

2.6 Optimizing Volume and Surface Area

From yesterday's activities we determine that the maximum volume and minimum surface area occur when you have a cube shape or a cylinder whose radius is half the height.

<u>Square</u>	<u>Cube</u>	<u>Cylinder</u>
$A = s^2$	$V = s^3$	$V = 2\pi r^3 *$
$P = 4s$	$S.A. = 6s^2$	$S.A. = 6\pi r^2$

$$V = \pi r^2 h$$

$$= \pi r^2 (2r)$$

$$V = 2\pi r^3$$

$$S.A. = 2\pi r^2 + 2\pi rh$$

$$= 2\pi r^2 + 2\pi r(2r)$$

$$= 2\pi r^2 + 4\pi r^2$$

$$= 6\pi r^2$$



Example 1:

a) Rosa constructs a rectangular prism using exactly 384 square inches of cardboard. It has the greatest volume possible. What are the ✓ dimensions of the prism? What is its volume?

Start with what you are given (knowing that you are aiming to have a cube):

$$S.A. = 6s^2$$

$$384 = 6s^2$$

$$\frac{384}{6} = s^2$$

$$64 = s^2$$

$$\sqrt{64} = s$$

$$8 = s$$

∴ The dimensions are 8" x 8" x 8"

$$V = s^3$$

$$= 8^3$$

$$= 512 \text{ sq. inches.}$$

b) Liam constructs a rectangular prism with a volume of exactly 1331 square metres. It has the least surface area possible. What are the dimensions of the prism? ✓

Again, remember you are working toward a cube:

CUBE

∴ The dimensions are 11m x 11m x 11m

$$\begin{aligned}V &= s^3 \\1331 &= s^3 \\ \sqrt[3]{1331} &= s \\ 11 &= s\end{aligned}$$

Example 2: a) Naveed is designing a can with volume 350 mL. What is the minimum surface area of the can? Determine the dimensions of a can with the minimum surface area.

*Note $1 \text{ mL} = 1 \text{ cm}^3$

$$V = 2\pi r^3$$
$$350 = 2\pi r^3$$
$$\frac{350}{(2\pi)} = r^3$$

$$55.7 = r^3$$

$$\sqrt[3]{55.7} = r$$
$$3.8 = r$$

\therefore The dimensions are a can with radius 3.8 cm and a height $2 \times 3.8 = 7.6 \text{ cm}$

b) What is the maximum volume for a cylinder which has a surface area of 13400cm^2

$$\begin{aligned}S.A. &= 6\pi r^2 \\13400 &= 6\pi r^2 \\ \frac{13400}{6\pi} &= r^2 \\ 710.89 &= r^2 \\ \sqrt{710.89} &= r \\ 26.7 &= r\end{aligned}$$

$$\begin{aligned}V &= 2\pi r^3 \\ &= 2\pi (26.7)^3 \\ &= 119595\text{cm}^3\end{aligned}$$

Success Criteria for Optimization Problems:

① MAX/MIN
→ Special Formulas

Homework:

p. 110 #1,2,6,10,13

Formative Assessment:
p. 112 #15
Hand in before you leave--
we will look at answers
tomorrow

If you haven't completed p. 86 do so ASAP -- Hand in to MSIP teacher this afternoon if you can.