.0	260.8
5	9.6	79.8	180.4	6.3	250.0	260.2
6	9.8	83.2	176.7	6.0	250.0	259.9
7	10.0	86.6	173.2	5.8	250.0	259.8
8	10.2	90.1	169.8	5.5	250.0	259.9
9	10.4	93.7	166.5	5.3	250.0	260.2

a) The students used this formula to determine the base area of the prism.
Base area $= \frac{\sqrt{3}s^2}{4}$
Use trigonometry to verify that the formula calculates the area of an equilateral triangle with side length *s*.

b) What are the dimensions of the triangular prism with the least surface area? How do you know?

c) What is the surface area of the prism in part b?

d) How do the side length and height of the prism seem to be related?

C

18. A pop manufacturer creates a can with volume 355 cm^3. Twelve cans are then packaged in cardboard boxes for distribution.

a) Determine the dimensions of a can with minimum surface area.

b) Determine the minimum amount of cardboard that can be used for each case of pop.

1 yard = 36 inches

The triangular prism with the greatest volume will have an equilateral triangle as the base.

19. A manufacturer is designing a new shipping container for powdered chemicals. The container could be a rectangular prism, triangular prism, or cylinder. The surface area for each design is to be 1 square yard.

a) Determine the dimensions of each container with the maximum volume.

b) Which container would be able to hold the most material?

c) Which container would you recommend? Justify your answer. Remember to think about other factors such as ease of use and ease of manufacturing.

In Your Own Words

Describe a situation when it would be important to design an object with a minimal surface area for its volume. Explain whether you would recommend a cylinder or rectangular prism in this situation.

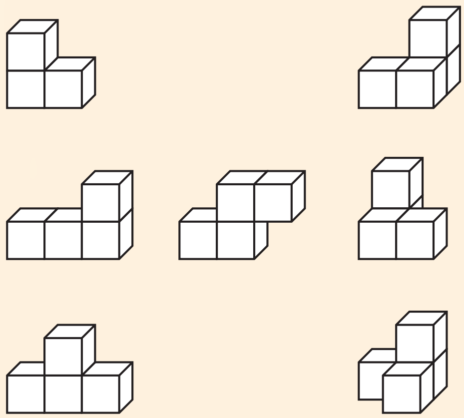
PUZZLE

Cube Creations

Materials

- 27 linking cubes

Use linking cubes to create these 7 objects:



Connect all 7 objects to form a larger composite object.

Assume each cube has side length 1 unit.

Calculate the volume and surface area of the composite object.

You may find it helpful to create each object using a different colour.

- Is it possible to create an object with lesser volume?
- Is it possible to create an object with lesser surface area?
- Is it possible to connect these objects to form a rectangular prism?

Repeat the activity to find out.

Reflect

- What strategies did you use to create an object with lesser surface area?
- Describe the composite object with volume 27 cubic units and the least surface area. Were you able to create this object?

2.7 Optimizing Surface Area Using a Spreadsheet

Two objects are constructed from the same material and have the same volume. The object with the greater surface area will lose heat more quickly. Engineers often try to minimize surface area when designing objects where heat loss is a problem.

Inquire

Materials
- *Microsoft Excel*
- saopt.xls

Comparing Prisms and Cylinders

Work with a partner.

A container is being designed with a volume of 500 cm^3.
What are the dimensions of the container with the minimum surface area?
The container could be a rectangular prism, cylinder, or triangular prism.

- Open the file *saopt.xls*. There are three sheets in the file. Move to any sheet by clicking on the tabs in the lower left corner.

1. Move to sheet **Rect. Prism**.
   a) What is the initial side length of the base of the prism?
   b) By what increment does the side length increase?
   c) Verify the formulas for calculating the height, volume, and surface area of the prism in cells B4, C4, and D4.

	A	B	C	D
1	**Minimum Surface Area of a Rectangular Prism with Volume 500 cm^3**			
2	**Base: Square**			
3	**Side length of base (cm)**	**Height of prism (cm)**	**Volume (cm^3)**	**Surface area (cm^2)**
4	1	=500/(A4^2)	=A4^2*B4	=2*A4^2+4*B4*A4
5	=A4+1			

- Select cells A5 to A23.
  Fill Down to show side lengths up to 20 cm.
  Select cells B4 to D23.
  Fill Down to calculate the corresponding heights, volumes, and surface areas.

