7.2 Geometric Sequences

A **Geometric Sequence** is a sequence that has a common ratio between the terms. (ie. you multiply by some number to move sequentially through the sequence)

2, 6, 18, ...

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Eg. 1) Given $t_n = 5(-2)^{n-1}$ find the first 4 terms of the sequence.

$t_1 = 5(-2)^{1-1}$
$= 5(-2)^0$
$= 5(1)$
$= 5$

$t_2 = 5(-2)^{2-1}$
$= 5(-2)^1$
$= 5(-2)$
$= -10$

$t_3 = 5(-2)^{3-1}$
$= 5(-2)^2$
$= 5(4)$
$= 20$

$t_4 = 5(-2)^{4-1}$
$= 5(-2)^3$
$= 5(-8)$
$= -40$

$t_5 = 5(-2)^{5-1}$
$= 5(-2)^4$
$= 5(16)$
$= 80$

*Note → if common ratio is negative, the terms alternate between positive and negative.

Eg. 2) Find the tenth term of the sequence $t_n = 3(2)^{n-1}$.

$t_{10} = 3(2)^{10-1}$

$= 3(2)^9$

$= 1536$

x^y y^x $\wedge$ on calculator

Eg. 3) Find the general term and recursive formula for the sequence 36, 18, 9, ...

$r = \frac{18}{36} = \frac{1}{2}$

Recursive

$t_n = r\, t_{n-1}$

$= \frac{1}{2} t_{n-1}$, $t_1 = 36$

General Term

$t_n = a r^{n-1}$

$t_n = 36\left(\frac{1}{2}\right)^{n-1}$

$r = \frac{1}{2}$

Eg. 4) Determine whether the following sequences are geometric, arithmetic or neither.

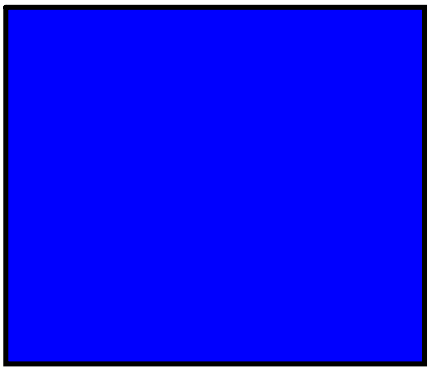
a) 99, 33, 11, ... Geometric $r = \frac{1}{3}$

b) -2, 10, 22, 34 Arithmetic $d = 12$

c) 3, 3, 3, ... Geometric $r = 1$
Arithmetic $d = 0$

d) 47, 44, 40, 32

Neither



Eg. 5) Determine the number of terms in the sequence 3, 6, 12, ..., 384.

$t_n = ar^{n-1}$

$384 = 3(2)^{n-1}$

$\frac{384}{3} = 2^{n-1}$

$128 = 2^{n-1}$

$2^7 = 2^{n-1}$

$\therefore 7 = n-1$

$7+1 = n$

$8 = n$

$r = \frac{6}{3} = 2$

$a = 3$

Rewrite with base the same as ratio.

$\therefore$ There are 8 terms in the sequence.

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Eg. 6) In a geometric sequence $t_6 = -2048$ and $t_{11} = -2097152$. Find the general term and the first 3 terms of the sequence.

$$\frac{-2097152}{-2048} = r^{11-6}$$

$$1024 = r^5$$

$$\sqrt[5]{1024} = r$$

$$4 = r$$

$t_n = ar^{n-1}$

$-2048 = a(4)^{6-1}$

$-2048 = a(4)^5$

$-2048 = 1024a$

$$\frac{-2048}{1024} = a$$

$-2 = a$

$$\boxed{t_n = -2(4)^{n-1}}$$

$t_1 = -2$

$t_2 = -2(4)^{2-1}$

$= -2(4)$

$= -8$

$t_3 = -2(4)^{3-1}$

$= -2(4)^2$

$= -2(16)$

$= -32$

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Eg. 7) Your boss comes to you and offers you a new pay structure:

Click Here to see Pay Structure *

Which plan would you choose? Justify your answer mathematically.

Pay 30th Day:

$t_n = ar^{n-1}$

$t_{30} = 0.01(2)^{30-1}$

$= \$5\,368\,709.12$

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Dec 11-1:01 PM