Answers

Chapter 1

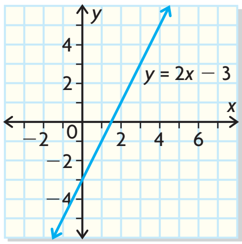
Getting Started, p. 2

1. **a)** $-2x + 8y$ **b)** $16x^2 - y^2$ **c)** $-x^2 + 2$ **d)** $2x^2 + 16x$

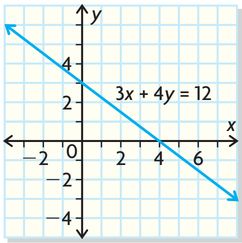
2. **a)** -46 **b)** 119 **c)** -7 **d)** 66

3. **a)** $x = 3$ **b)** $x = -2$ **c)** $y = -36$ **d)** $x = -2$

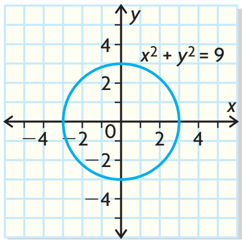
4. **a)**



b)



5. **a)**



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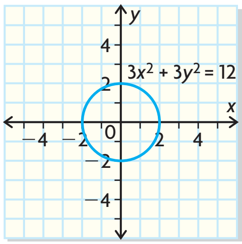
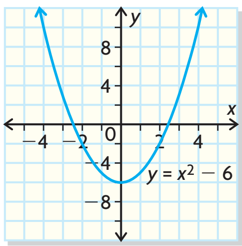
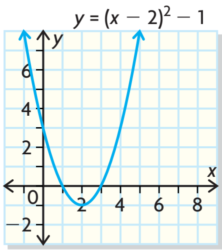


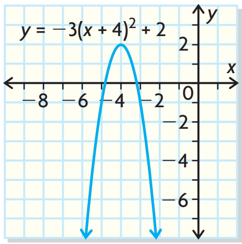
6. **a)**



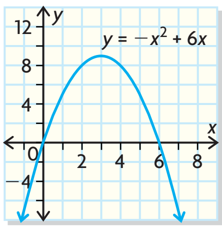
b)



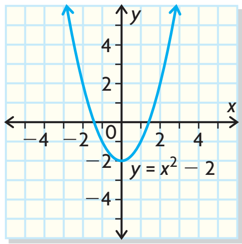
c)



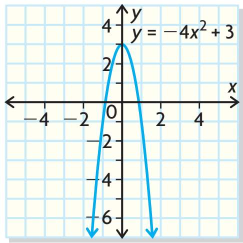
d)



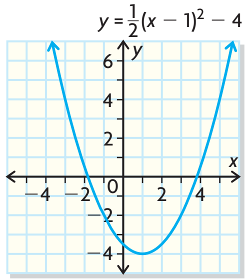
7. **a)** Translate down 2 units



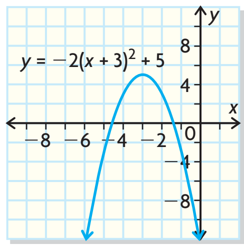
b) Reflect in x-axis, then vertical stretch, scale factor 4, then translate up 3 units



c) Vertical compression, scale factor, then translate right 1 unit and down 4 units



d) Reflect in x-axis, then vertical stretch, scale factor 2, then translate left 3 units and up 5 units



8. **a)** $x = 2$ or 3 **b)** $x = \pm 5$

9. Similarities: none of the equations for the relations involve powers higher than 2. Differences: linear and quadratic relations assign one y-value to each x-value, but circles may assign 0, 1, or 2 y-values; the relations have different shapes and model different types of problems; linear relations may only enter 2 or 3 quadrants.

Property	Linear Relations	Circles	Quadratic Relations
Equation(s)	$y = mx + c$ or $Ax + By = C$	$(x - h)^2 + (y - k)^2 = r^2$	$y = ax^2 + bx + c$ or $y = a(x - h)^2 + k$
Shape of graph	Straight line	Circle	Parabola
Number of quadrants graph enters	2 or 3	1, 2, 3, or 4	1, 2, 3, or 4
Descriptive features of graph	Slope is constant; crosses each axis at most once	Graph has upper and lower parts	Graph has a single lowest or highest point (vertex); crosses y-axis once, x-axis 0, 1, or 2 times
Types of problems modelled by the relation	Direct and partial variation	Constant distance from a point	Some economic functions; motion of a projectile; area

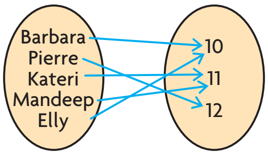
Lesson 1.1, pp. 10–12

1. **a)** Function; each x-value has only one y-value
b) Not a function; for $x = 1$, $y = -3$ and 0
c) Not a Function; for $x = 0$, $y = 4$ and 1
d) Function; each x-value has only one y-value

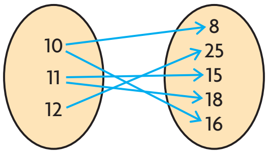
2. **a)** Not a function **c)** Function **e)** Function
b) Not a function **d)** Not a function **f)** Not a function

3. For $y = x^2 - 5x$, each x-value gives a single y-value. For $x = y^2 - 5y$, each x-value gives a quadratic equation in y, which may have two solutions.

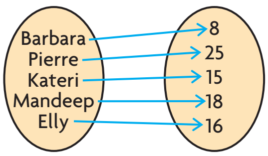
4. **a)** {(Barbara, 10), (Pierre, 12), (Kateri, 11), (Mandeep, 11), (Elly, 10)}



{(10, 8), (12, 25), (11, 15), (11, 18), (10, 16)}



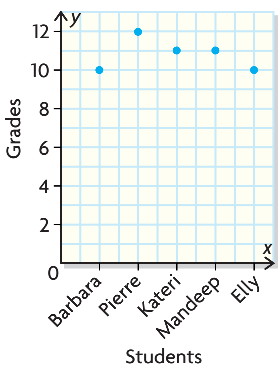
{(Barbara, 8), (Pierre, 25), (Kateri, 15), (Mandeep, 18), (Elly, 16)}



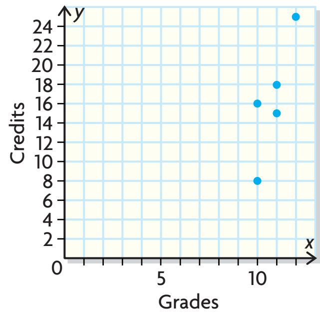
b) students, grades: domain = {Barbara, Pierre, Kateri, Mandeep, Elly}, range = {10, 11, 12} grades, credits: domain = {10, 11, 12}, range = {8, 15, 16, 18, 25} students, credits: domain = {Barbara, Pierre, Kateri, Mandeep, Elly}, range = {8, 15, 16, 18, 25}

c) Only grades-credits relation is not a function; it has repeated range values for single domain values.

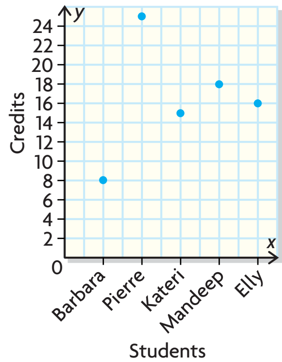
5. students, grades:



grades, credits:

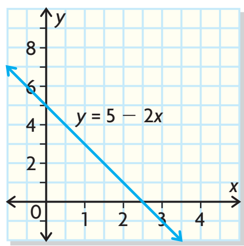


students, credits:

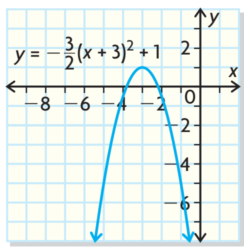


6. $y = 3$: horizontal line; function (passes vertical-line test).
$x = 3$: vertical line; not a function (fails vertical-line test)

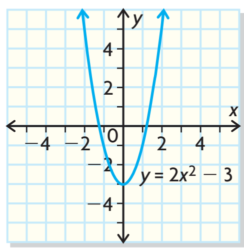
7. **a)** Linear, function



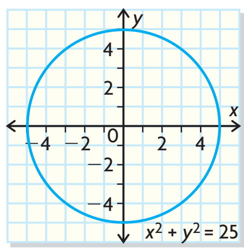
c) Quadratic, function



b) Quadratic, function



d) Circle, not a function



8. a) i) 1.25; 2.75 ii) ± 2; 0 iii) 2; -2 iv) 0; $\pm\sqrt{2}$
b) Functions: (i), (iii)
c) Graph relation and apply vertical-line test, or solve equation for y and check for multiple values

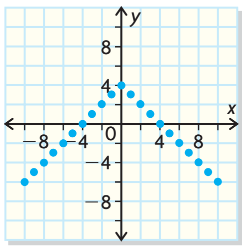
9. Functions: (a), (b), (d)

10. Not a function; for example, when $x = 6$, $y = 2$ or -2; graph fails vertical-line test

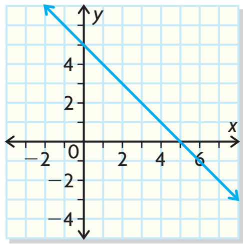
11. Functions: (a), (b)

12. a) domain $= \{x \in \mathbf{R} \mid x \geq 0\}$, range $= \{y \in \mathbf{R} \mid y \geq 44\}$
b) Distance cannot be negative, cost cannot be lower than daily rental charge.
c) Yes, it passes the vertical line test.

13. a) Answers may vary; for example:



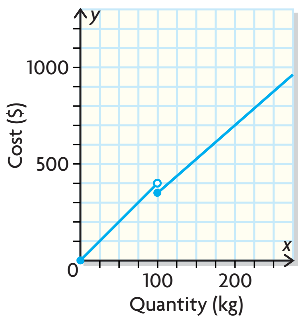
b) Answers may vary; for example:



14. Answers may vary; for example:

Definition: A relation with only one y-value for each x-value	**Characteristics:** A vertical line crosses the graph in at most one place
Function	
Examples: $3x + y = 2$; $y = -2x^2 + 7$	**Non-examples:** $x^2 + y^2 = 16$; $y = \pm\sqrt{x - 7}$

15. a) Each order quantity determines a single cost.
b) domain $= \{x \in \mathbf{R} \mid x \geq 0\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$
c)



d) Answers may vary. For example, the company currently charges more for an order of 100 kg ($350) than for an order of 99 kg ($396). A better system would be for the company to charge $50 plus $3.50 per kilogram for orders of 100 kg or more. This would make the prices strictly increasing as the weight of the order increases.

Lesson 1.2, pp. 22–24

1. a) -4 c) 14 e) $2 - 3a$
b) 2 d) $\frac{1}{2}$ f) $2 - 9b$

2. a) 2 b) 4 c) -5 d) -3 or -4

3. a) $f(x) = 1200 - 3x$
b) 840 mL c) 3:10 pm

4. a) 8, 0, -0.75 b) $-5, -25, -2.5$

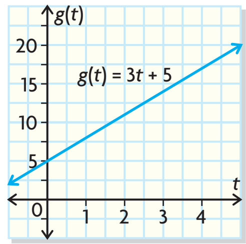
5. a) $-\frac{1}{6}$ b) undefined c) $\frac{1}{3}$ d) $2\frac{2}{3}$

6. a) domain $= \{-2, 0, 2, 3, 5, 7\}$, range $= \{1, 2, 3, 4, 5\}$
b) i) 4 ii) 2 iii) 5 iv) -2

7. a) $2a - 5$ b) $2b - 3$ c) $6c - 7$ d) $-10x - 1$

8. a)

t	$g(t)$
0	5
1	8
2	11
3	14
4	17
5	20



b) i) 5 ii) 14 iii) 3 iv) 3 v) 3 vi) 3

9. a)

s	$f(s)$
0	9
1	4
2	1
3	0

b) i) 9 ii) 4 iii) 1 iv) 0 v) 2 vi) 2
c) They are the same; they represent the second differences, which are constant for a quadratic function.

10. a) 49
b) The y-coordinate of the point on the graph with x-coordinate -2
c) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -1\}$
d) It passes the vertical-line test.

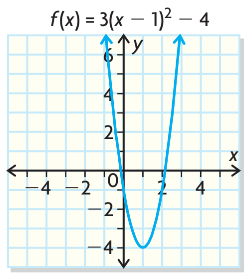
11. a) 2 b) 0.4 c) 0.8 d) $\frac{17}{25}$

12. a) $f(x) = 0.15x + 50$ b) $120.80 c) 200 km

13. a) $f(x) = (24 - 3x)x$ b) 45, -195, -60 c) 48

14. $f(x) \doteq 0.0034x(281 - x)$

15. a)



b) The y-coordinate of the point on the graph with x-coordinate -1; start from -1 on x-axis, move up to curve, then across to y-axis
c) i) 3 ii) 9 iii) $3x^2 - 4$

16. a) 3, -5 b) 1, -3 c) -1

17. a) $\frac{1}{4}$ b) $\frac{1}{3}$, -1

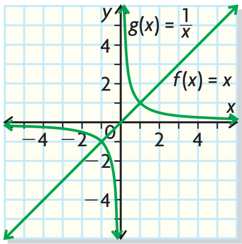
18. Answers may vary; for example: $f(x)$ is defined as equal to an expression involving x, for each x-value in the functions domain; the graph of $f(x)$ is the set of all points $(x, f(x))$ for which x is in the domain. Student examples will vary.

19. **a)** $f(x) = \frac{2}{3}x + 10$ **b)** $73\frac{1}{3}, 126\frac{2}{3}, 153\frac{1}{3}, 180$

20. **a)** 8, 27, 64, 125, 216 **b)** cube of x or x^3

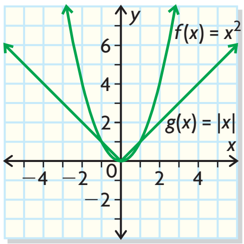
Lesson 1.3, p. 28

1.



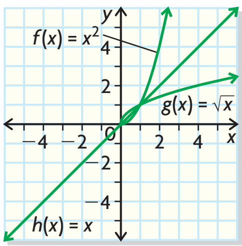
Both graphs lie in quadrants 1 and 3; graph of $g(x)$ is in two curved parts, and does not intersect axes. Vertical asymptote of $g(x)$: $x = 0$; horizontal asymptote of $g(x)$: $g(x) = 0$.

2.



Both graphs lie in quadrants 1 and 2; graph of $f(x)$ curves, but graph of $g(x)$ is formed by two straight half-lines.

3.



Graph of $g(x)$ is reflection of graph of $f(x)$ in graph of $h(x)$.

Lesson 1.4, pp. 35–37

1. **a)** domain $= \{1900, 1920, 1940, 1960, 1980, 2000\}$, range $= \{47.3, 54.1, 62.9, 69.7, 73.7, 77.0\}$
   b) domain $= \{-5, -1, 0, 3\}$, range $= \{9, 15, 17, 23\}$
   c) domain $= \{-4, 0, 3, 5\}$, range $= \{-1, 0, 3, 5, 7\}$

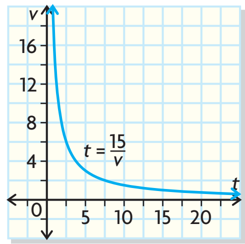
2. **a)** domain $= \{0, \pm 2, \pm 4, \pm 6, \pm 8, \pm 10\}$, range $= \{-8, -7, -6, -5, -4, -2, 0, 4, 8\}$
   b) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$
   c) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -8\}$
   d) domain $= \{x \in \mathbf{R} \mid -6 \leq x \leq 6\}$, range $= \{y \in \mathbf{R} \mid -6 \leq y \leq 6\}$
   e) domain $= \{x \in \mathbf{R} \mid x \leq 6\}$, range $= \{y \in \mathbf{R} \mid y \geq -2\}$
   f) domain $= \{x \in \mathbf{R} \mid x \geq -10\}$, range $= \{y \in \mathbf{R} \mid y = -6, -2 \leq y < 2, y > 4\}$

3. **1.** (a), (b); **2.** (b), (c), (e), (f)

4. domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -3\}$

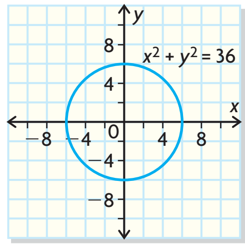
5. **a)** Even at masses when the price changes, a single price (the lower one) is assigned. It would not make sense to assign two or more prices to the same mass.
   b) domain $= \{x \in \mathbf{R} \mid 0 < x \leq 500\}$, range $= \{0.52, 0.93, 1.20, 1.86, 2.55\}$

6.



Graph passes vertical line test; domain $= \{v \in \mathbf{R} \mid v > 0\}$, range $= \{t \in \mathbf{R} \mid t > 0\}$

7. **a)**

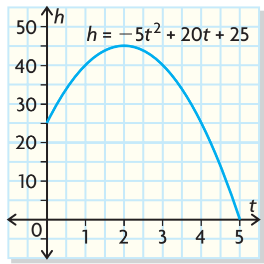


   b) domain $= \{x \in \mathbf{R} \mid -6 \leq x \leq 6\}$, range $= \{y \in \mathbf{R} \mid -6 \leq y \leq 6\}$
   c) No; fails vertical line test

8. $V(t) = t$; domain $= \{t \in \mathbf{R} \mid 0 \leq t \leq 2500\}$, range $= \{V \in \mathbf{R} \mid 0 \leq V \leq 2500\}$

9. **a)** domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$
   b) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 4\}$
   c) domain $= \{x \in \mathbf{R} \mid x \geq 1\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$
   d) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -5\}$
   e) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$
   f) domain $= \{x \in \mathbf{R} \mid x \leq 5\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$

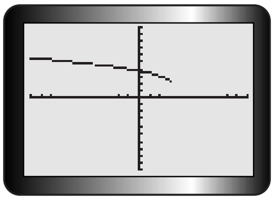
10. **a)**



   b) domain $= \{t \in \mathbf{R} \mid 0 \leq t \leq 5\}$, range $= \{h \in \mathbf{R} \mid 0 \leq h \leq 45\}$
   c) $h = -5t^2 + 20t + 25$

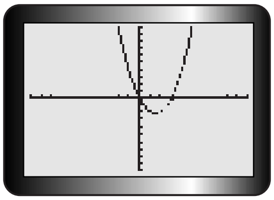
11. **a)** domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$
   b) domain $= \{x \in \mathbf{R} \mid x \geq 2\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$
   c) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -4\}$
   d) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq -5\}$

12. a)



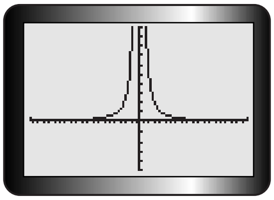
domain $= \{x \in \mathbf{R} \mid x \leq 3\}$, range $= \{y \in \mathbf{R} \mid y \geq 2\}$

b)



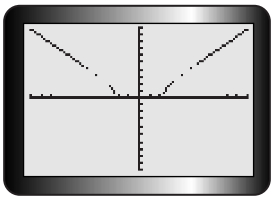
domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -2.25\}$

c)



domain $= \{x \in \mathbf{R} \mid x \neq 0\}$, range $= \{y \in \mathbf{R} \mid y > 0\}$

d)



domain $= \{x \in \mathbf{R} \mid x \leq -\sqrt{5}, x \geq \sqrt{5}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$

13. a) $A = \left(\dfrac{450 - 3w}{2}\right)w$

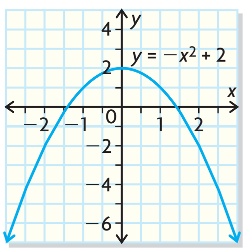
b) domain $= \{w \in \mathbf{R} \mid 0 < w < 150\}$,
range $= \{A \in \mathbf{R} \mid 0 < A \leq 8437.5\}$

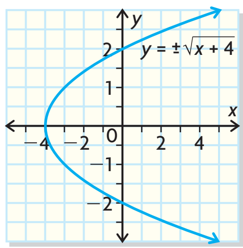
c) $l = 112.5$ m, $w = 75$ m

14. a) $\{-14, -3.5, 4, 7, 13\}$ **b)** $\{1, 6, 28, 55\}$

15. The domain is the set of x-values for a relation or function; the range is the set of y-values corresponding to these x-values. Domain and range are determined by values in x-column and y-column; x-coordinates and y-coordinates of graph; x-values for which relation or function is defined, and all possible corresponding y-values. Students' examples will vary.