2. **a)** What are the approximate dimensions of the rectangular prism with the minimum surface area?
b) What is the approximate surface area?

You will probably want to use an initial value slightly less than the side length you calculated in question 2, part a.

- For more precise dimensions, adjust the initial value and change the formula in cell A5 to = **A4** + **0.1**. Copy this formula down through row 23.

3. **a)** What are the approximate dimensions of the rectangular prism with the minimum surface area?
b) What is the approximate surface area?

Refer to page 98 for more instructions on using the **Chart** feature.

- Use the **Chart** feature to create a scatter plot of length and surface area. Connect the points with a smooth curve.

4. **a)** Describe the shape of the graph.
b) How does the graph show which edge length will produce a rectangular prism with the least surface area?

5. Move to sheet **Cylinder**.
a) What is the initial radius of the cylinder?
b) By what increment does the radius increase?
c) Verify the formulas for calculating the height, volume, and surface area of the cylinder in cells B4, C4, and D4.

	A	B	C	D
1		Minimum Surface Area of a Cylinder with Volume 500 cm^3		
2	Base: Circle			
3	Radius (cm)	Height of cylinder (cm)	Volume (cm^3)	Surface area (cm^2)
4	1	=500/(PI()*A4^2)	=PI()*A4^2*B4	=2*PI()*A4^2+(PI()*2*A4*B4)
5	=A4+1			

- Select cells A5 to A23.
  Fill Down to show radii up to 20 cm.
  Select cells B4 to D23.
  Fill Down to calculate the corresponding heights, volumes, and surface areas.

6. Repeat questions 2, 3, and 4 for the cylinder.

7. Move to sheet **Tri. Prism**.
  a) What is the initial side length of the base of the prism?
  b) By what increment does the side length increase?
  c) Verify the formulas for calculating the base area, height, volume, and surface area of the prism in cells B4, C4, D4, and E4.

	A	B	C	D	E
1	**Minimum Surface Area of a Triangular Prism with Volume 500 cm^3**				
2	**Base: Equilateral triangle**				
3	**Side length of base (cm)**	**Area of base (cm)**	**Height of prism (cm)**	**Volume (cm^3)**	**Surface area (cm^2)**
4	3	=A4^2*SQRT(3)/4	=500/B4	=B4*C4	=2*B4+3*A4*C4
5	=A4+1				

- Select cells A5 to A21.
  Fill Down to show side lengths up to 20 cm.
  Select cells B4 to D21.
  Fill Down to calculate the corresponding base areas, heights, volumes, and surface areas.

8. Repeat questions 2, 3, and 4 for the triangular prism.

9. Compare your answers to questions 3, 6, and 8.
  Which of these objects can enclose 500 cm^3 using the least surface area?
  Which requires the most surface area?

10. A container is being designed with volume 750 cm^3.
  The container could be a rectangular prism, cylinder, or triangular prism. Change the spreadsheet to determine the dimensions for a container of each shape with the minimum surface area.

11. Refer to the dimensions of the optimal containers you calculated in questions 3, 6, and 8.

a) How do the length, width, and height of a rectangular prism with the minimum surface area appear to be related?

b) How do the diameter and height of a cylinder with the minimum surface area appear to be related?

c) How do the side length of the base and the height of a triangular prism with the minimum surface area appear to be related?

12. Minimizing heat loss is an important factor in the design of water heaters.

1 gallon $\doteq$ 4546 cm^3

a) Modify the spreadsheet to determine the least surface area possible for a 50-gallon water heater tank if it is a rectangular prism, a cylinder, or a triangular prism.

b) Which object would you recommend for a water heater tank?

c) A typical water heater tank is a cylinder with height almost three times the diameter. How does this compare to the object you recommended?
Why might the taller, narrower tank be preferred?

Reflect

- What are the advantages of using a spreadsheet to optimize surface areas?
- Suppose you are designing a container to hold a given volume. The container will be made from sheets of an expensive metal. Would you design a rectangular prism, cylinder, or triangular prism? Justify your answer.

