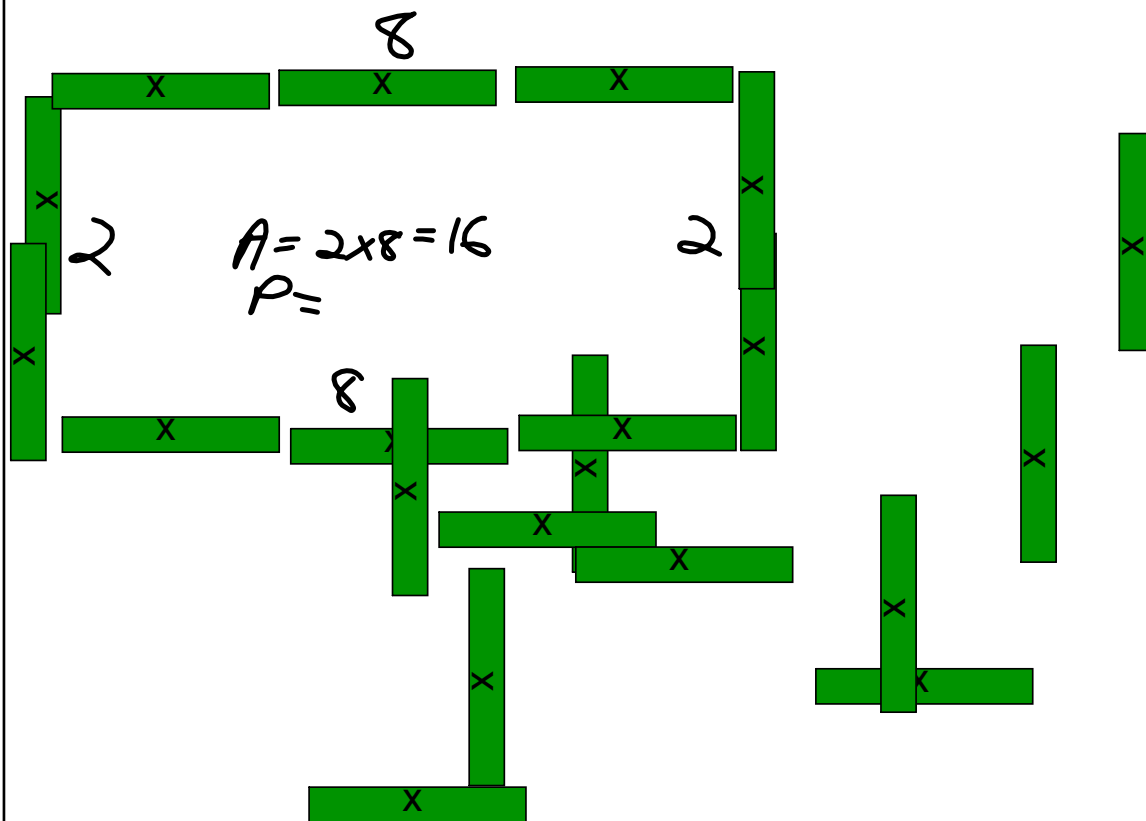


2.3 Optimizing Perimeter and Area

Arrange the 20 pieces of fencing to give you the largest enclosed space (area)



Given the area of 36 sq. units, what dimensions will give the minimum perimeter?

Area	length	Width	Perimeter
36	1	36	74
36	2	18	40
36	3	12	30
36	4	9	26
36	6	6	24
36	9	4	26
36	12	3	30

OPTIMIZATION

Write this down:

The shape that gives you the largest enclosed area (maximum area) **and** the minimum perimeter is a **square**.

Knowing this allows us to calculate maximum areas and minimum perimeters.

Example 1: What are the dimensions of a rectangle with perimeter 20 m and the maximum area? What is the maximum area?

*Maximum area occurs when the shape is a square. Start with what you are given to find side length:

$$P = 4s \quad 20 = 4s$$

$$\frac{P}{4} = s$$

$$\frac{20}{4} = s$$

$$5m = s$$

Area of the square: $A = s^2$

$$= 5^2$$

$$= 25m^2$$

Therefore the dimensions that give the maximum area are 5m by 5m with an area of 25 metres squared. $25m^2$.

Example 2: Determine the dimensions of a rectangle with area $45m^2$ and the minimum perimeter? What is the minimum perimeter?

Again, we know the shape is a square.

$$A = s^2 \quad 45 = s^2$$

$$s = \sqrt{A}$$

$$s = \sqrt{45}$$

$$s \doteq 6.71m$$

p. 94 #1,2ab,3ab,6

The rectangle is a square with side length 6.7 m.

$$P = 4s$$

$$= 4(6.71)$$

$$= 26.84m$$

It's perimeter is about 26.8 m.

Example 3: A hobby farmer is creating a fenced exercise yard for her horses. She has 900m of flexible fencing and wishes to maximize the area. She is going to fence a rectangular or a circular area. Determine which figure encloses the greater area.

Begin with what you are given: (perimeter -- same as circumference for a circle)

$$P = 4s$$

$$900 = 4s$$

$$\frac{900}{4} =$$

$$225 = s$$

$$A = s^2$$

$$= 225^2$$

$$= 50625 \text{ m}^2$$

$$C = 2\pi r$$

$$900 = 2\pi r$$

$$\frac{900}{(2\pi)} = r$$

$$143.2 = r$$

$$A = \pi r^2$$

$$A = \pi (143.2)^2$$

$$A = 64422 \text{ m}^2$$

p. 94 #1, 2, 3, 6-11, 14

Answer: The circular pen encloses the greater area.

Finish homework:

What if there are constraints?

A rectangular Garden is to be fenced using the wall of a house as one side of the garden. The garden should have an area of 40m squared.

Determine the minimum perimeter and dimensions of the garden if the dimensions must be whole numbers of metres.

Solution:

Only 3 sides of the garden need to be fenced.

So $P = l + 2w$. The area is $40m^2$

$$A = lw$$

$$40 = lw$$

$$40 \div l = w$$

Use a table:

length	Width	Perimeter
40	1	42
20	2	24
10	4	18
8	5	18
5	8	21

Therefore the minimum perimeter occurs twice:
When the length is 10 m and the width is 4 m or
when the length is 4 m and the width is 10 m.

Homework

p. 94 #1,2ab,3ab,6,7,8,9,10,11,14