16. a) Answers may vary; for example: **b)** Answers may vary; for example:





17. a) $A = x^2 + (10 - x)^2$ or $2x^2 - 20x + 100$

b) domain $= \{x \in \mathbf{R} \mid 0 \leq x \leq 10\}$,
range $= \{A \in \mathbf{R} \mid 50 \leq A \leq 100\}$

c) $P = 4\sqrt{x^2 + (10 - x)^2}$ or $\sqrt{2x^2 - 20x + 100}$

d) domain $= \{x \in \mathbf{R} \mid 0 \leq x \leq 10\}$,
range $= \{P \in \mathbf{R} \mid 50\sqrt{2} \leq P \leq 400\}$

Mid-Chapter Review, p. 40

1. a) Not a function

b) Function; each x-value goes to a single y-value

c) Function; passes vertical line test

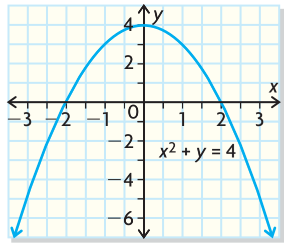
d) Not a function

e) Function; each x-value determines a single y-value

f) Function; each x-value determines a single y-value

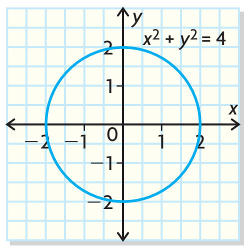
2. $x^2 + y = 4$:

x	y
−3	−5
−2	0
−1	3
0	4
1	3
2	0
3	−5

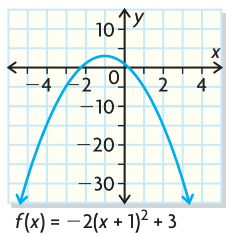


$x^2 + y^2 = 4$:

x	y
−2	0
0	±2
2	0



3. a)



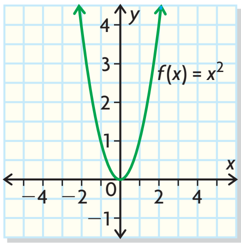
$f(x) = -2(x + 1)^2 + 3$

b) -5

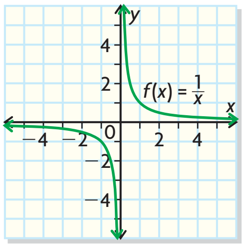
c) y-coordinate of the point on the graph with x-coordinate -3

d) i) -6 **ii)** -50 **iii)** $-2(3 - x)^2 + 3$

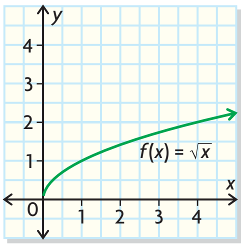
4. a) $f(x) = (20 - 5x)x$ **b)** $15, -25, -105$ **c)** 20

5. a) 

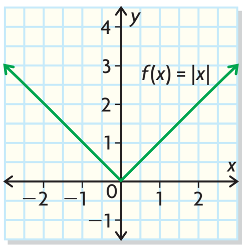
domain = $\{x \in \mathbf{R}\}$,
range = $\{y \in \mathbf{R} \mid y \geq 0\}$

b) 

domain = $\{x \in \mathbf{R} \mid x \neq 0\}$,
range = $\{y \in \mathbf{R} \mid y \neq 0\}$

c) 

domain = $\{x \in \mathbf{R} \mid x \geq 0\}$,
range = $\{y \in \mathbf{R} \mid y \geq 0\}$

d) 

domain = $\{x \in \mathbf{R}\}$,
range = $\{y \in \mathbf{R} \mid y \geq 0\}$

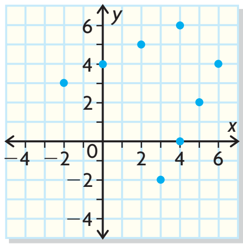
6. a) domain = $\{1, 2, 4\}$, range = $\{2, 3, 4, 5\}$
b) domain = $\{-2, 0, 3, 7\}$, range = $\{-1, 1, 3, 4\}$
c) domain = $\{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$,
range = $\{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$
d) domain = $\{x \in \mathbf{R} \mid x \geq -3\}$, range = $\{y \in \mathbf{R}\}$
e) domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R} \mid y \leq 5\}$
f) domain = $\{x \in \mathbf{R} \mid x \geq 4\}$, range = $\{y \in \mathbf{R} \mid y \geq 0\}$

7. a) $A = \left(\dfrac{600 - 4w}{2}\right)w$
b) domain = $\{w \in \mathbf{R} \mid 0 < w < 150\}$,
range = $\{A \in \mathbf{R} \mid 0 < A \leq 11\,250\}$
c) $l = 150$ m, $w = 75$ m

8. a) domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R} \mid y \leq 5\}$
b) domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R} \mid y \geq 4\}$
c) domain = $\{x \in \mathbf{R} \mid -7 \leq x \leq 7\}$,
range = $\{y \in \mathbf{R} \mid -7 \leq y \leq 7\}$
d) domain = $\{x \in \mathbf{R} \mid -2 \leq x \leq 6\}$,
range = $\{y \in \mathbf{R} \mid 1 \leq y \leq 9\}$

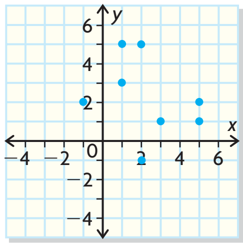
Lesson 1.5, pp. 46–49

1. a) $\{(3, -2), (4, 0), (5, 2), (6, 4)\}$

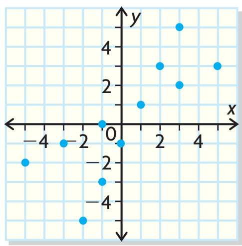


Both relation and inverse relation are functions.

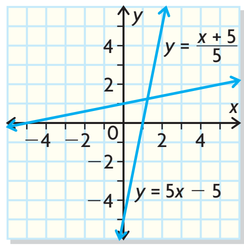
b) $\{(5, 2), (-1, 2), (1, 3), (1, 5)\}$



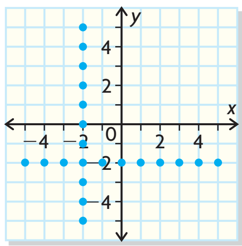
Both relation and inverse relation are not functions.

2. a) 

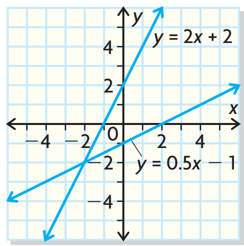
Function, point (1, 1) is common

d) 

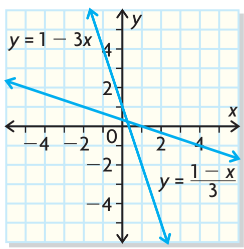
Function, point (1.25, 1.25) is common

b) 

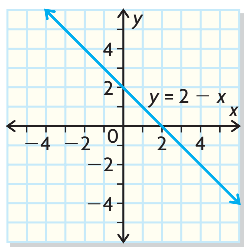
Not a function, point $(-2, -2)$ is common

e) 

Function, point $(-2, -2)$ is common

c) 

Function, point (0.25, 0.25) is common

f) 

Function, all points are common. The function and its inverse are the same graph.

3. a) No b) Yes

4. a) $y = \dfrac{x+3}{4}$ c) $y = \dfrac{6-4x}{3}$

b) $y = 2(2 - x)$ d) $y = \dfrac{2x-10}{5}$

5. a) $f^{-1}(x) = x + 4$ d) $f^{-1}(x) = 2(x+1)$

b) $f^{-1}(x) = \dfrac{x-1}{3}$ e) $f^{-1}(x) = \dfrac{6-x}{5}$

c) $f^{-1}(x) = \dfrac{1}{5}x$ f) $f^{-1}(x) = \dfrac{4}{3}(x-2)$

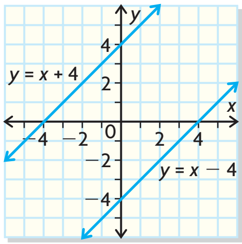
6. a) $f^{1}(x) = x - 7$ d) $f^{1}(x) = -5(x+2)$

b) $f^{1}(x) = 2 - x$ e) $f^{1}(x) = x$

c) $x = 5$ f) $f^{-1}(x) = 4x + 3$

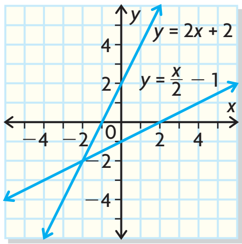
7. Question 5.

a)



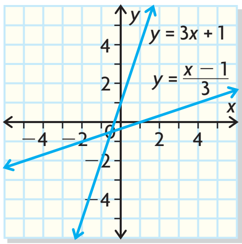
Function, linear

d)



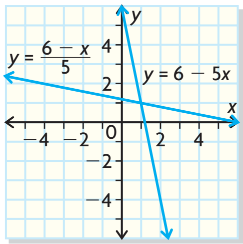
Function, linear

b)



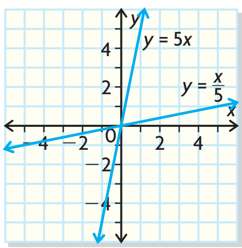
Function, linear

e)



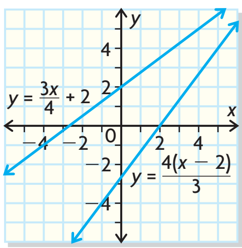
Function, linear

c)



Function, linear

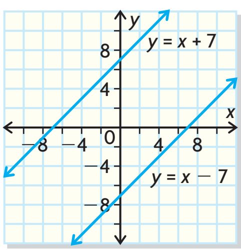
f)



Function, linear

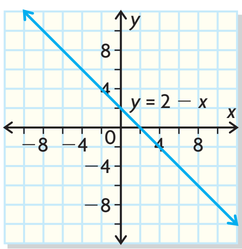
Question 6.

a)



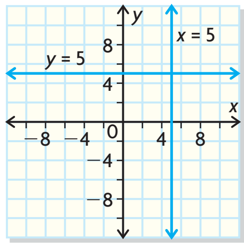
Function, linear

b)



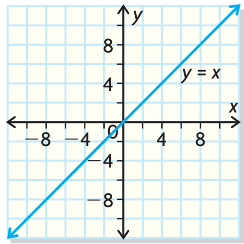
Function, linear
The function and its inverse are the same graph.

c)



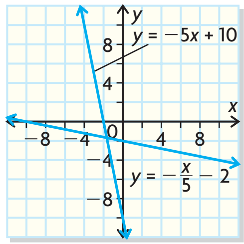
Not a function, linear

e)



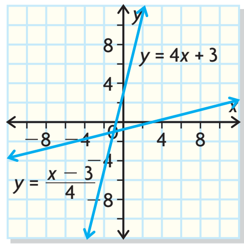
Function, linear
The function and its inverse are the same graph.

d)



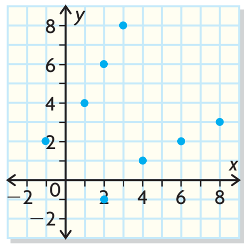
Function, linear

f)



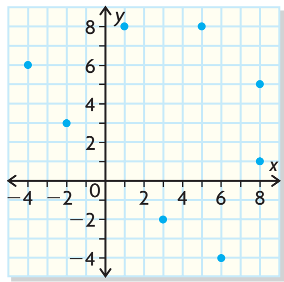
Function, linear

8. a) {(2, −1), (4, 1), (6, 2), (8, 3)}



Function: domain = {−1, 1, 2, 3}, range = {2, 4, 6, 8}; inverse: domain = {2, 4, 6, 8}, range = {−1, 1, 2, 3}; domain, range are interchanged

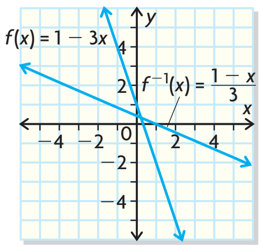
b) {(6, −4), (3, −2), (8, 1), (8, 5)}



Function: domain = {−4, −2, 1, 5}, range = {3, 6, 8}; inverse: domain = {3, 6, 8}, range = {−4, −2, 1, 5}; domain, range are interchanged

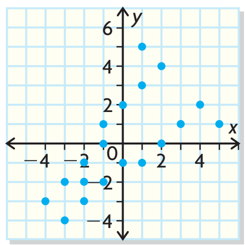
Answers

c) $f^{-1}(x) = \dfrac{1 - x}{3}$



Function: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$;
inverse: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$;
domain, range are identical for both

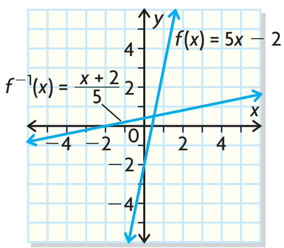
d) $\{(-3, -4), (-2, -3), (-2, -2), (-2, -1), (-1, 0), (-1, 1), (0, 2), (1, 3), (2, 4), (1, 5)\}$



Function: domain $= \{0, \pm 1, \pm 2, \pm 3, \pm 4, 5\}$, range $= \{0, \pm 1, \pm 2, -3\}$; inverse: domain $= \{0, \pm 1, \pm 2, -3\}$, range $= \{0, \pm 1, \pm 2, \pm 3, \pm 4, 5\}$; domain, range are interchanged

9. a) $f^{-1}(x) = \dfrac{x + 2}{5}$

b)

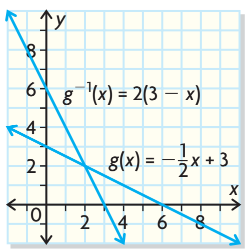


c) Function equation is linear in x; or, graph is a straight line.

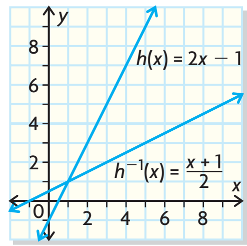
d) $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$

e) Slopes are reciprocals of each other.

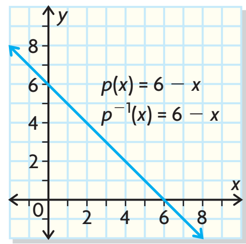
f) $g^{-1}(x) = 2(3 - x)$



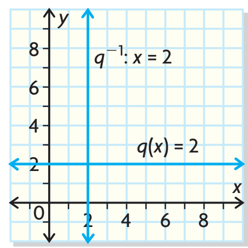
Function equation is linear in x, or, graph is a straight line;
(2, 2); slopes are reciprocals of each other. $h^{-1}(x) = \dfrac{x + 1}{2}$



Function equation is linear in x, or, graph is a straight line;
(1, 1); slopes are reciprocals of each other. $p^{-1}(x) = 6 - x$



Function equation is linear in x, or, graph is a straight line; all points on graphs; slopes are equal. q^{-1} is the relation $x = 2$



In this case, q^{-1} is not a function, but its graph is a straight line; (2, 2); slopes are 0 and undefined.

10. a) 37 b) 19 c) 3 d) 5 e) 3 f) $\dfrac{1}{3}$

11. c) is slope of $g(t)$; f) is slope of $g^{-1}(t)$

12. a) $f(x) = 2x + 30$

b) Subtract 30, then halve; a Canadian visiting the United States

c) $f^{1}(x) = \dfrac{1}{2}(x - 30)$

d) $f(14) = 2(14) + 30 = 58\ °F$

e) $f^{1}(70) = \dfrac{1}{2}(70 - 30) = 20\ °C$

13. a) Multiply by 10, then divide by 4

b) A Canadian visiting the U.S.

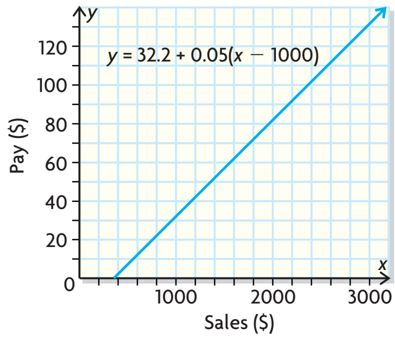
c) $g(x) = \dfrac{4x}{10}$; $g^{-1}(x) = \dfrac{10x}{4}$

d) $g(15) = \dfrac{4(15)}{10} = 6$ in.

e) $g^{-1}(70) = \dfrac{10(70)}{4} = 175$ cm

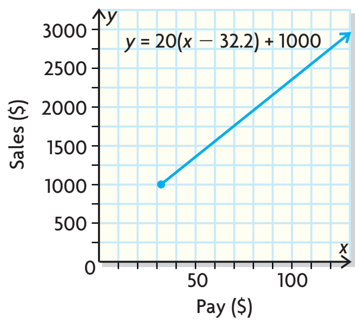
14. $y = 0.38x + 0.50$

15. **a)**



b) $f(x) = 32.2 + 0.05(x - 1000)$

c)



d) $f^{1}(x) = 1000 + 20(x - 32.2)$

e) $f^{-1}(420) = 1000 + 20(420 - 32.2) = \$8756$

16. Because (1, 2) cannot belong to f, as well as (1, 5).

17. $k = 2$

18.

Definition:	**Methods:**
Inverse of a function of form $f(x) = mx + c$	Switch x and y and solve for y Take reciprocal of slope, switch x- and y-intercepts
Inverse of a Linear Function	
Examples:	**Properties:**
$f(x) = 3x + 2, f^{-1}(x) = \frac{x-2}{3}$ $g(x) = \frac{5}{7}(2 - 5x), g^{-1}(x) = \frac{2}{5} - \frac{7}{25}x$	Has form $f^{-1}(x) = mx + c$ or $x = c$ Graph is straight line

19. Answers may vary; for example: $y = x$; $y = -x$; $y = 1 - x$

20. $(f^{-1})^{-1}(x) = 3x + 4$

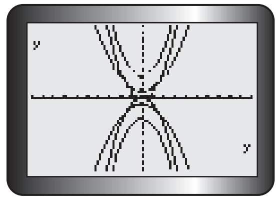
Lesson 1.6, p. 51

1. **a)** Upper half-parabola, opening right, vertex at (1, 2)
b) V-shape, opening up, vertex at (1, 2)
c) Hyperbola, asymptotes $x = 1$ and $y = 2$, graph lying to upper right and lower left of asymptotes

2. **a)** Graph of $y = \sqrt{x}$ is upper half, graph of $y = -\sqrt{x}$ lower half of parabola opening right.
b) Graph of $y = |x|$ opens up, graph of $y = -|x|$ opens down.
c) Graph of $y = \frac{1}{x}$ lies to upper right and lower left of asymptotes, graph of $y = -\frac{1}{x}$ lies to lower right and upper left.

3. **a)** Graph of $y = 2\sqrt{x}$ is narrower (steeper) than graph of $y = \sqrt{x}$.
b) Graph of $y = 2|x|$ is narrower (steeper) than graph of $y = |x|$.
c) Graph of $y = \frac{2}{x}$ is narrower (steeper) than graph of $y = \frac{1}{x}$.