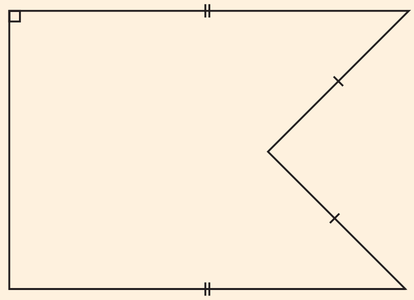
Home Quit

Study Guide

Composite Figures

- A composite figure is made up of simpler figures.
- To find the area of a composite figure, calculate the areas of the simpler figures. Add the areas. Subtract the areas of any figures that were removed.
- The perimeter of a figure is the distance around it.

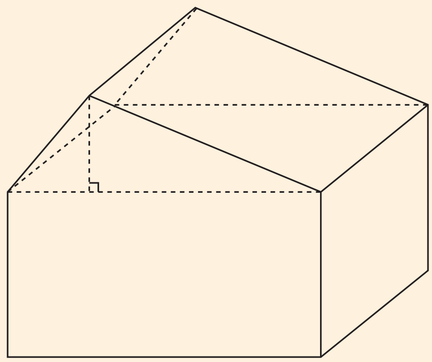
This figure is a rectangle with an isosceles triangle removed from the right side. To determine its area, subtract the area of the triangle from the area of the rectangle.



Composite Objects

- To calculate the volume of a prism with a base that is a composite figure, use the formula: V = base area $\times$ height
- To calculate the volume of a composite object, break the composite object into simpler objects. Calculate the volumes of the simpler objects. Add the volumes. Subtract the volumes of any objects that were removed.
- The surface area of an object is the sum of the areas of the faces.

This is a rectangular prism with a triangular prism on top. To determine its volume, calculate the volume of each smaller prism and add them. Or, calculate the area of the 5-sided base and multiply by the length.



Optimizing

- To optimize area or volume, consider the greatest area or volume that can be enclosed by a given amount of material.
- To optimize perimeter or surface area, consider the least amount of material required to enclose a given area or volume.
- When there are no constraints, the optimal rectangle is a square and the optimal rectangular prism is a cube.

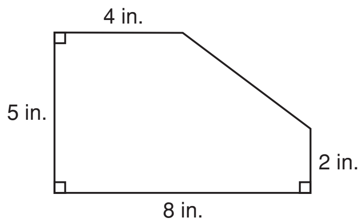
Square	*Cube*
$A = s^2$	$V = s^3$
$P = 4s$	$SA = 6s^2$

Chapter Review

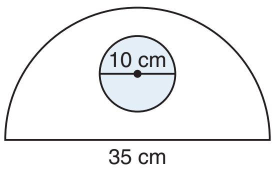
2.1

1. Determine the area of each composite figure. The curve is a semicircle.

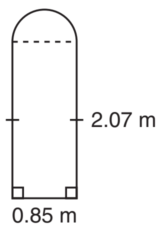
a)



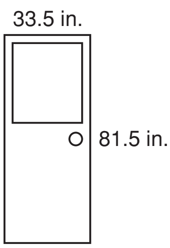
b)



c)



2. The window in this door is 25.5 inches wide and 35.0 inches high. The hole for the lock is 2.0 inches in diameter.



1 foot = 12 inches

a) Determine the area of the wooden part of the door, in square inches.

b) Salim is painting the front and back of four of these doors. He will apply two coats of paint. Each can of paint will cover 125 square feet. How many cans should he buy? Justify your answer.

3. The owner of a small art gallery is framing a painting for a client. The painting measures $15\frac{1}{2}$ inches by $15\frac{1}{2}$ inches. She leaves a border around the painting that is $2\frac{1}{2}$ inches wide on each side and $3\frac{1}{2}$ inches wide on the top and bottom.

a) Sketch the situation. What will be the length and width of the painting and border together?

b) Determine the area of glass that will be needed to cover the painting and border.

2.2

4. A cylindrical tank has radius 4.5 m and height 6.1 m.

a) Determine the volume of liquid it can hold, in litres.

$1 \text{ m}^3 = 1000 \text{ L}$

b) Jamal is painting the tank with rust paint. Determine the surface area he must paint, in square metres. Explain any assumptions you make.

c) Suppose each can of rust paint will cover 650 square feet. How many cans should Jamal buy? Justify your answer.

$1 \text{ m} \doteq 3.2808 \text{ feet}$

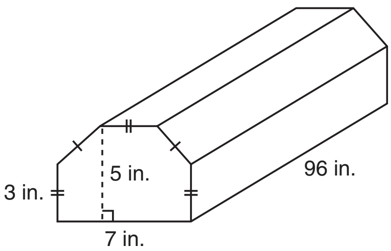
5. Shipping containers are used to transport goods across the sea, over the rail system, or by transport truck. The outer dimensions of a container are 40 feet by 8 feet by 8 feet 6 inches. The walls are 2 feet thick on all sides.

a) The outside of a shipping container requires painting. Determine the surface area that needs to be painted.

b) Determine the maximum volume of storage space.

6. Concrete parking curbs are often used in parking lots.

a) Determine the volume of this curb.



b) Suppose Leo has to make 100 curbs like this. How many cubic yards of concrete will he need?

1 yard = 36 inches

1 foot = 12 inches

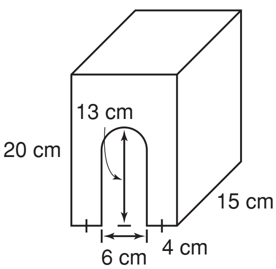
c) Curbs are sometimes painted to make them more visible to drivers. Determine the surface area to be painted on one curb. Explain which face you would omit in your calculation.

d) A can of paint covers 600 square feet of concrete. How many cans would Leo need to paint 100 curbs? Justify your answer.

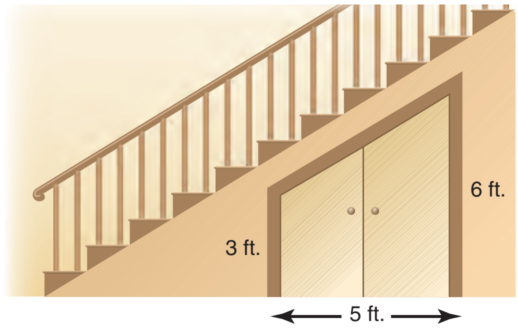
7. Nasmin will construct this object from sheet metal, then coat it with enamel.

a) Determine the volume of the object.

b) Determine the surface area to be covered with enamel. All exposed faces are to be covered.



8. Amrit builds this storage unit under the stairs. The unit has depth 3 feet. He will paint the front of the unit.



a) What is the volume of air inside the storage unit?

b) What area will Amrit paint?

2.3
2.4

9. For each perimeter, what are the dimensions of the rectangle with the maximum area? What is the area?

a) 40 cm **b)** 110 feet

c) 25 m **d)** 87 inches

10. For each area, what are the dimensions of the rectangle with the minimum perimeter? What is the perimeter?

a) 25 square feet **b)** 81 m^2

c) 144 cm^2 **d)** 169 square inches

11. A car dealership fences in a rectangular area behind their building to secure unsold vehicles. One length will be the back wall of the dealership. What is the maximum parking area that can be created if they have 2 km of fencing to use?

12. Jim is setting up a rectangular dog run in his backyard. He buys six 3-foot sections of fencing and a 3-foot wide gate. What are the dimensions of the dog run with the greatest area in each situation?
   a) Jim uses the yard fence for one side of the dog run.
   b) Jim uses the corner of the yard fence for two sides of the dog run.

13. A marine biologist is collecting data. She has 100 m of rope with buoys to outline a rectangular or circular research area on the surface of the water.
   a) Which figure will enclose a greater area?
   b) How much extra area will be enclosed by using the more efficient shape?

2.5 14. A cube is the rectangular prism with the least surface area for a given volume. What do you think a rectangular prism with the greatest surface area for a given volume would look like? Use the file *DynamicVolumeInvestigations.gsp* to explore your predictions.

2.6 2.7 15. Yasmin is constructing a rectangular prism with volume exactly 216 m^3. It will have the least possible surface area.
   a) Describe the prism. What will be its dimensions?
   b) What will be its surface area?

16. Linda is a potter. She has a slab of clay with area 150 cm^2. She will make a ceramic box from this slab by cutting out and joining six rectangular faces. Linda wants the volume of the box to be as great as possible.
   a) Describe the prism. What will be its dimensions?
   b) What will its volume be?