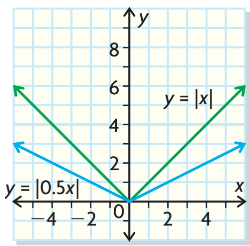
4. Answers will vary. For example,



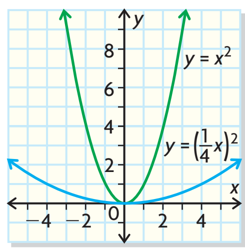
Lesson 1.7, pp. 58–60

1. **a)** $y = (3x)^2$ **b)** $y = \sqrt{-\frac{1}{2}x}$

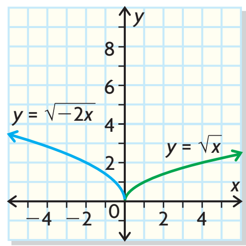
2. **a)** $y = |x|$; horizontal stretch factor 2



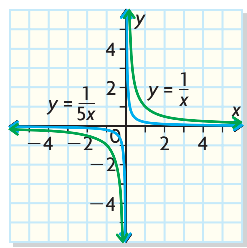
b) $y = x^2$; horizontal stretch factor 4



c) $y = \sqrt{x}$; horizontal compression factor $\frac{1}{2}$ and reflection in y-axis

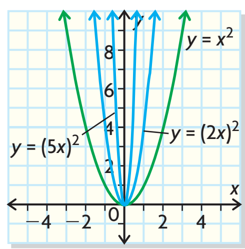


d) $y = \frac{1}{x}$; horizontal compression factor $\frac{1}{5}$



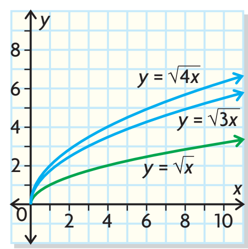
3. **a)** (1.5, 4) **b)** (6, 4) **c)** (9, 4) **d)** (−0.75, 4)

4. **a)**



Horizontal compression factor $\frac{1}{2}$, (0, 0); horizontal compression factor $\frac{1}{5}$, (0, 0)

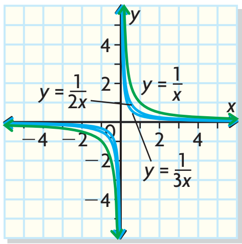
b)



Horizontal compression factor $\frac{1}{3}$, (0, 0); horizontal compression factor $\frac{1}{4}$, (0, 0)

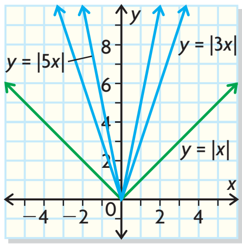
Answers

c)



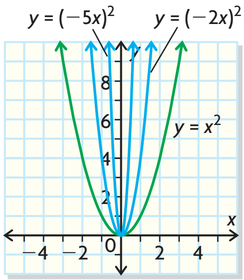
Horizontal compression factor $\frac{1}{2}$, no invariant points; horizontal compression factor $\frac{1}{3}$, no invariant points

d)



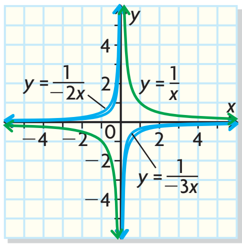
Horizontal compression factor $\frac{1}{3}$, (0, 0); horizontal compression factor $\frac{1}{5}$, (0, 0)

5. a)



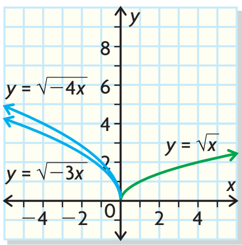
Horizontal compression factor $\frac{1}{2}$ and (optional) reflection in *y*-axis, (0, 0); horizontal compression factor $\frac{1}{5}$ and (optional) reflection in *y*-axis, (0, 0)

c)



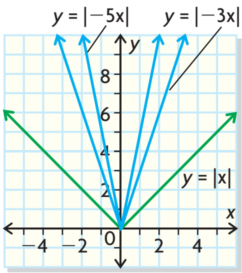
Horizontal compression factor $\frac{1}{2}$ and reflection in *y*-axis, no invariant points; horizontal compression factor $\frac{1}{3}$ and reflection in *y*-axis, no invariant points

b)



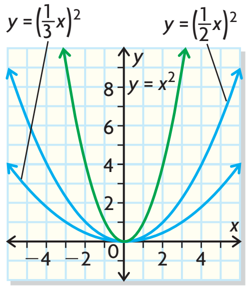
Horizontal compression factor $\frac{1}{3}$ and reflection in *y*-axis, (0, 0); horizontal compression factor $\frac{1}{4}$ and reflection in *y*-axis, (0, 0)

d)



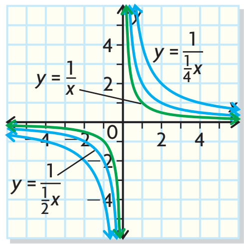
Horizontal compression factor $\frac{1}{3}$ and (optional) reflection in *y*-axis, (0, 0); horizontal compression factor $\frac{1}{5}$ and (optional) reflection in *y*-axis, (0, 0)

6. a)



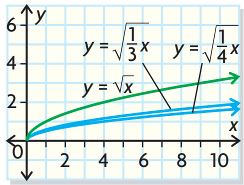
Horizontal stretch factor 2, (0, 0); horizontal stretch factor 3, (0, 0)

c)



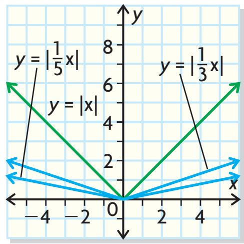
Horizontal stretch factor 2, no invariant points; horizontal stretch factor 4, no invariant points

b)



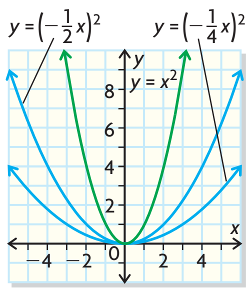
Horizontal stretch factor 2, (0, 0); horizontal stretch factor 3, (0, 0)

d)



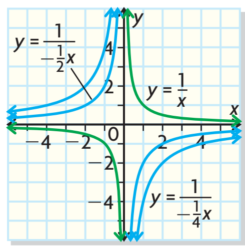
Horizontal stretch factor 3, (0, 0); horizontal stretch factor 5, (0, 0)

7. a)



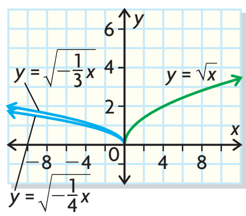
Horizontal stretch factor 2 and (optional) reflection in *y*-axis, (0, 0); horizontal stretch factor 3 and (optional) reflection in *y*-axis, (0, 0)

c)



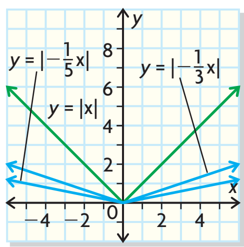
Horizontal stretch factor 2 and reflection in *y*-axis, no invariant points; horizontal stretch factor 4 and reflection in *y*-axis, no invariant points

b)



Horizontal stretch factor 2 and reflection in *y*-axis, (0, 0); horizontal stretch factor 3 and reflection in *y*-axis, (0, 0)

d)



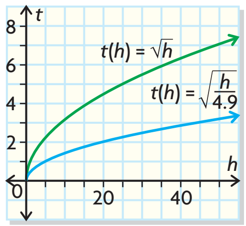
Horizontal stretch factor 3 and (optional) reflection in *y*-axis, (0, 0); horizontal stretch factor 5 and (optional) reflection in *y*-axis, (0, 0)

8. **a)** $g(x) = |2x|$ **b)** $g(x) = \frac{1}{-2x}$ **c)** $g(x) = \left(\frac{1}{4}x\right)^2$

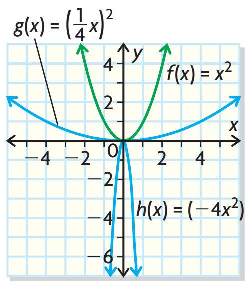
d) $g(x) = \sqrt{-3x}$

9. **a)** domain: $\{h \in \mathbf{R} \mid h \geq 0\}$; range: $\{t \in \mathbf{R} \mid t \geq 0\}$

b)

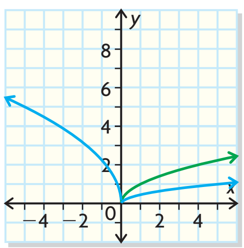


10. **a)**



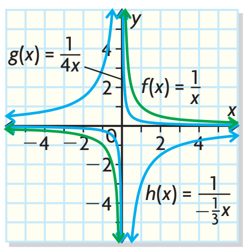
$g(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 0\}$

b)



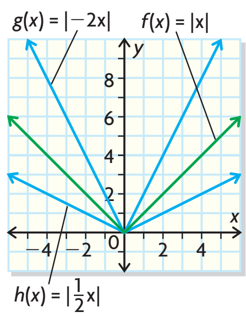
$g(x)$: domain $= \{x \in \mathbf{R} \mid x \geq 0\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R} \mid x \leq 0\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$

c)



$g(x)$: domain $= \{x \in \mathbf{R} \mid x \neq 0\}$, range $= \{y \in \mathbf{R} \mid y \neq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R} \mid x \neq 0\}$, range $= \{y \in \mathbf{R} \mid y \neq 0\}$

d)



$g(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$

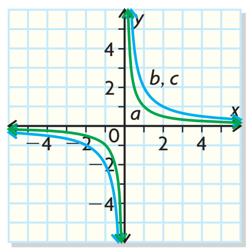
11. **a)** $\frac{1}{4}$ **b)** 2 **c)** -1 **d)** -5

12. **a)** 1.5, -1 **b)** 9, -6 **c)** $-1, \frac{2}{3}$

13. **a)** For $k > 1$, effect is a horizontal compression with scale factor $\frac{1}{k}$; for $0 < k < 1$, a horizontal stretch with scale factor $\frac{1}{k}$; for $k < 0$, reflection in the y-axis, then a horizontal compression or stretch with scale factor $\frac{1}{|k|}$. Apply these transformations to the graph of $y = f(x)$ to sketch the graph of $y = f(kx)$.

b) Answers may vary; for example: A horizontal compression or stretch is equivalent to a vertical stretch or compression, respectively; scale factors are reciprocals of each other for some functions, such as $f(x) = ax$, $f(x) = a|x|$, and $f(x) = \frac{a}{x}$, but not for others, such as $f(x) = ax^2 + bx + c$ and $f(x) = a\sqrt{x - d} + c$.

14. **a)–c)**



c) The horizontal and vertical stretches give the same graph.

d) $y = \frac{1}{\frac{1}{2}x}$, $y = 2\left(\frac{1}{x}\right)$; both equations simplify to $y = \frac{2}{x}$

15. Translation 4 units left, then horizontal compression factor $\frac{1}{2}$; yes; check students' parent function investigations

Lesson 1.8, pp. 70–73

1. A: vertical stretch, factor 5; B: reflection in y-axis; C: horizontal compression, factor 5; D: translation 2 units right; E: translation 4 units up

2. Divide the x-coordinates by 3: C; Multiply the y-coordinates by 5: A; Multiply the x-coordinates by -1: B; Add 4 to the y-coordinate: E; Add 2 to the x-coordinate: D

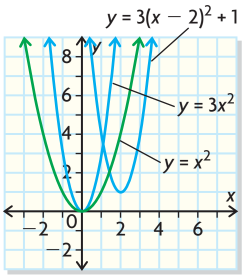
Answers

3.

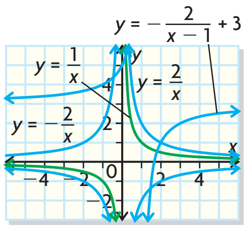
$f(x)$	$f(3x)$	$f(-3x)$	$5f(-3x)$	$5f[-3(x-2)]+4$
$(1, 1)$	$\left(\frac{1}{3}, 1\right)$	$\left(-\frac{1}{3}, 1\right)$	$\left(-\frac{1}{3}, 5\right)$	$\left(1\frac{2}{3}, 9\right)$

4. **a)** Vertical stretch, factor 3, then translation 1 unit down
b) Translation 2 units right and 3 units up
c) Horizontal compression, factor $\frac{1}{2}$, then translation 5 units down
d) Reflection in x-axis, horizontal stretch with factor 2, and then translation 2 units down
e) Vertical compression, factor $\frac{2}{3}$, then translation 3 units left and 1 unit up
f) Vertical stretch with factor 4, reflection in y-axis, and then translation 4 units down

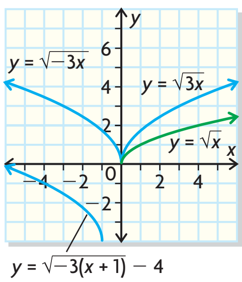
5. **a)**



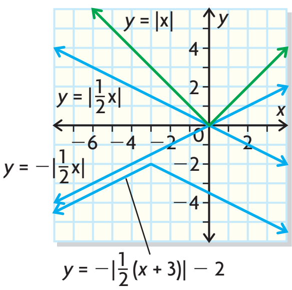
c)



b)

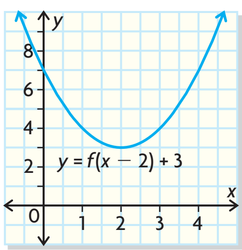


d)



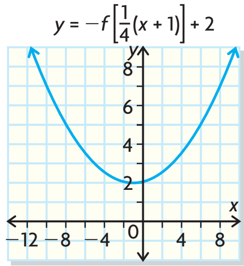
6. **a)** Horizontal stretch, factor 3, then translation 4 units left
b) Vertical stretch with factor 2, reflection in y-axis, and then translation 3 units right and 1 unit up
c) Reflection in x-axis, vertical stretch with factor 3, horizontal compression with factor $\frac{1}{2}$, then translation 1 unit right and 3 units down

7. **a)**



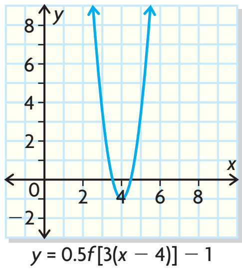
domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 3\}$

b)



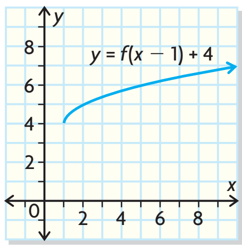
domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 2\}$

c)



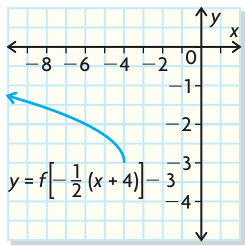
domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq -3\}$

8. **a)**



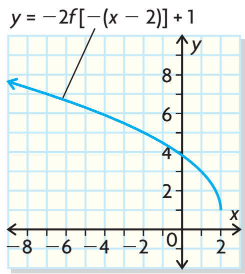
domain $= \{x \in \mathbf{R} \mid x \geq 1\}$, range $= \{y \in \mathbf{R} \mid y \geq 4\}$

b)



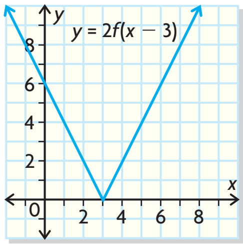
domain $= \{x \in \mathbf{R} \mid x \leq -4\}$, range $= \{y \in \mathbf{R} \mid y \geq -3\}$

c)



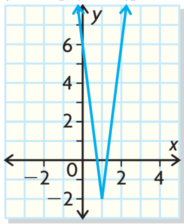
domain $= \{x \in \mathbf{R} \mid x \leq 2\}$, range $= \{y \in \mathbf{R} \mid y \leq 1\}$

9. a)



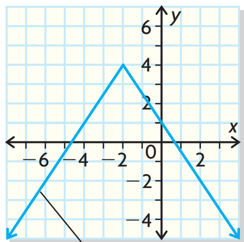
domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$

b) $y = 4f[2(x-1)] - 2$



domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -2\}$

c)

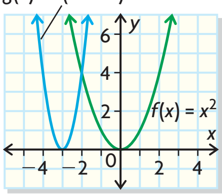


$y = -\frac{1}{2}f[3(x+2)] + 4$

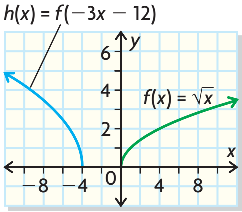
domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 4\}$

10. a) Translation right 2
b) Translation up 2
c) Vertical compression, factor 0.5
d) Vertical stretch, factor 2
e) Horizontal compression, factor 0.5
f) Reflection in x-axis

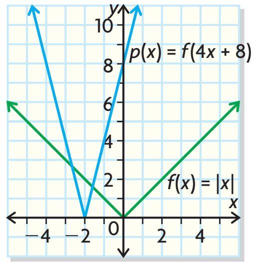
11. $g(x) = f(2x + 6)$



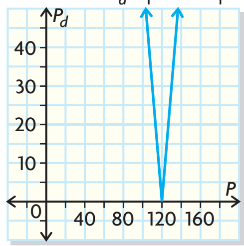
12. $h(x) = f(-3x - 12)$



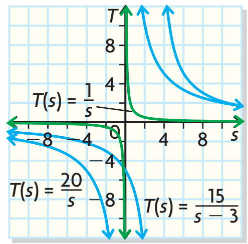
13.



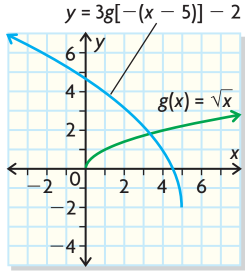
14. $P_d = |P - 120|$



15.



16. $y = 3g[-(x-5)] - 2$

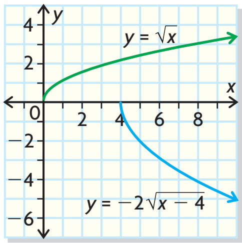


$y = 3\sqrt{-(x-5)} - 2$

17. $g(x) = 3f[-(x+1)] + 2$

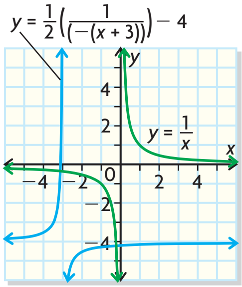
18. a) C; parent graph is $y = \frac{1}{x}$, asymptotes are translated 2 units right and 1 unit up, and graph has been reflected in one of the axes
b) E; parent graph is $y = |x|$, and vertex is translated 3 units right and 2 units down
c) A: parent graph is $y = \sqrt{x}$, graph has been reflected in y-axis, and vertex is translated 3 units left and 2 units down
d) G: parent graph is $y = x^2$, and vertex is translated 2 units right and 3 units down
e) F; parent graph is $y = \frac{1}{x}$, asymptotes are translated 3 units down, and graph has been reflected in one of the axes
f) D; parent graph is $y = |x|$, graph has been reflected in y-axis, and vertex is translated 4 units left and 2 units up
g) H: parent graph is $y = \sqrt{x}$, graph has been reflected in x- and y-axes, and vertex is translated 1 unit right and 1 unit up
h) B: parent graph is $y = x^2$, graph has been reflected in y-axis, and vertex is translated 4 units left and 1 unit up

19. a) $a = -2, k = 1, c = 0, d = 4$



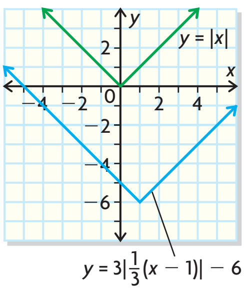
domain $= \{x \in \mathbf{R} \mid x \geq 4\}$, range $= \{y \in \mathbf{R} \mid y \leq 0\}$

b) $a = \frac{1}{2}, k = -1, c = -3, d = -4$



domain $= \{x \in \mathbf{R} \mid x \neq -3\}$, range $= \{y \in \mathbf{R} \mid y \neq -4\}$

c) $a = 3, k = \frac{1}{3}, c = -6, d = 1$



domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -6\}$

20. a) $2, -5$ b) $2, -5$ c) $-\frac{2}{3}, 1\frac{2}{3}$ d) $-4, 3$

21. A. Sketch parent function; B. Apply reflections in x-axis if $a < 0$ and in y-axis if $k < 0$; apply vertical stretch or compression with factor $|a|$, and stretch or compression with factor $\frac{1}{|k|}$; D. Translate c units right (or $-c$ units left if $c < 0$) and d units up (or $-d$ units down if $d < 0$). Transformations in steps B and C can be done in any order, but must precede translation in step D.

22. a) Reflection in x-axis, vertical compression factor $\frac{1}{4}$ [or horizontal stretch factor 2], and then translation 3 units left and 1 unit up

b) $y = -\frac{1}{4}(x+3)^2 + 2$ $\left[\text{or } y = -\left[\frac{1}{2}(x+3)\right]^2 + 2\right]$

23. Graphs are both based on a parabola, but open in different directions, and graph of $g(x)$ is only an upper half-parabola. Reflect right half of graph of $f(x)$ in the line $y = x$.

Chapter Review, pp. 76–77

1. a) domain $= \{-3, -1, 0, 4\}$, range $= \{0, 1, 5, 6\}$; not a function, because two y-values are assigned to $x = 0$

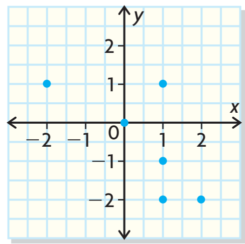
b) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$; function, because each x-value has only one y-value assigned

c) domain $= \{x \in \mathbf{R} \mid x \geq -4\}$, range $= \{y \in \mathbf{R}\}$; not a function, because each $x > -4$ has two y-values assigned

d) domain $= \{x \in \mathbf{R} \mid -4 \leq x \leq 4\}$, range $= \{y \in \mathbf{R} \mid -4 \leq y \leq 4\}$; not a function, because each x except ± 4 has two y-values assigned

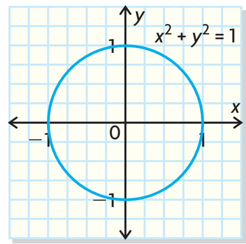
2. Vertical-line test

a)



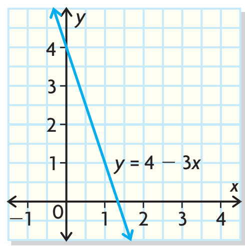
Not a function

d)



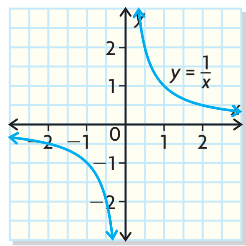
Not a function

b)



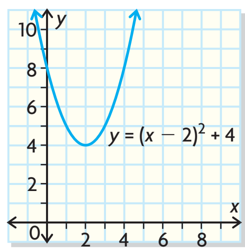
Function

e)



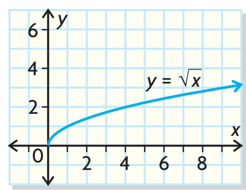
Function

c)



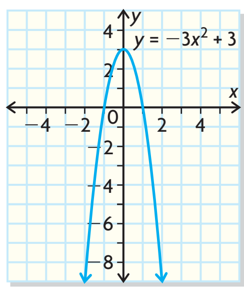
Function

f)



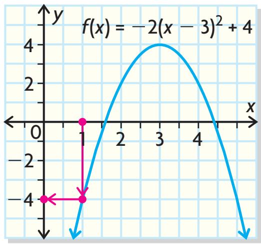
Function

3. Answers may vary; for example:



4. a) -7 b) -5 c) -2 d) $4b^2 + 6b - 5$ e) $-8a - 1$ f) 1 or -2

5. **a), b)**



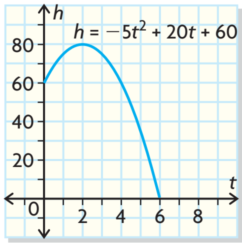
a) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 4\}$

b) $f(1)$ represents the y-coordinate corresponding to $x = 1$.

c) **i)** 2 **ii)** -1 **iii)** $-2(-x - 2)^2 + 4$

6. 5, -1

7. **a)**



b) domain $= \{t \in \mathbf{R} \mid 0 \leq t \leq 6\}$,
range $= \{h \in \mathbf{R} \mid 0 \leq h \leq 80\}$

c) $h = -5t^2 + 20t + 60$

8. **a)** domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 3\}$

b) domain $= \{x \in \mathbf{R} \mid x \geq -2\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$

9. **a)** $A(w) = \left(\dfrac{540 - 3w}{2}\right)w$

b) domain $= \{w \in \mathbf{R} \mid 0 < w < 180\}$,
range $= \{A \in \mathbf{R} \mid 0 < A \leq 12\,150\}$

c) $l = 270$ m, $w = 90$ m

10. **a)** Graph $y = 2x - 5$, and reflect it in the line $y = x$ to get graph of inverse. Use graph to determine the slope-intercept form of inverse; slope is 0.5 and y-intercept is 2.5, so $f^{-1}(x) = 0.5x + 2.5$.

b) Switch x and y, then solve for y:

$$y = \frac{x + 3}{7}$$
$$x = \frac{y + 3}{7}$$
$$7x = y + 3$$
$$7x - 3 = y$$
$$f^{-1}(x) = 7x - 3$$

c) Reverse operations: for f, divide by 2 and subtract from 4, so for f^{-1}, subtract from 4 (operation is self-inverse) and multiply by 2. Therefore, $f^{-1}(x) = 2(4 - x)$.

11. **a)** $f(x) = 30x + 15\,000$

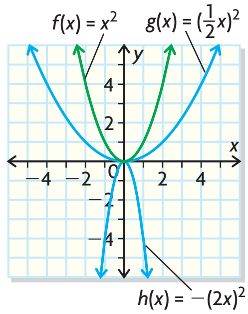
b) domain $= \{x \in \mathbf{R} \mid x \geq 0\}$, range $= \{y \in \mathbf{R} \mid y \geq 15\,000\}$; number of people cannot be negative, and income cannot be less than corporate sponsorship

c) $f^{-1}(x) = \dfrac{x - 15\,000}{30}$; domain $= \{x \in \mathbf{R} \mid x \geq 15\,000\}$

12. **a)** $y = \sqrt{4x}$

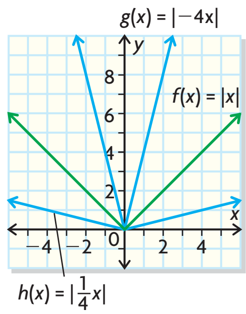
b) $y = \dfrac{1}{-\frac{1}{5}x}$

13. **a)**



$f(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$g(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 0\}$

b)



$f(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$g(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$

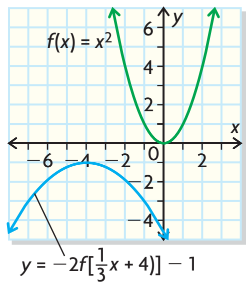
14. **a)** Yes; translations must be done last.

b) Yes: vertical stretch with factor 2, translation 4 units down, and translation 3 units right

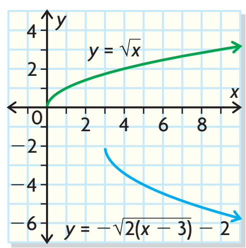
15. $(-5, 10)$

16. **a)** Reflection in x-axis, vertical stretch factor 2, horizontal stretch factor 3, then translation left 4 and down 1

b)

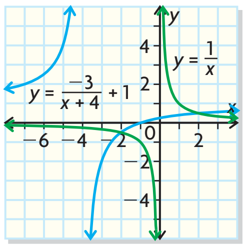


17. **a)** $y = -\sqrt{2(x - 3)} - 2$



domain $= \{x \in \mathbf{R} \mid x \geq 3\}$, range $= \{y \in \mathbf{R} \mid y \leq -2\}$

b) $y = \dfrac{-3}{x+4} + 1$



domain $= \{x \in \mathbf{R} \mid x \neq -4\}$, range $= \{y \in \mathbf{R} \mid y \neq 1\}$

18. **a)** 4, −3 **b)** 4, −3 **c)** −8, 6 **d)** −5, 2

19. **a)** domain $= \{x \in \mathbf{R} \mid x > -4\}$, range $= \{y \in \mathbf{R} \mid y < -2\}$
b) domain $= \{x \in \mathbf{R} \mid x < 4\}$, range $= \{y \in \mathbf{R} \mid y < -1\}$
c) domain $= \{x \in \mathbf{R} \mid x > -5\}$, range $= \{y \in \mathbf{R} \mid y < 1\}$
d) domain $= \{x \in \mathbf{R} \mid x < -1\}$, range $= \{y \in \mathbf{R} \mid y < 3\}$

Chapter Self-Test, p. 78

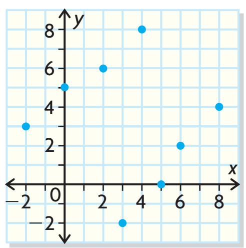
1. **a)** domain $= \{-5, -2, 0, 3\}$, range $= \{-1, 1, 7\}$; function, because each x-value has only one y-value assigned
b) domain $= \{x \in \mathbf{R} \mid x \geq -2\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$; function, same reason as part (a)

2. **a)** $f(x) = 0.004x + 0.65$, $g(x) = 0.001x + 3.50$
b) f: domain $= \{x \in \mathbf{R} \mid x \geq 0\}$, range $= \{y \in \mathbf{R} \mid y \geq 0.65\}$; g: domain $= \{x \in \mathbf{R} \mid x \geq 0\}$, range $= \{y \in \mathbf{R} \mid y \geq 3.50\}$
c) 950 h
d) Regular bulb costs $3.72 more than fluorescent.

3. **a)** domain $= \{x \in \mathbf{R} \mid x \neq 2\}$, range $= \{y \in \mathbf{R} \mid y \neq 0\}$
b) domain $= \{x \in \mathbf{R} \mid x \leq 3\}$, range $= \{y \in \mathbf{R} \mid y \geq -4\}$
c) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 3\}$

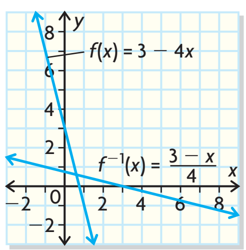
4. The inverse of a linear function is either the linear function obtained by reversing the operations of the original function, or if the original function is $f(x) = c$ constant, the relation $x = c$. Domain and range are exchanged for the inverse.

5. **a)** $\{(3, -2), (5, 0), (6, 2), (8, 4)\}$



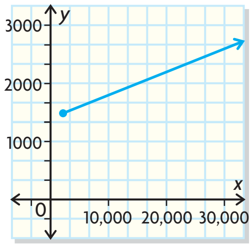
Function: domain $= \{-2, 0, 2, 4\}$, range $= \{3, 5, 6, 8\}$; inverse: domain $= \{3, 5, 6, 8\}$, range $= \{-2, 0, 2, 4\}$

b) $f^{-1}(x) = \dfrac{3-x}{4}$

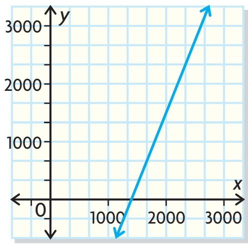


Function: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$; inverse: domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$

6. **a)** $f(x) = 0.04(x - 2500) + 1500$



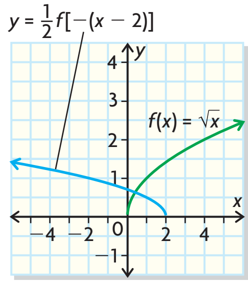
b) $f(x) = 0.04(x - 2500) + 1500$ for $x \geq 2500$
c) $f^{-1}(x) = 25(x - 1500) + 2500$



d) $f^{-1}(x) = 25(x - 1500) + 2500$ for $x \geq 1500$
e) $f^{-1}(1740) = 25(1740 - 1500) + 2500 = \$51\,000$

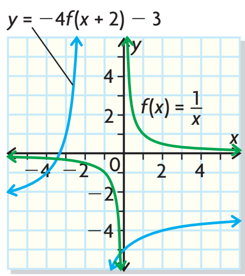
7. **a)** $\dfrac{1}{5}$ **b)** −3

8. **a)** $a = \dfrac{1}{2}$, $k = -1$, $c = 0$, $d = 2$



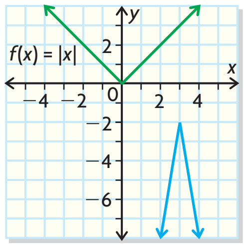
domain $= \{x \in \mathbf{R} \mid x \leq 2\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$

b) $a = -4$, $k = 1$, $c = -3$, $d = -2$



domain $= \{x \in \mathbf{R} \mid x \neq -2\}$, range $= \{y \in \mathbf{R} \mid y \neq -3\}$

c) $a = -\frac{3}{2}, k = 4, c = -2, d = 3$



$y = -\frac{3}{2}f[4(x-3)] - 2$

domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq -2\}$

Chapter 2

Getting Started, p. 82

1.

	Type	Degree
a)	binomial	1
b)	monomial	0
c)	binomial	2
d)	monomial	2
e)	trinomial	2

2. a) $9x - 2$ c) $8x^2 - 2x - 15$
b) $2x^2 - 4x - 9$ d) $4x^2 - 4x + 1$

3. Factoring is the opposite of expanding. To expand a polynomial, you multiply using the distributive property. To factor, you try to determine the polynomials that multiply together to give you the given polynomial. e.g., $(x + 2)(3x - 1) = 3x^2 + 5x - 2$

$\xrightarrow{\text{expanding}}$

$\xleftarrow{\text{factoring}}$

4. a) $2xy^3(3 - 4x)$ d) $(3 - 5x)(3 + 5x)$
b) $(a - 2)(a - 5)$ e) not possible
c) $(4n + 5)(3n - 2)$ f) $(y - 9)(y + 4)$

5. a) $\frac{11}{12}$ b) $-\frac{1}{2}$ c) $\frac{8}{15}$ d) -10

6. a) $\frac{5x^5}{6}$ b) $\frac{15}{2x^2}$ c) $8x^3y^5$ d) $5x^3y^2$

7. a) $\{x \in \mathbf{R}\}$ d) $x = 0, \{x \in \mathbf{R} | x \neq 0\}$
b) $\{x \in \mathbf{R}\}$ e) $x = 4, \{x \in \mathbf{R} | x \neq 4\}$
c) $x < 0, \{x \in \mathbf{R} | x \geq 0\}$ f) $x < -10, \{x \in \mathbf{R} | x \geq -10\}$

8.

Definition:	**Characteristics:**
A polynomial is any algebraic expression that contains one or more terms.	• usually contains variables • can contain both like and unlike terms • exponents must be whole numbers
Polynomial	
Examples: $3x^2$ $4x - 3$ $5x^3 - 2xy + 6y$	**Non-examples:** $\frac{5-x}{3+x}$ $5x^{-2} + 4x + 3$ $\sqrt{x}$

Lesson 2.1, pp. 88–90

1. a) $4x^2 - 8x + 8$ b) $2x^2 - 4$ c) $2x^2 - x$

2. $f(x) = 7x - 2$
$g(x) = 7x - 2$

3. Answers may vary. For example, $f(1) = -10$; $g(1) = -20$

4. a) $9a - 5c + 5$ d) $3x^2 - 9x - 3$
b) $2x^2 + 3x + 4y + z$ e) $2x^2 - 5xy + 2y^2$
c) $3x - 3y + 1$ f) $5x^2 - y^2 - 1$

5. a) $m - 4n + p + 7$ d) $-2m^2 - 5mn + 15n^2$
b) $-8m - 4q + 1$ e) $-x^2 + 4y^2 + 15$
c) $-a^3 + 4a^2 - 2a$ f) $3x^2 + 50$

6. a) $11x - 7y$ d) $2x + xy - 4y + yz$
b) $4x^2 - 16x - 3$ e) $\frac{3}{10}x + \frac{4}{3}y$
c) $4x^2$ f) $\frac{1}{12}x + \frac{1}{4}y + 1$

7. i) $(3x^2 - x) - (5x^2 - x)$
$= -2x^2$
$\neq -2x^2 - 2x$

ii) Answers will vary. For example, if $x = 1$,
$(3x^2 - x) - (5x^2 - x)$
$= (3 - 1) - (5 - 1)$
$= -2$
but $-2x^2 - 2x$
$= -2 - 2$
$= -4$

8. a) $f(x) = 2x^2 + 4x - 9$ and $g(x) = 2x^2 + 4x - 5 \therefore f(x) \neq g(x)$
b) $s_1(1) = 27$ and $s_1(1) = 9 \therefore s_1(t) \neq s_2(t)$
c) e.g., if $x = -1$, then $y_1 = 2$ and $y_2 = 0$
$\therefore y_1 \neq y_2$
d) $f(n) = 2n^2 + 2n - 9$ and $g(n) = 2n^2 + 2n - 9$
$\therefore f(n) = g(n)$
e) $p = 1, q = 1, y_1 = 9; y_2 = 5 \therefore y_1 \neq y_2$
f) $f(2) = 6$
$g(2) = 14$
$\therefore f(m) \neq g(m)$

9. Answers will vary. For example, $f(x) = 2x$ and $g(x) = x^2$

10. a) $25 - x - y$ b) $5x + y + 25$ c) 95

11. $3x + 3y + 2$

12. a) $P(x) = -50x^2 + 2350x - 9500$ b) \$11 500

13. a) cannot be determined d) cannot be determined
b) cannot be determined e) equivalent
c) not equivalent

14. a) yes b) Replace variables with numbers and simplify.

15. a) $x + (x + 7) + (x + 14) + (x + 15) + (x + 16) = 5(x + 14) - 18$
b) 26
c) $5x - 18$

16. a) $19 + 20 + 21 + 22 + 23$
b) $n = (m - 2) + (m - 1) + m + (m + 1) + (m + 2)$
c) $10 + 11 + 12 + 13 + 14 + 15 + 16$

17. a) both functions are linear; a pair of linear functions intersect at only one point, unless they are equivalent; since the functions are equal at two values, they must be equivalent
b) both functions are quadratic; a pair of quadratic functions intersect at most in two points, unless they are equivalent; since the functions are equal at three values, they must be equivalent

Lesson 2.2, pp. 95–97

1. a) $6x^2 - 10x^3 + 8xy$ c) $x^2 + 8x + 16$
   b) $6x^2 + 7x - 20$ d) $x^3 + 3x^2 - x - 3$
2. a) no; for $x = 1$, left side is 25, right side is 13
   b) $9x^2 + 12x + 4$
3. a) $6x^3 + 24x^2 + 14x - 20$ b) same as (a)
4. a) $25x^3 + 15x^2 - 20x$ d) $n^2 - 13n + 72$
   b) $2x^2 - 7x - 30$ e) $-68x^2 - 52x - 2$
   c) $16x^2 - 53x + 33$ f) $5a^2 - 26a - 37$
5. a) $4x^3 - 100x$ d) $-6x^3 + 31x^2 - 23x - 20$
   b) $-2a^3 - 16a^2 - 32a$ e) $729a^3 - 1215a^2 + 675a - 125$
   c) $x^3 - 5x^2 - 4x + 20$ f) $a^2 - 2ad - b^2 + 2bc - c^2 + d^2$
6. a) yes c) no e) no
   b) yes d) yes f) yes
7. All real numbers. Expressions are equivalent.
8. a) Both methods give $285x^2 + 209x - 266$.
   b) Answer may vary. For example, I preferred multiplying the last two factors together first. Multiplying the first two factors together first meant that I had to multiply larger numbers in the second step.
9. a) $16x^2 + 8\pi x$ b) $8\pi x^3 + 4\pi x^2 - 2\pi x - \pi$
10. a) yes
    b) no, $x - 3 = -(3 - x)$. A negative number squared is positive (the same); a negative number cubed is negative (different).
11. a) $x^4 + 4x^3 + 2x^2 - 4x + 1$ c) $x^6 - x^4 - 2x^3 - 3x^2 - 2x - 1$
    b) $8 - 12a + 6a^2 - a^3$ d) $-16x^2 + 43x - 13$
12. 0
13. a) $\frac{1}{2}mv^2 + \frac{1}{2}xv^2$ b) $\frac{1}{2}mv^2 + mvy + \frac{1}{2}my^2$
14. a) 6 $2 \times 3 = 6$; $(x^7 + x^6)(x^9 + x^4 + 1)$ has 6 terms
    b) Multiply the number of terms in each polynomial
15. a) i) 8 ii) 12 iii) 6 iv) 1
    b) i) 8 ii) 96 iii) 384 iv) 512
    c) i) 8 ii) $12(n - 2)$ iii) $6(n - 2)^2$ iv) $(n - 2)^3$
    d) same answers
16. a) Answers may vary. For example,
    115: $11^2 + 11 = 132$
    $115^2 = 13\ 225$
    b) $(10x + 5)^2 = 100x^2 + 100x + 25$ and $(x^2 + x)100 + 25$ are both the same

Lesson 2.3, pp. 102–104

1. a) $(x - 9)(x + 3)$ c) $(2x + 5)^2$
   b) $(5x + 7)(5x - 7)$ d) $(2x + 1)(3x - 2)$
2. a) $(c - d)(a + b)$
   b) $(x + y + 1)(x - y + 1)$
   c) $(x - y - 5)(x + y + 5)$
3. a) $(x - 7)(x + 4)$ c) $(3x - 7)^2$
   b) $(6x - 5)(6x + 5)$ d) $(2x + 3)(x - 5)$
4. a) $2x(2x - 1)(x - 1)$ d) $(x + 1)(7x^2 - x + 6)$
   b) $3xy^2(x^2 - 3xy^2 + y)$ e) $(x - 4)(3x - 1)$
   c) $(a + 1)(4a - 3)$ f) $-2t(t - 13)(t - 1)$
5. a) $(x - 7)(x + 2)$ d) $(2y + 7)(y - 1)$
   b) $(x + 5y)(x - y)$ e) $(4a - 7b)(2a + 3b)$
   c) $6(m - 6)(m - 9)$ f) $2(2x + 5)(4x + 9)$
6. a) $(x - 3)(x + 3)$ d) $(3y - 8)(3y + 2)$
   b) $(2n - 7)(2n + 7)$ e) $-12(2x - 3)(x - 3)$
   c) $(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$ f) $-(pq + 9)(pq - 9)$
7. a) $(x + y)(a + b)$ d) $(4 - x)(x - 2)$
   b) $(b + 1)(2a - 3)$ e) $(a - b + 5)(a + b + 5)$
   c) $(x + 1)^2(x - 1)$ f) $2(m + n)(m - n + 5)$
8. no; $(x - y)(x^2 + y^2) = x^3 - x^2y + xy^2 - y^3$
9. a) $(x - 3)(2x - 7)$ d) $(y - x + 7)(y + x - 7)$
   b) $(x + 5)(y + 6)$ e) $3(2x - 7)(x - 2)$
   c) $(x - 1)(x - 2)(x + 2)$ f) $(2m^2 - 5)(6m - 7)$
10. $f(n) = (n^2 + 3)(2n + 1)$. Since n is a natural number, $2n + 1$ is always odd and greater than 1. Because $(2n + 1)$ is a factor of $f(n)$, the condition is always true.
11. a) $a^2 = (c - b)(c + b)$ b) $a = \sqrt{33}$ m, $b = 4$ m, $c = 7$ m
12. Saturn
    a) i) $\pi(r_2 - r_1)(r_2 + r_1)$
    ii) $\pi(r_3 - r_1)(r_3 + r_1)$
    iii) $\pi(r_3 - r_2)(r_3 + r_2)$
    b) The area of the region between the inner ring and outer ring
13. 1. Always do common factor first.
    2. Do difference of squares for 2 square terms separated by a minus sign.
    3. Do simple trinomials for 3 terms with $a = 1$ or a prime.
    4. Do complex trinomials for 3 terms with $a \neq 1$ or a prime.
    5. Do grouping for a difference of squares for 4 or 6 terms with 3 or 4 squares.
    6. Do incomplete squares for 3 terms when you can add a square to allow factoring; e.g.,
    1. $5x + 10 = 5(x + 2)$
    2. $4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$
    3. $x^2 - x - 20 = (x - 5)(x + 4)$
    4. $12x^2 - x - 20 = (4x + 5)(3x - 4)$
    5. $x^2 + 6x + 9 - y^2 = (x + 3 + y)(x + 3 - y)$
    6. $x^4 + 5x^2 + 9 = (x^2 + 3 + x)(x^2 + 3 - x)$
14. a) $(x^2 - 3x + 6)(x^2 + 3x + 6)$ b) $(x^2 - 3x - 7)(x^2 + 3x - 7)$
15. a) $x^4 - 1 = (x - 1)(x^3 + x^2 + x + 1)$
    b) $x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$
    c) $x^n - 1 = (x - 1)\ (x^{(n-1)}y^0 + x^{(n-2)} + \ldots + x^0)$
    d) $x^n - y^n = (x - y)(x^{(n-1)})y^0 + x^{(n-2)} + x^{(n-3)}y^2 + \ldots + x^0y^{(n-1)}$
16. a) $2^6 + 2^4 + 2^2 - 2^4 - 2^2 - 2^0 = 2^6 - 1$;
    $2^6 + 2^3 - 2^3 - 2^0 = 2^6 - 1$
    b) $35 = 5 \times 7$
    $\therefore 2^{35} - 1 = (2^5 - 1)(2^{30} + 2^{25} + 2^{20} + 2^{15} + 2^{10} + 2^5 + 2^0)$ or
    $2^{35} - 1 = (2^7 - 1)(2^{28} + 2^{21} + 2^{14} + 2^7 + 2^0)$
    c) Yes. If m is composite, then let $m = a \times b$, where a and/or b cannot equal 1.
    $2^{m-1} = 2^{ab-1} = \frac{2^{ab}}{2^1} = \frac{(2^a)^b}{2^1}$
    This result will always have two factors: $(2^{a-1})(2^a)^{b-1}$
    Neither of these will ever equal 1, so 2^{m-1} is composite.