17. Jake designs personalized self-adhesive notes. Each note is rectangular. The notes come in blocks that are rectangular prisms.
   a) Determine the minimum surface area of a block with volume 8 cubic inches.
   b) Determine the dimensions of each note.
   c) Each note is 0.13 mm thick. Determine the number of notes in a block with minimum surface area.

18. Giulia is designing glass storage jars with surface area 675 cm^2. The jars are cylinders. She wants to maximize the volume of each jar to save on the materials cost.
   a) What dimensions should Giulia use?
   b) What will the volume be?

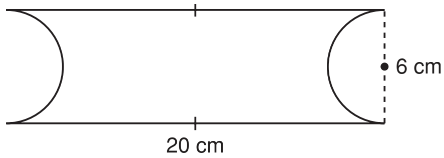
19. Armin is designing a bottle for a new shampoo. The bottle should have a volume of 450 cm^3. Armin wants to minimize the amount of plastic used in the bottle. Determine the dimensions of the bottle with the minimum surface area in each case. Explain your method.
   a) The bottle is a cylinder.
   b) The bottle is a triangular prism with a base that is an equilateral triangle.

Practice Test

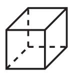
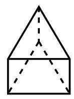
Multiple Choice: Choose the correct answer for questions 1 and 2. Justify each choice.

1. The approximate area of this composite figure is:

A. 63 cm^2 **B.** 92 cm^2
C. 120 cm^2 **D.** 148 cm^2



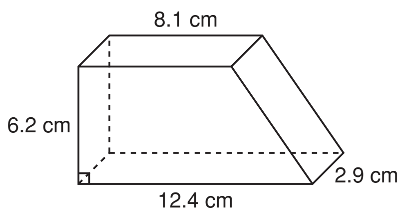
2. These objects have the same volume. Which one has the least surface area?

A. 
B. 
C. 
D. 

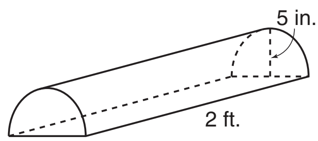
Show your work for questions 3 to 6.

3. **Knowledge and Understanding** Determine the volume and surface area of each three-dimensional object.

a)



b)



4. **Communication** A rectangular patio is being built against the side of a house using 60 congruent square tiles. Determine the arrangement of tiles that requires the least amount of edging. Justify your answer, including a description of any constraints that affected your solution strategy.

5. **Application** A flowerbed is a rectangle with a semicircle at each end. The total length of the flowerbed is 50 feet and the width is 14 feet.

a) Sketch the flowerbed and determine its area.

b) A gardener can plant 10 plants per square yard. How many plants should he buy for this flowerbed?

c) The fertilizer he will apply should be spread at a rate of 2 kg per 30 m^2. About how much fertilizer will he need for this garden?

1 m ≐ 3.2808 feet

6. **Thinking** A dairy sells ice cream in 11.4-L cardboard containers.

a) If the dairy uses a rectangular container optimized for surface area, what would the dimensions and surface area be?

b) If the dairy uses a cylindrical container optimized for surface area, what would the dimensions and surface area be?

c) How much cardboard is saved by using the more efficient container?

1 L = 1000 cm^3

Chapter Problem A Winning Design

A company is developing a new power drink.
Imagine you are trying to win the design contract for the product. You must create a name for the product and design a container, label, and shipping carton. Here are the guidelines you must follow:

- The name of the drink should appeal to people in your age group and suggest that the drink boosts energy and tastes good.
- The container of an individual drink can be made from aluminum, plastic, or glass, or it can be a Tetra Pak™. To minimize material costs, the container must use no more than 80 square inches of material. Describe the constraints and explain how you chose the shape and dimensions of the container and the type of packaging material.
- The label must display the product name and capacity, and have room for the ingredients, UPC code, and a 2-cm by 5-cm rectangular area for the nutrition facts table.
- The shipping carton must be a cardboard case that holds between 18 and 30 individual containers. Calculate the amount of cardboard used per case and the total amount required to package 720 containers. Justify your choices.

The winner will be chosen from those designs that provide optimal packaging solutions in interesting and appealing ways.