Mid–Chapter Review, p. 107

1. a) $6a^2 - 7$ d) $18x^2 - 12x + 3xy$
   b) $x^2 - 11xy + 9y^2$ e) $6a^2 + 20a - 10ab + 6b - 18$
   c) $-6c^2 + cd - 8d^2 - d$ f) $14x^3 + 6x^2 - 48x + 9xy$
2. a) No, e.g., $g(0) = -32$ and $h(0) = 32$
   b) Yes, $f(x) = 2x^2 - 7x + 5$ and $g(x) = 2x^2 - 7x + 5$
   c) Yes
   d) No, e.g., $b(0) = 1$ and $c(0) = -1$
3. The resulting polynomial will be a cubic because you add the exponents of the highest terms (linear = 1 and quadratic = 2).
4. $3x - 94$
5. a) $6x^2 - 38x + 40$
   b) $27x^3 - 27x^2 + 9x - 1$
   c) $-2x^4 + 12x^3 - 34x^2 + 48x - 32$
   d) $11x^2 - 25x + 11$
   e) $-30x + 30$
   f) $-x^3 + 3x^2y - 3xy^2 + y^3$

6. **a)** 2 **b)** $2w - l - 2$

7. **a)** $(x-2)(x-3)$ **b)** $(x-7)(x-4)$ **c)** $(3a+2)(a-4)$ **d)** $3(5x+1)(2x-1)$ **e)** $(4-5x)(4+5x)$ **f)** $-(5+2a)(13-2a)$

8. **a)** $(n-3m)(2+5n)$ **b)** $(y-3-x)(y-3+x)$ **c)** $(y-b)(1-y+b)$ **d)** $2(x+2-2y)(x+2+2y)$ **e)** $(w-a)(w+b)$ **f)** $(b+6)(a+b)$

9. $20x + 8$

10. Many answers are possible; for example, $k = -60, -42, -34, -20, -14, -4, 0, 6, 8, 10$. All answers for k are of the form $k = 11b - 3b^2$, $b \in \mathbf{I}$.

Lesson 2.4, pp. 112–114

1. **a)** $3 - 2t$ **b)** $\frac{3}{2x}, x \neq 0$ **c)** $\frac{b^2}{3a^2}, a \neq 0, b \neq 0$

2. **a)** $\frac{5}{x-3}, x \neq -3, 3$ **b)** $3, x \neq \frac{3}{2}$ **c)** $\frac{2ab}{2a-b}, a \neq \frac{1}{2}b$

3. **a)** $\frac{x-3}{x+2}, x \neq -2, 1$

b) $\frac{(x+1)}{5x-4)}, x \neq \frac{4}{5}$

c) $\frac{x-5y}{x+3y}, x \neq 2y, -3y$

4. **a)** $2x^2 - x + 3, x \neq 0$ **b)** $-\frac{x^2}{2y}, x \neq 0, y \neq 0$ **c)** $-\frac{2}{5t}, t \neq 0, 5$ **d)** $\frac{1}{a}(3a^2 - 2b), a \neq 0, \sqrt{\frac{2}{3}b}, b \neq 0$ **e)** $-\frac{2x}{3}, x \neq -5$ **f)** $-\frac{2}{3}, a \neq 0, b \neq 3$

5. **a)** $\frac{1}{a} - 1, a \neq -4, 1$ **b)** $-x - \frac{3}{5}, x \neq 3$ **c)** $\frac{x-3}{x+5}, x \neq -5, 2$ **d)** $\frac{2+p}{5+p}, p \neq -5, 5$ **e)** $\frac{t-4}{t(t-3)}, t \neq 0, 3$ **f)** $\frac{3t-2}{t-1}, t \neq -\frac{1}{2}, 1$

6. **a)** the denominator equals 0; $\mathbf{R}, x \neq 0$ **b)** $\mathbf{R}, x \neq 0, 2$ **c)** $\mathbf{R}, x \neq -5, 5$ **d)** $\mathbf{R}, x \neq -1, 1$ **e)** $\mathbf{R}$ **f)** $\mathbf{R}, x \neq -1, 1$

7. **a)** yes **b)** no, not the same domain

8. **a)** $\frac{2}{5}, x > -\frac{1}{3}$

b) Because $x \leq -\frac{1}{3}$ would imply sides of length 0 or less, therefore this would not be a triangle.

9. $\frac{25}{24}$

10. **a)** $(t+1)(4t-1), t \neq 0$ **b)** $\frac{5}{4(2x-1)}, x \neq \frac{1}{2}$ **c)** $\frac{5-x}{4+x}, x \neq -4, 4$ **d)** $\frac{2x+y}{x-y}, x \neq y$

11. $\frac{3w}{7}$

12. Answers will vary. For example,
$\frac{(3x-2)(x-4)}{(x-4)}; \frac{5(3x-2)(x-4)}{5(x-4)}$

13. Answers will vary. For example, $\frac{5}{(x-1)(x-2)(x-3)}$

14. **a)** Answers will vary. For example,

i) $\frac{(2x+1)(x+1)}{(x-4)(x+1)}$ **ii)** $\frac{x(2x+1)}{x(x-4)}$ **iii)** $\frac{(2x+1)(3x-2)}{(x-4)(x+1)}$ **iv)** $\frac{(2x+1)(2x+1)}{(x-4)(2x+1)}$

b) yes; $\frac{(2x+1)(x-4)}{(x-4)^2}$

15. yes; $\frac{(x+1)(x+2)}{(x+1)(x+3)}$ and $\frac{(x+4)(x+2)}{(x+4)(x+3)}$

16. **a)** $\lim_{x \to \infty} f(x) = 0$ **b)** $\lim_{x \to \infty} g(x) = \frac{4}{5}$ **c)** $\lim_{x \to \infty} h(x) = -\infty$

17. **a)** $\frac{2(3t^2-1)}{(1+t^2)^3}$; no restrictions **b)** $\frac{(2x+1)(6x-11)}{(3x-2)^4}; x \quad \frac{2}{3}$

Lesson 2.5, p. 116

1. Answers will vary. For example, $y = \frac{2(x-2)(x+3)}{(x-2)(x+3)}$

2. Answers will vary. For example, $y = \frac{1}{x^2}$

3. Answers will vary. For example, $y = \frac{2}{x(x-2)} + 2$

Lesson 2.6, pp. 121–123

1. **a)** $\frac{5}{12}$ **b)** $\frac{3x^3}{20y}, y \neq 0$ **c)** $\frac{(x+1)}{2}, x \neq -4, 5$ **d)** $\frac{(3x)}{5}, x \neq -\frac{1}{2}, 0$

2. **a)** $\frac{10}{(3x)}, x \neq 0$ **b)** $\frac{5}{4}, x \neq 7$ **c)** $\frac{(3x)}{(x-7)}, x \neq -2, 6, 7$ **d)** $-6(x-1), x \neq -1, 2$

3. **a)** $\frac{(x^2-1)}{(x+3)^2}, x \neq -3, -1, 1$

b) $\frac{2}{(x-2)(x-5)}, x \neq -5, 2, 5$

4. **a)** $6x, x \neq 0$ **b)** $\frac{5}{6a}, a \neq 0$ **c)** $\frac{3x}{2y^2}, x \neq 0, y \neq 0$ **d)** $\frac{7a}{3}, a \neq 0, b \neq 0$

5. **a)** $\frac{(x-1)}{9}, x \neq -1$ **b)** $3, a \neq -2, 2$ **c)** $-\frac{8x}{3}, x \neq 0, 2$ **d)** $\frac{21(m+4)(m+2)}{5(2m+1)}, m \neq -4, -2, -\frac{1}{2}$

6. **a)** $\frac{2(x+1)}{(x+2)(x+3)}, x \neq -3, -2, 3$

b) $\frac{2(n-2)}{5}, n \neq -2, 2, 3, 4$

c) $\dfrac{(x-1)(3x-1)(2x-1)}{(x-3)(x+2)(4x+5)}, x \neq -2, -\dfrac{5}{4}, -\dfrac{1}{2}, 3$

d) $\dfrac{-3(3y-2)}{2(3y+2)}, y \neq -\dfrac{2}{3}, 3$

7. a) $\dfrac{x+y}{x+7y}, x \neq -y, -7y, y, 4y$

b) $\dfrac{(a-3b)}{2a}, a \neq 0, 2b, 3b, 5b$

c) $\dfrac{4(5x-y)}{3x(3x-y)}, x \neq 0, -\dfrac{1}{2}y, -\dfrac{1}{3}y, \dfrac{1}{3}y$

d) $\dfrac{-(3m-n)(m-n)}{2(m+n)}, m \neq \dfrac{1}{7}n, -\dfrac{2}{5}n, -n$

8. $\dfrac{3x^2}{(x-1)(x+2)}$, restrictions: $x \neq -3, -2, 0, \dfrac{1}{2}, 1, 2$

9. $\dfrac{10x^2}{(x-9)(x+3)}, x \neq 9, 7, -3$

10. $\dfrac{3p+1}{p-1}, p \neq -1, -\dfrac{1}{3}, 1$

11. If $x = y$, then Liz is dividing by 0.

12. a) Then you can simplify and cancel common factors

b) Sometimes you cannot factor and you need to take into account the factors you cancel because they could make the denominator equal to 0.

c) Yes. Dividing is the same as multiplying by the reciprocal.

13. $\dfrac{m(m+2n)}{(m+n)(4m+n)}, m \neq -\dfrac{n}{3}, -\dfrac{3n}{2}, 2n, n, -n, -2n, -\dfrac{n}{4}$

14. 22 933.7

Lesson 2.7, pp. 128–130

1. a) $\dfrac{19}{12}$

b) $\dfrac{17x}{5}$

c) $\dfrac{35x+4}{28x^3}, x \neq 0$

d) $\dfrac{2x+6}{x^2}, x \neq 0$

2. a) $-\dfrac{1}{9}$

b) $\dfrac{7y}{6}$

c) $\dfrac{25-21x^2}{15x^2}, x \neq 0$

d) $\dfrac{6y-15x}{3xy^2}, x, y \neq 0$

3. a) $\dfrac{8x+18}{(x-3)(5x-1)}, x \neq \dfrac{1}{5}, 3$

b) $\dfrac{2x+1}{(x-3)(x+3)}, x \neq -3, 3$

c) $\dfrac{-4x+22}{(x-3)(x-1)^2}, x \neq 1, 3$

4. a) $\dfrac{13}{8}$ b) $\dfrac{3x+11}{(x-3(x+3)}$ c) $\dfrac{13}{8}$; same

5. a) $\dfrac{5x}{4}$

b) $\dfrac{30+5t^2-6t^3}{10t^4}, t \neq 0$

c) $\dfrac{40xy^3-15x^2y+36}{60y^4}, y \neq 0$

d) $\dfrac{n^2+m^2-m^2n}{mn}, m \neq 0, n \neq 0$

6. a) $\dfrac{9a-8}{a(a-4)}, a \neq 0, 4$

b) $\dfrac{18x-8}{3x-2}, x \neq \dfrac{2}{3}$

c) $\dfrac{12x+43}{(x+4)(x+3)}, x \neq -4, -3$

d) $\dfrac{-2n-18}{(2n-3)(n-5)}, n \neq \dfrac{3}{2}, 5$

e) $\dfrac{10x^2-30x}{(x+4)(x-6)}, x \neq -4, 6$

f) $\dfrac{78x-129}{10(x-3)(2x-3)}, x \neq \dfrac{3}{2}, 3$

7. a) $\dfrac{3x-8}{(x+1)(x-4)}, x \neq -1, 4$

b) $\dfrac{2t^2+3t}{(t-4)(t+4)}, t \neq -4, 4$

c) $\dfrac{8t-1}{(t+3)^2(t-2)}, t \neq -3, 2$

d) $\dfrac{x^2-32x}{(x+2)(x+4)(x-5)}, x \neq 5, -2, -4$

e) $\dfrac{2x^2+7x+23}{(x-3)(x+3)(x-2)}, x \neq -3, 2, 3$

f) $\dfrac{9t^2-14t+2}{4(t-3)(t-4)(t+1)}, t \neq -1, 3, 4$

8. a) $\dfrac{7x+4}{(x+1)(4x+3)^2}, x \neq -1, -\dfrac{3}{4}$

b) $\dfrac{a^2+4a-7}{(a-3)(a-5)(2a+1)}, a \neq -\dfrac{1}{2}, 3, 5$

c) $\dfrac{10x^2-5x+7}{(2x-1)(2x+1)^2}, a \neq -\dfrac{1}{2}, \dfrac{1}{2}$

9. a) $\dfrac{9x^3-10y^2}{15xy}, x \neq 0, y \neq 0$

b) $43x^2-84x-\dfrac{136}{4(x-3)(x+1)(x-2)}, x \neq -1, 2, 3$

c) $\dfrac{p^3+5p^2-25p-65}{(p+5)(p+7)(p-5)}, p \neq -7, -5, -4, 3, 5, 6$

d) $\dfrac{15m^2-2mn-3m-n^2-15n}{(2m+n)(3m+n)}, m \neq -\dfrac{1}{2}n, -\dfrac{1}{3}n, -5n, \dfrac{1}{2}n$

10. a) $\dfrac{23m+20}{10}$

b) $\dfrac{20x-3}{4x^3}, x \neq 0$

c) $\dfrac{-y-7}{(y+1)(y-2)}, y \neq -1, 2$

d) $\dfrac{2x^2+13x+15}{(x+3)(x-2)(x+4)}, x \neq -4, -3, 2$

11. $\dfrac{-s+t-1}{t+s}, t \neq -s$

12. a) $\dfrac{x-300}{6}$ b) $0 \leq x < 300$

13. $\dfrac{2kdx+kx^2}{d^2(d+x)^2}, d \neq 0, -x$

14. a) Answers will vary. For example,

i) $\dfrac{1}{2}+\dfrac{1}{4}$

ii) $\dfrac{1}{3}+\dfrac{1}{4}$

iii) $\dfrac{1}{4}+\dfrac{1}{6}$

b) Factor the quadratic denominators and find the common denominator from these factors.

15. a) $\dfrac{1}{n} - \dfrac{1}{n+1}$
$= \dfrac{1(n+1)}{n(n+1)} - \dfrac{1(n)}{n(n+1)}$
$= \dfrac{n+1-n}{n(n+1)}$
$= \dfrac{1}{n(n+1)}$
b) Answers may vary. For example, 4, 12, 3 or 5, 20, 4

16. a) Let x be the smaller of the two consecutive even or odd numbers. Then $x + 2$ is the larger of the two.
$\dfrac{1}{x} + \dfrac{1}{x+2} = \dfrac{x + (x+2)}{x(x+2)}$
$= \dfrac{2x+2}{x(x+2)}$
$(2x+2)^2 + (x^2+2x)^2$
$= 4x^2 + 8x + 4 + x^4 + 4x^3 + 4x^2$
$= x^4 + 4x^3 + 8x^2 + 8x + 4$
$= (x^2 + 2x + 2)^2$
So, $2x + 2$, $x^2 + 2x$, and $x^2 + 2x + 2$ are Pythagorean triples.
b) Answers may vary. For example, 16, 63, 65

Chapter Review, pp. 132–133

1. a) $12x^2 - 12x + 13$ b) $8a^2 - 6ab + 3b^2$
2. Answers will vary. For example, $f(x) = x^2 + x$, $g(x) = 2x$
3. a) no b) probably c) 32 or 33
4. a) $-84x^2 + 207x - 105$
b) $-3y^4 + 17y^3 - 38y^2 + 23y + 21$
c) $2a^3 + 6a^2b + 6ab^2 + 2b^3$
d) $12x^6 - 36x^5 - 21x^4 + 144x^3 - 60x^2 - 144x + 108$
5. $V = \left(\dfrac{1}{3}\right)\pi(r+x)^2(h+2x)$
$V = \left(\dfrac{1}{3}\right)\pi(2hrx + r^2h + 2r^2x + 4rx^2 + 2x^3 + x^2h)$
6. a) $8x^4 - x^3 + x^2 - 1$
b) $2x^4 + 5x^3 - 10x^2 - 20x + 8$
c) $-13x^3 + 8x^2 - 11x$
d) $-5x^6 - 6x^5 - 2x^4 + 22x^3 - 4x^2$
e) $-20x$
f) $2x^3 + 29x^2 - 4x - 252$
g) $x^4 + 10x^3 + 19x^2 - 30x + 9$
7. a) $6m^2n^2(2n + 3m)$ d) $2(5x + 6)(5x - 6)$
b) $(x - 5)(x - 4)$ e) $(3x - 1)^2$
c) $3(x + 3)(x + 5)$ f) $(2a - 1)(5a + 3)$
8. a) $2x^2y(y^3 - 3x^3y^2 + 4x)$ d) $(5x - 6)(3x - 7)$
b) $(x + 4)(2x + 3)$ e) $(a^2 + 4)(a + 2)(a - 2)$
c) $(x + 2)(x - 5)$ f) $-(m + 4n)(3m + 2n)$
9. a) $-2a^2 - 3c^2$; $b \neq 0$ c) $1 - z$; $x \neq 0, y \neq 0$
b) $3y^3 - 2z^2 + 5$; $x \neq 0$ d) $2r - 3p + 5k$; $m \neq 0, n \neq 0$
10. a) $\dfrac{8xy^3 + 12x^2y^2 - 3x^2}{y}$, $x \neq 0, y \neq 0$
b) $\dfrac{7}{2}$, $a \neq 2b$
c) $\dfrac{1}{m+7}$, $m \neq -7, -3$
d) $\dfrac{2x+1}{2x+3}$, $x \neq -\dfrac{3}{2}, \dfrac{3}{2}$
e) $\dfrac{3x(x-7)}{7(x-3)(x-1)}$, $x \neq 1, 3$
f) $\dfrac{x-y}{x+y}$, $x \neq -\dfrac{1}{3}y, -y$
11. Perhaps, but probably not. e.g., $\dfrac{x+1}{x-1}$ and $\dfrac{x+2}{x-1}$ are not equivalent.
12. a) $\dfrac{y}{2}$; $x \neq 0, y \neq 0$ c) $\dfrac{2b}{3c^2}$; $a \neq 0, b \neq 0, c \neq 0$
b) m; $m \neq 0, n \neq 0$ d) $\dfrac{5}{2p}$; $p \neq 0, q \neq 0$
13. a) $\dfrac{x}{36y}$; $x \neq 0, y \neq 0$
b) $\dfrac{(x-3)(x+1)}{(x-1)}$; $x \neq \pm 1, \pm 2, 5$
c) $\dfrac{x(1-x)}{-y(y+1)}$; $x \neq 0, -1, y \neq 0, \pm 1$
d) $x - y$; $y \neq \pm 2x, -x, -\dfrac{2}{3}x$
14. a) $\dfrac{2}{15x}$; $x \neq 0$
b) $\dfrac{3x-7}{(x+1)(x-1)}$; $x \neq 1, -1$
c) $\dfrac{2(x-2)}{(x+4)(x-1)(x-3)}$; $x \neq -4, 1, 3$
d) $\dfrac{5}{(x-2)(x+3)(x-3)}$; $x \neq \pm 3, 2$
15. a) $\dfrac{3x^2 - 14x + 24}{6x^3}$, $x \neq 0$
b) $\dfrac{7x^2 - 10x}{(x+2)(x-6)}$, $x \neq -2, 6$
c) $\dfrac{3x^2 + 30x}{(x-3)(x+4)(x-2)}$, $x \neq -4, 2, 3$
d) $\dfrac{6}{x+1}$, $x \neq -1, -5, 2$
e) $\dfrac{(x-2y)(x+y)}{(x-y)(x+3y)}$, $x \neq -3y, -y, y, 2y$
f) $\dfrac{3b^2 + 8b - 5}{(b-5)(b+3)}$, $x \neq 5, -3, -6$
16. 65
17. a) $\dfrac{6}{n^2 - 3n + 2}$ or $\dfrac{6}{(n-1)(n-2)}$, $n \neq 0, 1, 2$
b) i) $\dfrac{1}{2}$ ii) 1

Chapter Self-Test, p. 134

1. a) $x^2 - 5x$ b) $4m^2 - 3mn + 5n^2$
2. a) $72a^2 - 198a + 70$ c) $20x^3 - 57x^2 - 11x + 6$
b) $-7x^3y^3 + 10x^4y^2 - 12x^2y^4$ d) $9p^4 + 6p^3 - 11p^2 - 4p + 4$
3. no
4. a) $24n^2 + 48n + 26$ b) 866
5. a) $m(m - 1)$ d) $(x + 3)(3x - 2y - 1)$
b) $(x - 3)(x - 24)$ e) $(y - 2)(5x - 3)$
c) $(5x + y)(3x - 2y)$ f) $(p - m + 3)(p + m - 3)$
6. $x = -1, 1, 4$

7. a) $\frac{14}{3b^2}, a \neq 0, b \neq 0$

b) $\frac{2}{(x-2)(x+3)}, x \neq -3, 2, 4$

c) $\frac{6t-49}{(t+2)(t-9)}, t \neq -2, 9$

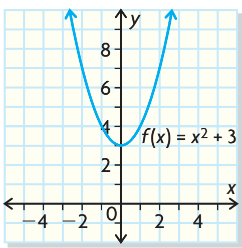
d) $\frac{-x^2-18x}{(3x+2)(2x+3)(2x-3)}, x \neq -\frac{3}{2}, -\frac{2}{3}, \frac{3}{2}$

8. yes (as long as there are no restrictions that were factored out)
9. yes

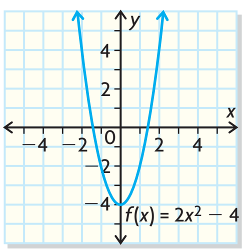
Chapter 3

Getting Started, p. 138

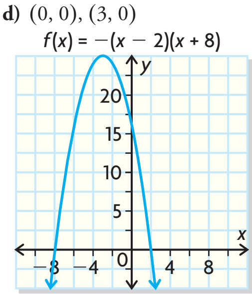
1. a) 0 b) -21 c) 0 d) -1 e) $-3k^2 + 4k - 1$ f) $-3k^2 - 4k - 1$
2. a) $f(x) = x^2 + 2x - 15$ b) $f(x) = 2x^2 + 12x$ c) $f(x) = -3x^2 - 12x - 9$ d) $f(x) = x^2 - 2x + 1$
3. a) vertex $(-3, -4)$, $x = -3$, domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq -4\}$
   b) vertex $(5, 1)$, $x = 5$, domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 1\}$
4. a) vertex $(0, 4)$, $x = 0$, opens up
   b) vertex $(4, 1)$, $x = 4$, opens up
   c) vertex $(-7, -3)$, $x = -7$, opens down
   d) vertex $(1.5, 36.75)$, $x = 1.5$, opens down
5. a) $x = 3$ or 8 b) $x = 0.55$ or 5.45 c) $x = -1$ or 1.67 d) $x = 0.5$ or 3
6. a) $(-3, 0)$, $(3, 0)$ b) $(-1.83, 0)$, $(9.83, 0)$ c) $(1.33, 0)$, $(2, 0)$ d) $(0, 0)$, $(3, 0)$
7. a)



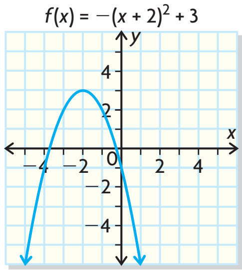
b)



c)



d)



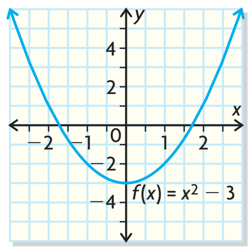
8.

Definition:	Characteristics:
equation is of form $y = ax^2 + bx + c$ or equivalent	graph is a parabola function has two, one, or no zeros second differences are constant
Examples:	**Non-examples:**
$y = x^2$ $\quad y = -4(x + 3)^2 - 5$	$y = 5 - 4x$ $y = 2\sqrt{x} - 5$

Quadratic Function

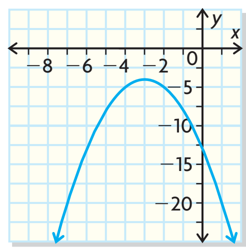
Lesson 3.1, pp. 145–147

1. a) linear, first differences are constant
   b) quadratic, second differences are constant
   c) linear, first differences are constant
   d) quadratic, second differences are constant
2. a) opens up b) opens down c) opens down d) opens up
3. a) zeros $x = 2$ or -6 b) opens down c) $x = -2$
4. a) vertex $(-2, 3)$ b) $x = -2$
   c) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 3\}$
5. a)



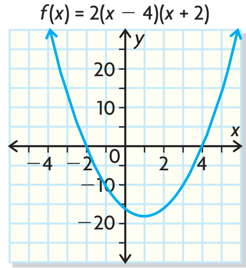
opens up, vertex $(0, -3)$, $x = 0$

b)



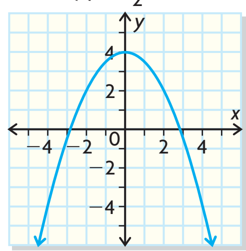
opens down, vertex $(-3, -4)$, $x = -3$

c)



opens up, vertex $(1, -18)$, $x = 1$

d)



opens down, vertex $(0, 4)$, $x = 0$

6. a) $f(x) = -3x^2 + 6x + 3$, $(0, 3)$
   b) $f(x) = 4x^2 + 16x - 84$, $(0, -84)$
7. a) opens down
   b) vertex $(-1, 8)$
   c) $(-3, 0)$, $(1, 0)$
   d) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 8\}$
   e) negative; parabola opens down
   f) $f(x) = -2(x + 1)^2 + 8$ or $f(x) = -2(x + 3)(x - 1)$
8. a) opens up
   b) vertex $(1, -3)$
   c) $x = 1$
   d) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -3\}$
   e) positive; parabola opens up

9. a) $x = 0$ c) $x = 12$ e) $x = -1.5$

b) $x = -7$ d) $x = -2$ f) $x = -\dfrac{5}{16}$

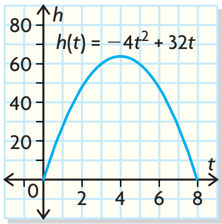
10. a)

x	-2	-1	0	1	2
$f(x)$	3	4	3	0	-5

b) First differences: 1, -1, -3, -5; Second differences: -2; parabola opens down

c) $f(x) = -(x + 1)^2 + 4$

11. a)



b) 8 s; height starts at 0 m and is 0 m again after 8 s.

c) $h(3) = 60$ m

d) 64 m

12. $y = 30$

13. Similarities: both are quadratic; both have axis of symmetry $x = 1$. Differences: $f(x)$ opens up, $g(x)$ opens down; $f(x)$ has vertex $(1, -2)$, $g(x)$ has vertex $(1, 2)$

14.

x	-2	-1	0	1	2	3
$f(x)$	19	9	3	1	3	9
First Differences		-10	-6	-2	2	6
Second Differences		4	4	4	4	

15. \$56 250

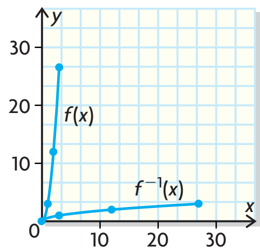
16. $y = -\dfrac{1}{110}(x + 7.5)^2 + \dfrac{1805}{88}$

Lesson 3.2, pp. 153–154

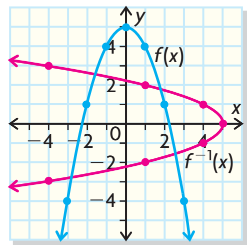
1. a) and c); (a) is negative.
2. a) vertex $(-5, -2)$, minimum value -2
   b) vertex $(4, 8)$, maximum value 8
3. a) maximum: 6 c) maximum: 8
   b) minimum: 0 d) minimum: -7
4. a) complete the square; minimum: -5
   b) factor or complete the square; minimum: -4
   c) factor or complete the square; minimum: -18
   d) factor or complete the square; maximum: 27
   e) use partial factoring; minimum: 2
   f) use vertex form; maximum: -5
5. a) i) $R(x) = -x^2 + 5x$ ii) maximum revenue: \$6250
   b) i) $R(x) = -4x^2 + 12x$ ii) maximum revenue: \$9000
   c) i) $R(x) = -0.6x^2 + 15x$ ii) maximum revenue: \$93 750
   d) i) $R(x) = -1.2x^2 + 4.8x$ ii) maximum revenue: \$4800
6. a) minimum: -2.08 b) maximum: 1.6
7. a) i) $P(x) = -x^2 + 12x - 28$ ii) $x = 6$
   b) i) $P(x) = -2x^2 + 18x - 45$ ii) $x = 4.5$
   c) i) $P(x) = -3x^2 + 18x - 18$ ii) $x = 3$
   d) i) $P(x) = -2x^2 + 22x - 17$ ii) $x = 5.5$
8. a) 70 m b) 2 s c) 50 m
9. \$562 500
10. Minimum value is 2, therefore $3x^2 - 6x + 5$ cannot be less than 1.
11. a) \$5 450 000
    b) Maximum profit occurs when \$40 000 is spent on advertising.
    c) \$22 971
12. Is possible, because maximum rectangular area occurs when rectangle is 125 m by $\dfrac{250}{\pi}$ m.
13. Possible response: Function is in standard form, so to find the minimum, we must find the vertex. Completing the square would result in fractions that are more difficult to calculate than whole numbers. Since this function will factor, putting the function in factored form and averaging the zeros to find the x-intercept of the vertex would be possible; however, there would still be fractions to work with. Using the graphing calculator to graph the function, then using CALC to find the minimum, would be the easiest method for this function.
14. $t = \dfrac{v_0}{9.8}$ seconds
15. \$9

Lesson 3.3, pp. 160–162

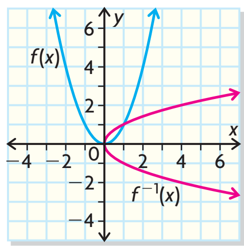
1. a)



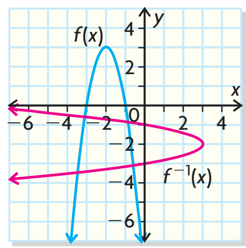
b)



2. a)



b)

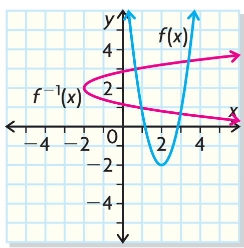


3. $f^{-1}(x) = \pm\sqrt{\dfrac{x + 1}{2}}$

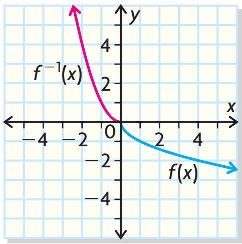
4. a) -1 c) 0 or 2

b) $1 \pm \sqrt{\dfrac{7 - x}{2}}$ d) $1 \pm \sqrt{-a}$

5. a), b)

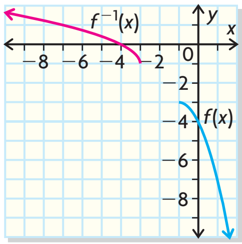


6. a), b)

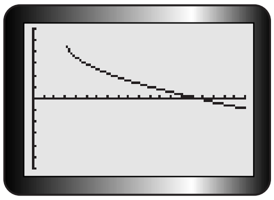


c) domain $= \{x \in \mathbf{R} \mid x \le 0\}$; range $= \{y \in \mathbf{R} \mid y \ge 0\}$
d) $g^{-1}(x) = (-x)^2$ or $g^{-1}(x) = x^2, x \le 0$

7. $f^{-1}(x) = -1 + \sqrt{-x-3}$



8. $f^{-1}(x) = 5 - \sqrt{2x-6}, x \le 3, x \le -3$



9. a) domain $= \{x \in \mathbf{R} \mid -2 < x < 3\}$;
range $= \{y \in \mathbf{R} \mid -3 \le y < 24\}$
b) $f^{-1}(x) = 1 + \sqrt{\dfrac{x+3}{3}}, -3 \le x \le 24$

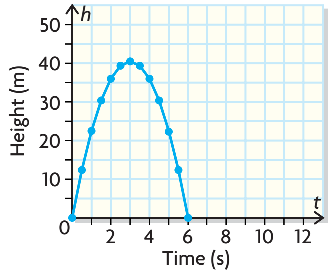
10. a) $h(t) = -5(t-1)^2 + 40$
b) domain $= \{t \in \mathbf{R} \mid 0 \le t \le 3.83\}$;
range $= \{h \in \mathbf{R} \mid 0 \le h \le 40\}$
c) $t = \begin{cases} 1 - \sqrt{\dfrac{40-h}{5}}, & 35 < h \le 40 \\ 1 + \sqrt{\dfrac{40-h}{5}}, & 0 \le h \le 35 \end{cases}$
d) domain $= \{h \in \mathbf{R} \mid 0 \le h \le 40\}$;
range $= \{t \in \mathbf{R} \mid 0 \le t \le 3.83\}$

11. a)

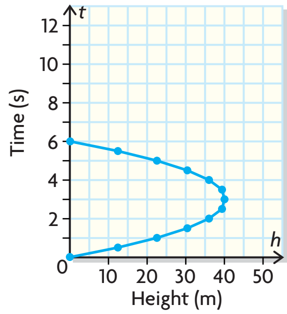
Time (s)	0	0.5	1	1.5	2	2.5
Height (m)	0	12.375	22.5	30.375	36.0	39.375

Time (s)	3	3.5	4	4.5	5	5.5	6
Height (m)	40.5	39.375	36.0	30.375	22.5	12.375	0

b)

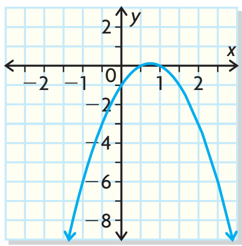


c)



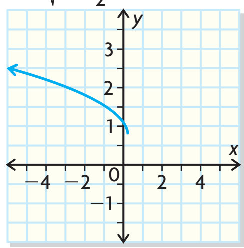
d) The inverse is not a function. It does not pass the vertical-line test.

12. a) (0.75, 0.125)
b)



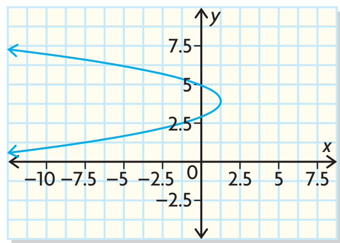
$f(x) = -2x^2 + 3x - 1$

c) $y = \sqrt{\dfrac{0.125 - x}{2}} + 0.75$



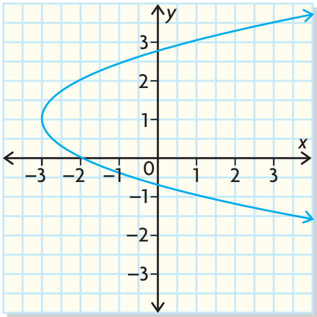
d) domain $= \{x \in \mathbf{R} \mid x \le 0.125\}$; range $= \{y \in \mathbf{R} \mid y \ge 0.75\}$
e) The y-values were restricted to ensure $f^{-1}(x)$ is a function.

13. a) i) $f(x) = -(x-4)^2 + 1$
ii)



iii) Domain of f should be restricted to $\{x \in \mathbf{R} \mid x \ge 4\}$ or $\{x \in \mathbf{R} \mid x \le 4\}$
iv) f^{-1} is $y = \pm\sqrt{-x-1} + 4$

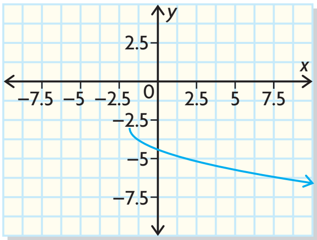
b) i) $f(x) = (x - 1)^2 - 3$

ii) 

iii) Domain of f should be restricted to $\{x \in \mathbf{R} \mid x \leq 1\}$ or $\{x \in \mathbf{R} \mid x \geq 1\}$

iv) f^{-1} is $y = \pm\sqrt{x + 3} + 1$

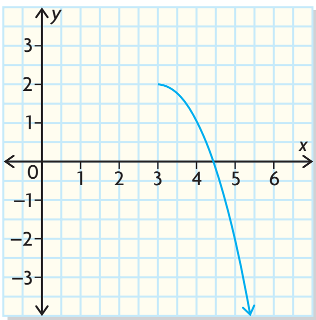
c) i) $f(x) = (x + 3)^2 - 2$ where $x \leq -3$

ii) 

iii) No restrictions necessary.

iv) $f^{-1}(x) = \pm\sqrt{x + 2} - 3$

d) i) $f(x) = 3 + \sqrt{2 - x}$

ii) 

iii) No restrictions necessary.

iv) f^{-1} is $y = -(x - 3)^2 + 2$ where $x \geq 3$

14. The original function must be restricted so that only one branch of the quadratic function is admissible. For example, if $f(x) = x^2$ had its domain restricted to $x \geq 0$, the inverse of $f(x)$ would be a function.

15. a) Possible response: Switch x and y and solve resulting quadratic equation for y, either by completing the square or by using the quadratic formula.

b) No, because the original function assigns some y-values to two x-values, so the inverse assigns two y-values to some x-values.

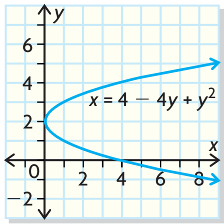
16. a) $P(x) = (x - 3.21)(14\,700 - 3040x)$

b) $P^{-1}(x) = 4.02 \pm \sqrt{\dfrac{-x + 2008}{3040}}$. This equation will take the total profit and determine the price per kilogram.

c) $P^{-1}(1900) = 4.02 \pm \sqrt{\dfrac{-1900 + 2008}{3040}}$
$= 4.21$ or 3.83

If the meat manager charges either \$4.21/kg or \$3.83/kg, she will make a profit of \$1900.

d) \$4.02/kg e) \$3.97/kg. Profit would be about \$2289.

17. a) 

b) domain $= \{x \in \mathbf{R} \mid x \geq 0\}$; range $= \{y \in \mathbf{R}\}$

c) $y = (x - 2)^2$

d) Yes, the inverse is a function. Its graph will be a parabola, so it will pass the vertical-line test.

Lesson 3.4, pp. 167–168

1. a) $3\sqrt{3}$ b) $5\sqrt{2}$ c) $7\sqrt{2}$ d) $4\sqrt{2}$

2. a) $\sqrt{35}$ b) $\sqrt{66}$ c) $10\sqrt{6}$ d) $-32\sqrt{39}$

3. a) $7\sqrt{5}$ b) $5\sqrt{7}$ c) $-\sqrt{3} + 19\sqrt{2}$ d) $-\sqrt{2}$

4. a) $6\sqrt{3}$ b) $-25\sqrt{5}$ c) $20\sqrt{10}$ d) $-\sqrt{15}$ e) $2\sqrt{5}$ f) $-18\sqrt{3}$

5. a) $2\sqrt{3} - \sqrt{15}$ b) $2\sqrt{14} + 6\sqrt{6}$ c) 32 d) $-24\sqrt{3}$ e) $36\sqrt{2}$ f) -140

6. a) $-2\sqrt{2}$ b) $-\sqrt{3} + 8\sqrt{2}$ c) $-9\sqrt{2}$ d) $15\sqrt{2}$ e) $2\sqrt{13}$ f) $16\sqrt{3} - 4\sqrt{7}$

7. a) $18 + 12\sqrt{10} - 3\sqrt{5} - 10\sqrt{2}$
b) $31 + 12\sqrt{3}$
c) -3
d) $-7 - 2\sqrt{6}$
e) $83 - 12\sqrt{35}$
f) $4 + 3\sqrt{6} - 8\sqrt{3} - 13\sqrt{2}$

8. $4\sqrt{2}$ cm

9. $15\sqrt{2}$ cm

10. $3\sqrt{10}$ cm

11. $6\sqrt{2}$

12. Perimeter $= 8\sqrt{2} + 4\sqrt{5}$, Area $= 12$

13. $(\sqrt{a} + \sqrt{b})^2$

14. Possible response: $2\sqrt{50}$, $5\sqrt{8}$, $10\sqrt{2}$; The last one is in simplest radical form because the number under the radical sign cannot be simplified any further.

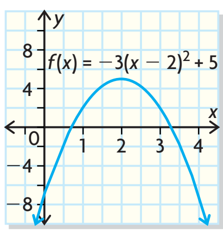
15. a) $a\sqrt{a}$ b) $x^2y^3\sqrt{x}$ c) $3n^3\sqrt{n}$ d) $-p + 2q - \sqrt{pq}$

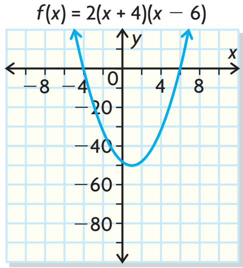
16. $2\sqrt{2}$

17. $x = 16$

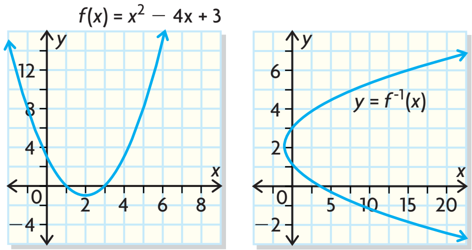
Mid-Chapter Review, p. 170

1. a) second differences $= -4$; quadratic
b) second differences $= 2$; quadratic

2. a) 

b) $f(x) = 2(x + 4)(x - 6)$


3. a) vertex $(2, 5)$, $x = 2$, domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 5\}$
   b) vertex $(1, -50)$, $x = 1$, domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -50\}$
4. a) $f(x) = -3x^2 + 12x - 7$
   b) $f(x) = 2x^2 - 4x - 48$
5. a) Minimum value of -7
   b) Minimum value of -50
   c) Maximum value of 12.5
   d) Minimum value of $-24.578\ 125$
6. Maximum profit is \$9000 when 2000 items are sold.
7. 2000 items/h
8. 64
9. a) $f^{-1}(x) = 2 \pm \sqrt{x + 1}$
   b) domain of $f(x) = \{x \in \mathbf{R}\}$, range of $f(x) = \{y \in \mathbf{R} \mid y \geq -1\}$; domain of $f^{-1}(x) = \{x \in \mathbf{R} \mid x \geq -1\}$, range of $f^{-1}(x) = \{y \in \mathbf{R}\}$
   c) $f(x) = x^2 - 4x + 3$



10. $x = 10 + \sqrt{\dfrac{R - 15}{-2.8}}$
11. Usually, the original function assigns some y-values to two x-values, so the inverse assigns two y-values to some x-values.
12. a) $\{x \in \mathbf{R} \mid x = -3\}\ \{y \in \mathbf{R} \mid y \leq 0\}$
    b) $f^{-1}(x) = x^2 - 3, x \geq 0\}$
13. a) $4\sqrt{3}$ b) $2\sqrt{17}$ c) $6\sqrt{5}$ d) $-15\sqrt{3}$ e) $35\sqrt{2}$ f) $-16\sqrt{3}$
14. a) $7\sqrt{2}$ b) $30\sqrt{3}$ c) $-5\sqrt{3}$ d) $9\sqrt{7} - 19\sqrt{2}$ e) $14 + 3\sqrt{3}$ f) $70 + 55\sqrt{2}$

Lesson 3.5, pp. 177–178

1. a) $x = -1$ or -4 b) $x = 2$ or 9 c) $x = \pm\dfrac{3}{2}$ d) $x = -\dfrac{1}{2}$ or 4
2. a) $x = 5.61$ or -1.61
   b) $x = 1.33$ or -2
   c) no real roots
   d) $x = -1.57$ or 5.97
3. a) $x = -1$ or -0.25 b) $x = 1$ or 4.5
4. a) i) Solve by factoring, function factors ii) $x = 0$ or 10
   b) i) Quadratic formula, function does not factor
      ii) $x = \dfrac{-3 \pm \sqrt{5}}{4}$
   c) i) Quadratic formula, function does not factor
      ii) $x = -2 \pm \sqrt{7}$
   d) i) Quadratic formula, function does not factor
      ii) $x = -4 \pm \sqrt{7}$
   e) i) Solve by factoring, function factors
      ii) $x = -1$ or 10
   f) i) Quadratic formula, function does not factor
      ii) $x = 2 \pm \sqrt{19}$
5. a) $(2.59, 0)$, $(-0.26, 0)$ b) $(1, 0)$, $\left(\dfrac{21}{4}, 0\right)$
6. a) 14 000 b) 4000 or 5000 c) 836 or 10 164 d) 901 or 11 099
7. 1.32 s
8. a) 50 000 b) 290 000 c) 2017
9. 15 m by 22 m
10. -19, -18 or 18, 19
11. base = 8 cm, height = 24 cm
12. 2.1 m
13. a) after 1.68 s and again at 17.09 s
    b) The rocket will be above 150 m for $17.09 - 1.68 = 15.41$ s.
14. \$2.75
15. Factoring the function and finding the zeros; substituting the values of a, b, and c into the quadratic formula; graphing the function on a graphing calculator and using **CALC** to find the zeros
16. 10 cm, 24 cm, 26 cm
17. $x = 0$ or $-\dfrac{2}{3}$

Lesson 3.6, pp. 185–186

1. a) vertex $(0, -5)$, up, 2 zeros
   b) vertex$(0, 7)$, down, 2 zeros
   c) vertex $(0, 3)$, up, no zeros
   d) vertex $(-2, 0)$, up, 1 zero
   e) vertex $(-3, -5)$, down, no zeros
   f) vertex $(4, -2)$, up, 2 zeros
2. a) 2 zeros b) 2 zeros c) 2 zeros d) 1 zero
3. a) 2 zeros b) no zeros c) 1 zero d) 1 zero
4. a) 2 zeros b) 2 zeros c) 2 zeros d) no zeros
5. a) 2 break-even points
   b) Cannot break even
   c) 1 break-even point
   d) Cannot break even
6. $k = \dfrac{4}{3}$
7. $k < -2$ or $k > 2$
8. $k > \dfrac{4}{3}$, $k = \dfrac{4}{3}$, $k < \dfrac{4}{3}$
9. $k = -4$ or 8
10. No, resulting quadratic has no solutions.
11. Answers may vary. For example,
    a) $y = -2(x + 1)(x + 2)$
    b) $y = 2x^2 + 1$
    c) $y = -2(x - 2)^2$
12. A: break-even $x = 4.8$
    B: break-even $x = 0.93$
    C: break-even $x = 2.2$
    Buy Machine B. It has the earliest break-even point.
13. a) no effect
    b) no effect
    c) no effect
    d) change from 1 to 2 zeros
    e) change from 1 to no zeros
    f) change from 1 to 2 zeros
14. 10.5
15. $f(x) = -(x - 3)(3x + 1) + 4$ is a vertical translation of 4 units up of the function $g(x) = -(x - 3)(3x + 1)$. Function $g(x)$ opens down and has 2 zeros. Translating this function 4 units up will have no effect on the number of zeros, so $f(x)$ has 2 zeros.
16. a) If the vertex is above the x-axis, the function will have 2 zeros if it opens down and no zeros if it opens up. If the vertex is below the x-axis, there will be 2 zeros if the function opens up and no zeros if it opens down. If the vertex is on the x-axis, there is only 1 zero.
    b) If the linear factors are equal or multiples of each other, there is 1 zero; otherwise, there are 2 zeros.
    c) If possible, factor and determine the number of zeros as in part (b). If not, use the value of $b^2 - 4ac$. If $b^2 - 4ac > 0$, there are 2 zeros, if $b^2 - 4ac = 0$, 1 zero, and if $b^2 - 4ac < 0$, no zeros.
17. $(x^2 - 1)k = (x - 1)^2$

$$kx^2 - k = x^2 - x - x + 1$$
$$kx^2 - k = x^2 - 2x + 1$$
$$0 = x^2 - kx^2 - 2x + 1 + k$$
$$0 = x^2(1 - k) - 2x + (1 + k)$$

The equation will have one solution when the discriminant is equal to zero.

$$\begin{aligned} b^2 - 4ac &= 0 \\ (-2)^2 - 4(1-k)(1+k) &= 0 \\ 4 - 4(1-k^2) &= 0 \\ 4 - 4 + 4k^2 &= 0 \\ 4k^2 &= 0 \\ k^2 &= 0 \\ k &= 0 \end{aligned}$$

Therefore, the function will have one solution when $k = 0$.

18. Function has 2 zeros for all values of k.

Lesson 3.7, pp. 192–193

1. Same zeros, 3 and -4

2. Same vertex, $(2, -4)$; stretched vertically by different factors, opening in different directions

3. $(0, -7)$ (the y-intercept)

4. **a)** $f(x) = -\frac{7}{6}(x+4)(x-3)$ **c)** $f(x) = \frac{1}{6}(x^2 - 7)$

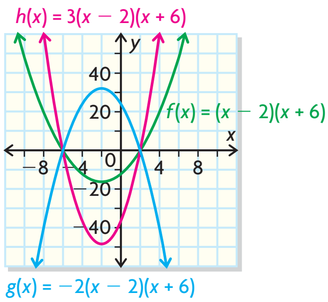
b) $f(x) = -\frac{6}{33}x(x-8)$ **d)** $f(x) = -4(x^2 - 2x - 1)$

5. **a)** $f(x) = -\frac{13}{36}(x+2)^2 + 5$ **c)** $f(x) = \frac{2}{25}(x-4)^2 - 5$

b) $f(x) = -13(x-1)^2 + 6$ **d)** $f(x) = \frac{8}{49}(x-4)^2$

6. $f(x) = 5.5x^2 - 6x - 7$

7. **a)–c)**

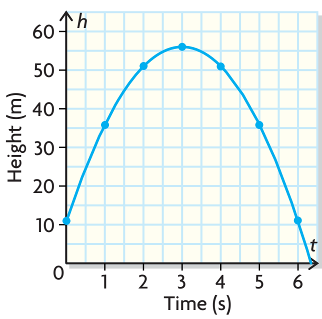


8. $f(x) = -\frac{6}{7}(x-4)(x+4)$

9. $f(x) = \frac{5}{33}(x^2 - 4x + 1)$

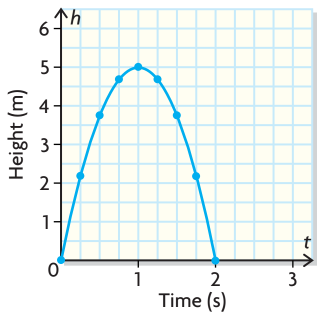
10. $f(x) = -\frac{3}{16}x(x-12)$ Yes, because at a height of 5 m the bridge is 6.11 m wide.

11. **a), b)**



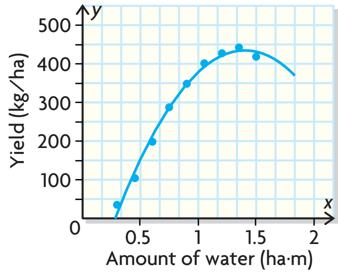
c) $h(t) = -5(t-3)^2 + 56$

12. **a), b)**



c) $h(t) = -5t^2 + 10t$

13. **a)**

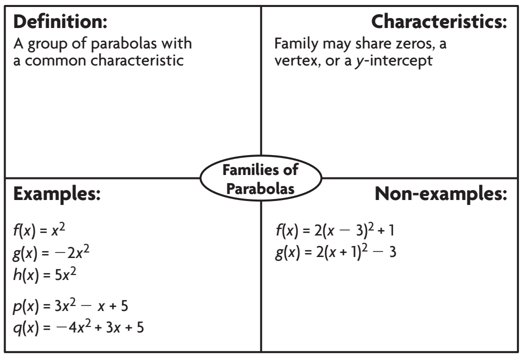


b) approximately $(1.35, 442)$

c) possible function (using $(0.60, 198)$ and vertex): $f(x) = 343(x - 1.35)^2 + 442$

14. $f(x) = -3(x+3)(x+1)$ or $f(x) = -3x^2 - 12x - 9$

15. Sample response:



16. 15.36 m

17. $f(x) = -\frac{1}{4}(x+3)(x-1)(x-5)$

Lesson 3.8, pp. 198–199

1. **a)** $(3, 9)$ $(-2, 4)$ **b)** $(0, 3)$ $(-0.25, 2.875)$ **c)** no solutions

2. **a)** $(4, 3)$ $(6, -5)$ **b)** $(2, 7)$ $(-0.5, -0.5)$ **c)** no solutions

3. one solution

4. **a)** $(1.5, 8)$ $(-7, -43)$

b) $(1.91, 8.91)$ $(-1.57, 5.43)$

c) no solutions

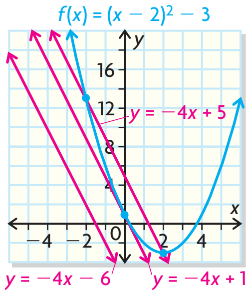
d) $(-0.16, 3.2)$ $(-1.59, -3.95)$

5. 3 and 5 or -1 and 1

6. \$3.00

Answers

7. **a)**



b) $y = -4x - 6$, $y = -4x + 1$, $y = -4x + 5$

c) y-intercepts are all less than 1

8. $k = -5$

9. $k > \frac{73}{12}$

10. 7.20 s

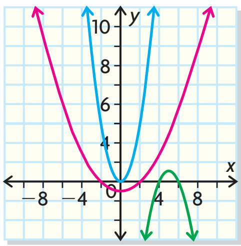
11. 10, -2

12. Yes, 0.18 s after kick at (0.18, 4.0)

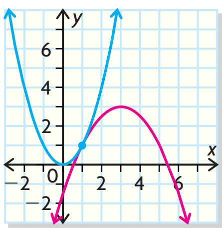
13. Plot graphs of functions and count points of intersection; calculate $b^2 - 4ac$, since there are two points of intersection when $b^2 - 4ac > 0$, one when $b^2 - 4ac = 0$, and none when $b^2 - 4ac < 0$

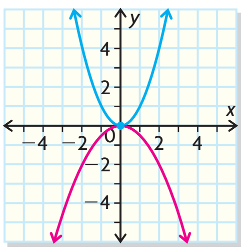
14. $(0, -2)$, $(4, -\frac{14}{3})$

15. Zero points of intersection:

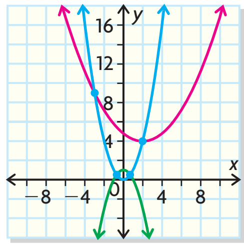


One point of intersection:





Two points of intersection:



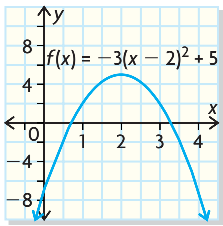
16. $y = 0.5x - 1$

Chapter Review, pp. 202–203

1. **a)** down, vertex (2, 5), $x = 2$

b) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 5\}$

c)

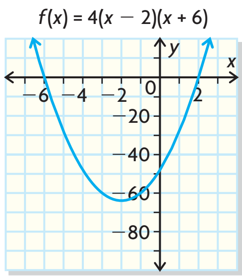


2. **a)** up, zeros 2 and -6

b) vertex $(-2, -64)$

c) domain $= \{x \in \mathbf{R}\}$; range $= \{y \in \mathbf{R} \mid y \geq -64\}$

d)



3. $x = -1$

4. **a)** Maximum value of 7 at $x = 4$

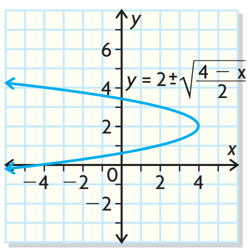
b) Minimum value of -36 at $x = -3$

5. 42 m after about 2.9 s

6. $g(x)$ and $h(x)$ are the two branches of the inverse of $f(x) = x^2$.

7. The inverse of a quadratic function is not a function, because it has two y-values for every x-value. It can be a function only if the domain of the original function has been restricted to a single branch of the parabola.

8. **a)**



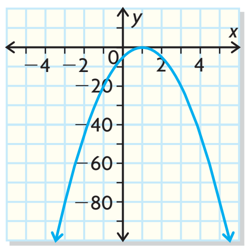
b) Domain $= \{x \in \mathbf{R} \mid x \leq 4\}$; Range $= \{y \in \mathbf{R}\}$

c) The inverse relation is not a function; it does not pass the vertical-line test.

9. a) $7\sqrt{2}$ c) $-4\sqrt{4}$
b) $-20\sqrt{2}$ d) $37 - 12\sqrt{7}$
10. $2\sqrt{66}$
11. $(9 + 3\sqrt{5})$ cm
12. $\left(\frac{5}{2}, 0\right)$, $(-3, 0)$
13. a) 52 428 b) 1990
14. 55.28 m by 144.72 m
15. Yes, because $14t - 5t^2 = 9$ has $b^2 - 4ac = 16 > 0$, so there are two roots. Because parabola opens down and is above t-axis for small positive t, at least one of these roots is positive.
16. $x < -0.5$ or $x > 3.5$
17. 4408 bikes
18. $f(x) = -\frac{5}{3}x^2 + \frac{20}{3}x - \frac{5}{3}$
19. The family of parabolas will all have vertex $(-3, -4)$; $f(x) = 10(x + 3)^2 - 4$
20. a) $f(x) = -\frac{7}{9}x^2 + 15$ b) 8.8 m
21. $(-5, 19)$, $(1.5, -0.5)$
22. Yes, after 4 s. Height is 15 m.
23. a) No, they will not intersect. The discriminant of $f(x) - g(x)$ is -47. There are no real solutions for $f(x) - g(x)$, meaning that $f(x)$ and $g(x)$ do not intersect.
b) Answers will vary. Use $g(x) = 3x - 5$.

Chapter Self-Test, p. 204

1. a) $f(x) = -5(x - 1)^2$, vertex $(1, 0)$
b) zero at $x = 1$, axis of symmetry $x = 1$, opens down
c) domain $= \{x \in \mathbf{R}\}$; range $= \{y \in \mathbf{R} \mid y \leq 0\}$
d)



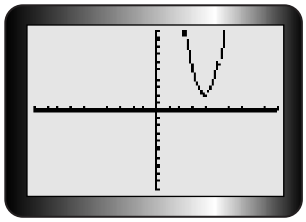
$f(x) = -5x^2 + 10x - 5$

2. a) Maximum value; complete the square.
b) Minimum value; average the zeros.
3. a) Vertex form; vertex is visible in equation.
b) Standard form; y-intercept is visible in equation.
c) Factored form; zeros are visible in equation.
d) Vertex form; use x-coordinate of vertex.
e) Vertex form.; use vertex and direction of opening.
4. 360 000 m^2
5. $f^{-1} = 1 \pm \sqrt{\frac{x + 3}{2}}$
6. a) $2 - 4\sqrt{2}$
b) $15 - 3\sqrt{10} + 5\sqrt{5} - 5\sqrt{2}$
c) $\sqrt{8}$ can be simplified to $2\sqrt{2}$. This resulted in like radicals that could be combined.
7. $k = -2$ or 2
8. Intersects in 2 places, since $2x^2 - 3x + 2 = 6x - 5$ has $b^2 - 4ac > 0$; $(1, 1)$, $(3.5, 16)$
9. $f(x) = -x^2 + 8x - 13$

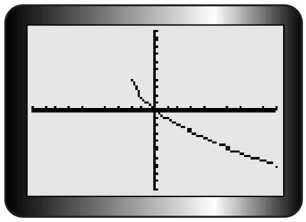
Cumulative Review Chapters 1–3, pp. 206–209

1. (c)	7. (d)	13. (d)	19. (d)	25. (b)	31. (b)
2. (b)	8. (b)	14. (b)	20. (d)	26. (c)	32. (c)
3. (b)	9. (a)	15. (a)	21. (c)	27. (d)	
4. (a)	10. (b)	16. (c)	22. (d)	28. (a)	
5. (b)	11. (c)	17. (d)	23. (b)	29. (b)	
6. (c)	12. (a)	18. (a)	24. (c)	30. (d)	

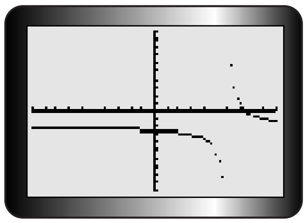
33. a) Domain: $\{x \in \mathbf{R}\}$, Range: $\{y \in \mathbf{R} \mid y \geq 2\}$; Parent function: $y = x^2$; Transformations: Vertical stretch by a factor of 3, horizontal translation 4 right, vertical translation 2 up; Graph:



b) Domain: $\{x \in \mathbf{R} \mid x \geq -2\}$, Range: $\{y \in \mathbf{R} \mid y \leq 5\}$; Parent function: $y = \sqrt{x}$; Transformations: Vertical stretch by a factor of 2, reflection in the x-axis, horizontal compression by a factor of $\frac{1}{3}$, horizontal translation 2 left, vertical translation 5 up; Graph:



c) Domain: $\{x \in \mathbf{R} \mid x \neq 6\}$, Range: $\{y \in \mathbf{R} \mid y \neq -2\}$; Parent function: $y = \frac{1}{x}$; Transformations: Horizontal stretch by a factor of 3, horizontal translation 6 to the right, vertical translation 2 down; Graph:



34. Jill: 3.6 km/h, 8 h 20 min; Sacha: 5 km/h; 6 h 20 min (including time to stop and talk with friend)
35. a) 8 students b) about \$700

Chapter 4

Getting Started, p. 212

1. a) 49 c) $\frac{1}{5}$ e) 10 000
b) 32 d) 1 f) $\frac{1}{8}$

2. a) 9 c) -16 e) -125
b) -27 d) 16 f) -125

3. $(-5)^{120}$ will result in a positive answer since the exponent is an even number.

4. a) 81 c) 4096 e) 256
b) 5 764 801 d) $-1\ 000\ 000$ f) -1

5. a) 49 b) 24 c) 8 d) $\frac{1}{3}$

6. a) $\frac{55}{24}$ c) $\frac{21}{16}$ e) $\frac{62}{75}$
b) $-\frac{25}{24}$ d) $-\frac{3}{10}$ f) $-\frac{39}{16}$

7. a) a^7 b) b^4 c) c^{12} d) d^{10}

8. a) $x = 2$ b) $m = \frac{8}{3}$ c) $a = 3$ d) $r = 8$

9. a) $V \doteq 94.248\ \text{cm}^3$ b) $V \doteq 65.45\ \text{cm}^3$

10. a) first differences are all -5; linear function
b) first differences are 1, 2, 3, 4, 5; second differences are all 1; quadratic function

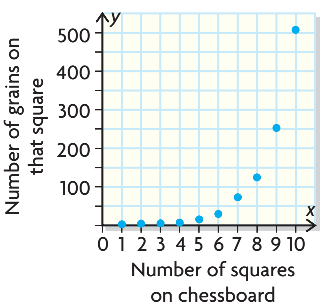
Lesson 4.1, p. 216

1. a) Both graphs decrease rapidly at the beginning, then continue to decrease less rapidly before levelling off.
b) 85 °C
c) 20 °C

2. a)

Number of Squares on the Chessboard	Number of Grains on that Square	First Differences
1	1	
2	2	1
3	4	2
4	8	4
5	16	8
6	32	16
7	64	32
8	128	64
9	256	128
10	512	256

b)



c) They are similar in their shape; that is both decrease rapidly at the beginning and then level off. They are different in that they have different y-intercepts and asymptotes.

Lesson 4.2, pp. 221–223

1. a) $\frac{1}{5^4}$ c) 2^4 e) $\frac{11}{3}$
b) $(-10)^3$ d) $-\left(\frac{5}{6}\right)^3$ f) $\frac{8}{7^2}$

2. a) $(-10)^0 = 1$ c) 2^{13} e) $-\frac{1}{9^4}$
b) $\frac{1}{6^2}$ d) $\frac{1}{11^8}$ f) $\frac{1}{7^{12}}$

3. $2^{-5} = \frac{1}{2^5}$ is less than $\left(\frac{1}{2}\right)^{-5} = 2^5$

4. a) $2^4 = 16$ c) $5^{-2} = \frac{1}{25}$ e) $4^3 = 64$
b) $(-8)^0 = 1$ d) $3^{-2} = \frac{1}{9}$ f) $7^{-2} = \frac{1}{49}$

5. a) $3^1 = 3$ c) $12^0 = 1$ e) $3^{-2} = \frac{1}{9}$
b) $9^0 = 1$ d) $5^0 = 1$ f) $9^1 = 9$

6. a) $10^3 = 1000$ c) $6^{-1} = \frac{1}{6}$ e) $2^{-3} = \frac{1}{8}$
b) $8^{-1} = \frac{1}{8}$ d) $4^2 = 16$ f) $13^1 = 13$

7. a) $-\frac{3}{16}$ c) 1 e) $\frac{1}{1000}$
b) $\frac{1}{2}$ d) 9 f) $-\frac{1}{12}$

8. a) $\frac{1}{400}$ c) $-\frac{1}{3}$ e) $\frac{1}{16}$
b) 2 d) $\frac{1}{9}$ f) 125

9. a) $-\frac{1}{64}$ c) $-\frac{1}{125}$ e) $-\frac{1}{216}$
b) $\frac{1}{16}$ d) $-\frac{1}{25}$ f) $-\frac{1}{36}$

10. 5^{-2}, 10^{-1}, 3^{-2}, 2^{-3}, 4^{-1}, $(0.1)^{-1}$; If the numerators of the numbers are all the same (1), then the larger the denominator, the smaller the number.

11. a) $\frac{1}{36}$ b) $-\frac{9}{2}$ c) $-\frac{2}{3}$ d) $\frac{1}{576}$

12. a) Erik: $3^{-1} \neq -\frac{1}{3}$ (negative exponents do not make numbers negative)
Vinn: $3 = 3^1$ and he did not add the exponents correctly.

b) Correct solution:

$$\boxed{\begin{aligned} &3^{-2} \times 3 \\ &= \frac{1}{3^2} \times 3 \\ &= \frac{1}{9} \times 3 \\ &= \frac{3}{9} \\ &= \frac{1}{3} \end{aligned}}$$

13. a) $\frac{1}{8}$ c) $\frac{9}{4}$ e) 6 g) $\frac{1}{6}$ i) $-\frac{1}{9}$
b) $\frac{1}{6}$ d) $\frac{17}{4}$ f) $\frac{1}{26}$ h) $\frac{57}{34}$

14. a) 4 c) $\frac{1}{9}$ e) 9 g) 9
b) 9 d) $\frac{2}{3}$ f) $\frac{1}{81}$ h) 9

15. a) $(-10)^3$ is -10 multiplied by itself three times. 10^{-3} is the reciprocal of 10 cubed.
b) $(-10)^4$ is -10 multiplied by itself four times. -10^4 is the negative of 10^4.

16. a) $x = -1$ c) $x = 0$ e) $n = -2$
b) $x = -2$ d) $n = -2$ f) $w = -2$

17. $10^{-y} = \frac{1}{5}$

18. a) x^{10-2r} c) b^{m+2n} e) a^{10-2p}
b) b^{4m-n} d) x^{21-2r} f) $3^{6-m}x^{24-5m}$

Lesson 4.3, pp. 229–230

1. a) $\sqrt{49} = 7$ c) $\sqrt[3]{-125} = -5$ e) $\sqrt[4]{81} = 3$
b) $\sqrt{100} = 10$ d) $\sqrt[4]{16} = 2$ f) $-\sqrt{144} = -12$

2. a) $512^{\frac{1}{9}} = 2$ d) $(-216)^{\frac{5}{3}} = 7776$
b) $(-27)^{\frac{1}{3}} = -3$ e) $\left(\frac{-32}{243}\right)^{\frac{1}{5}} = \frac{-2}{3}$
c) $27^{\frac{2}{3}} = 9$ f) $\left(\frac{16}{81}\right)^{\frac{-1}{4}} = \frac{3}{2}$

3. a) 8^1 c) $(-11)^{\frac{11}{4}}$ e) $9^{\frac{-13}{15}}$
b) $8^{\frac{1}{3}}$ d) 7 f) $10^{-\frac{7}{5}}$

4. a) 5 b) -2 c) 4 d) 3

5. a) 11 c) $\frac{47}{3}$ e) $\frac{253}{4}$
b) -18 d) $-\frac{255}{32}$ f) 3

6. a) $4^{\frac{1}{2}} = 2$ c) 4 e) $16^{-\frac{1}{4}} = \frac{1}{2}$
b) $100^{\frac{-1}{2}} = \frac{1}{10}$ d) $\frac{1}{3}$ f) 64

7. a) 4.996 c) 1.262 e) 5.983
b) 6.899 d) 2.999 f) 98.997

8. 0.25 m

9. $27^{\frac{4}{3}} = 81$, $27^{1.3333} \doteq 80.991\ 101\ 73$ The values are not equal as $\frac{4}{3} \neq 1.3333$.

10. $0.2 = \frac{1}{5}$, an odd root , $0.5 = \frac{1}{2}$, an even root. Even root of a negative number is not real.

11. $125^{\frac{-2}{3}} = \frac{1}{125^{\frac{2}{3}}} = \frac{1}{(125^{\frac{1}{3}})^2} = \frac{1}{5^2} = \frac{1}{25}$

12. a) -8 c) 0.0081 e) 0.008
b) 39.0625 d) 2.25 f) 1 679 616

13. $4^{2.5} = 4^{\frac{5}{2}} = (\sqrt{4})^5$. Change 2.5 to a fraction as $\frac{5}{2}$. This is the same as taking the square root of the four and then taking the fifth power of that result.

14. a) false c) false e) false
b) false d) true f) true

15. a) n cannot be zero or an even number. b) n cannot be zero.

16. The value of x can equal the value of y. Also $x = 4$ and $y = 2$.

17. Yes this works. The value of i is approximately 0.017.

18. a) $x = \frac{1}{6}$ b) $x = \frac{27}{2}$

Lesson 4.4, pp. 235–237

1. a) x^7 c) m^8 e) y^6
b) p^2 d) $\frac{1}{a^2}$ f) $\frac{1}{k^{12}}$

2. a) $\frac{1}{y^2}$ c) $\frac{1}{n^{24}}$ e) 1
b) x^4 d) $\frac{1}{w}$ f) $\frac{1}{b^{19}}$

3. a) 36 b) $x^2y^2 - 36$
c) Usually it is faster to substitute numbers into the simplified form.

4. a) p^2q c) $\frac{1}{a^2b^7}$ e) wx^4
b) $\frac{y^2}{x^6}$ d) $\frac{n^6}{m^4}$ f) $\frac{a^6}{b^4}$

5. a) $72x^8y^{11}$ c) $\frac{y^6}{150x^4}$ e) $\frac{r^4}{p^7}$
b) $\frac{a^5}{b^2}$ d) $\frac{3m^{10}}{4n^2}$ f) $\frac{y^4}{x^5}$

6. a) 1 c) $\frac{5}{6m^{11}}$ e) $\frac{1}{4}$
b) $\frac{9c}{2}$ d) $10x^4$ f) $\frac{1}{x}$

7. a) $4x^3y^2 = 32$ c) $\frac{9y^2}{8x} = \frac{45}{16}$
b) $\frac{1}{2p^3} = \frac{1}{54}$ d) $\frac{1}{35b} = \frac{1}{350}$

8. a) $1000x^{\frac{3}{4}} = 8000$ c) $\frac{-5}{8a^4} = \frac{-5}{8}$
b) $\frac{x^3}{16} = \frac{125}{16}$ d) $\frac{12n^3}{m^2} = \frac{3}{25}$

9. a) $18m^5n^5$ b) $\frac{27y}{2}$ c) $4a^{10}$ d) $\frac{1}{\sqrt{3x^{\frac{5}{2}}}}$

10. $M = \frac{x^4}{16y^9}$
a) Answers may vary. For example, $x = 2, y = 1$
b) Answers may vary. For example, $x = 3, y = 1$
c) Answers may vary. For example, $y = 1, x = 1$
d) impossible, always positive

11. a) $\frac{2h + 2r}{hr}$ b) $\frac{SA}{V} = \frac{8}{3} \doteq 2.67\ \text{cm}^{-1}$

12. These simplify to $\frac{y}{x} = -\frac{3}{2}, \frac{x^2}{y^2} = \frac{4}{9}, \frac{x^2}{y^3} = \frac{4}{27}$, respectively. Switch second and third for proper order.

13. Algebraic and numerical expressions are similar in the following way: when simplifying algebraic or numerical expressions, you have to follow the order of operations. When simplifying algebraic expressions, you can only add or subtract like terms—while unlike terms may be multiplied. In this way algebraic expressions are different than numerical expressions.

14. a) $r = \sqrt[3]{\frac{3V}{4\pi}} = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$ b) 4 m

15. x

Answers

Mid-Chapter Review, p. 239

1. **a)** 5^5 **c)** 9^{18} **e)** $\left(\frac{1}{10}\right)^2$

b) $-\frac{1}{8}$ **d)** 3^2 **f)** 7^2

2. **a)** $-\frac{1}{16}$ **b)** $\frac{24}{25}$ **c)** $\frac{4}{25}$ **d)** $\frac{-7}{64}$

3. **a)** $\frac{16}{49}$ **b)** $-\frac{8}{125}$ **c)** $-\frac{27}{8}$ **d)** -27

4. x cannot be zero or a negative number for $x^{\frac{-1}{2}}$. x can be zero for $x^{\frac{1}{2}}$, but not negative.

5. **a)** $\frac{7}{9}$ **c)** $\frac{3}{4}$ **e)** $\frac{1}{3}$

b) $\frac{10}{11}$ **d)** $-\frac{1}{125}$ **f)** 2

6.

	Exponential Form	Radical Form	Evaluation of Expression
a)	$100^{\frac{1}{2}}$	$\sqrt{100}$	10
b)	$16^{0.25}$	$\sqrt[4]{16}$	2
c)	$121^{\frac{1}{2}}$	$\sqrt{121}$	11
d)	$(-27)^{\frac{5}{3}}$	$\sqrt[3]{(-27)^5}$	-243
e)	$49^{2.5}$	$\sqrt{49^5}$	16 807
f)	$1024^{\frac{1}{10}}$	$\sqrt[10]{1024}$	2

7. **a)** -33.068 **b)** 31.147 **c)** 0.745 **d)** -63.096

8. $-8^{\frac{4}{3}} = -(\sqrt[3]{8})^4 = -(2)^4 = -16$ and $(-8)^{\frac{4}{3}} = (\sqrt[3]{-8})^4 = (-2)^4 = 16$

The second expression has an even root so the negative sign is eliminated.

9. **a)** $\frac{1}{x^5}$ **c)** $\frac{1}{y^6}$ **e)** $\frac{3}{a^8}$

b) n^4 **d)** $-x^5$ **f)** -8

10. **a)** $\frac{x^{0.2}}{y^{0.7}}$ **c)** $\frac{y^5}{x}$ **e)** 1

b) $\frac{n}{m}$ **d)** $\frac{4a^6b^{10}}{c^2}$ **f)** $\frac{2}{5}$

11. **a)** $\frac{2b}{a} = 3$ **b)** $\frac{3}{a^2b^2} = \frac{1}{12}$

12. **a)** a^{p+2} **b)** $2^{3-2m}x^{6-6m}$ **c)** 1 **d)** x^{n-4m}

Lesson 4.5, p. 243

1. **a)** quadratic **b)** exponential **c)** exponential **d)** linear

2. **a)** exponential; the values increase at a fast rate

b) exponential; the values increase at a fast rate

c) linear; straight line

d) quadratic; graph is a parabolic shape

Lesson 4.6, pp. 251–253

1. **a)** The function moves up 3 units. It is a vertical translation.

b) The function moves to the left 3 units. It is a horizontal translation.

c) The function values are decreased by a factor of $\frac{1}{3}$. It is a vertical compression.

d) The x-values are increased by a factor of 3. It is a horizontal stretch.

2. **a)** The base function is 4^x. Horizontal translation left 1 unit, vertical stretch of factor 3, and reflection in x-axis.

b) The base function is $\left(\frac{1}{2}\right)^x$. Vertical stretch factor 2, horizontal compression of factor $\frac{1}{2}$, and vertical translation of 3 units up.

c) The base function is $\left(\frac{1}{2}\right)^x$. Vertical stretch factor 7, and vertical translation of 1 unit down and 4 units to the right.

d) The base is 5^x. Horizontal compression by a factor of $\frac{1}{3}$ and a translation 2 units to the right.

3.

Function	y-intercept	Asymptote	Domain	Range
$y = 3^x + 3$	4	$y = 3$	$x \in R$	$y > 3, y \in R$
$y = 3^{x+3}$	27	$y = 0$	$x \in R$	$y > 0, y \in R$
$y = \frac{1}{3}(3^x)$	$\frac{1}{3}$	$y = 0$	$x \in R$	$y > 0, y \in R$
$y = 3^{\frac{x}{3}}$	1	$y = 0$	$x \in R$	$y > 0, y \in R$
$y = -3(4^{x+1})$	-12	$y = 0$	$x \in R$	$y < 0, y \in R$
$y = 2\left(\frac{1}{2}\right)^{2x} + 3$	5	$y = 3$	$x \in R$	$y > 3, y \in R$
$y = 7(0.5^{x-4}) - 1$	111	$y = -1$	$x \in R$	$y > -1, y \in R$
$y = 5^{3x-6}$	6.4×10^{-5}	$y = 0$	$x \in R$	$y > 0, y \in R$

4. **a)** horizontal compression of factor $\frac{1}{2}$, reflect in x-axis

b) vertical stretch of factor 5, reflect in y-axis, translate 3 units right

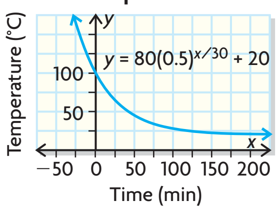
c) vertical stretch of factor 4, horizontal compression of factor $\frac{1}{3}$, reflect in the x-axis, translate 3 units left and 6 units down

5.

	Function	Transformations	y-intercept	Asymptote	Domain	Range
a)	$0.5f(-x) + 2$	• vertical compression by a factor of $\frac{1}{2}$ • reflection in the y-axis • translation of 2 units up	2.5	$y = 2$	$x \in \mathbf{R}$	$y > 2, y \in \mathbf{R}$
b)	$-f(0.25x + 1) - 1$	• reflection in the x-axis • horizontal stretch of 4 • translation 1 down and 4 left	-5	$y = -1$	$x \in \mathbf{R}$	$y < -1, y \in \mathbf{R}$
c)	$-2f(2x - 6)$	• reflection in the x-axis • vertical stretch of 2 • horizontal compression by factor of $\frac{1}{2}$ • translation 3 units right	$\frac{-2}{4^6}$	$y = 0$	$x \in \mathbf{R}$	$y < 0, y \in \mathbf{R}$
d)	$f(-0.5x + 1)$	• reflection in the y-axis • horizontal stretch of 2 • horizontal translation of 2 units right	4	$y = 0$	$x \in \mathbf{R}$	$y > 0, y \in \mathbf{R}$

6. Both functions have the y-intercept of 1 and the asymptote is $y = 0$. Their domains are all real numbers and their ranges are $y > 0$. The second function will increase at a faster rate.

7. **Water Temperature vs. Time**



The y-intercept represents the initial temperature of 100 °C. The asymptote is the room temperature and the lower limit on the temperature of the water.

8. a) If the doubling time were changed to 9, the exponent would change from $\frac{t}{3}$ to $\frac{t}{9}$. The graph would not rise as fast.

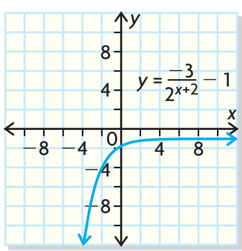
b) domain is $t \geq 0, t \in \mathbf{R}$; range is $N > N_0, N \in \mathbf{R}$

9. a) (iii) b) (ii) c) (iv) d) (i)

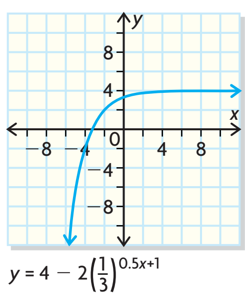
10. a) $y = -2^{2x} + 6$ b) $y = 2^{-x-3} - 2$

11. Translate down 5 units, translate right 1 unit, and vertically compress by factor $\frac{1}{4}$.

12.



13.



14. Reflect in the y-axis and translate 2 units up.

Lesson 4.7, pp. 261–264

1. a) 407.22 b) 35.16 c) 378.30 d) 13 631.85

2.

	Function	Exponential Growth or Decay?	Initial Value	Growth or Decay Rate
a)	$V = 20(1.02)^t$	growth	20	2%
b)	$P = (0.8)^n$	decay	1	−20%
c)	$A = 0.5(3)^x$	growth	0.5	200%
d)	$Q = 600\left(\frac{5}{8}\right)^w$	decay	600	−37.5%

3. a) 1250 persons; it is the value for a in the general exponential function.

b) 3%; it is the base of the exponent minus 1. c) 1730 d) 2012

4. a) \$1500; it is the value for a in the general exponential function.

b) −5%; it is the base of the exponent minus 1.

c) \$437.98

d) 10th month after purchase

5. a) 6% c) 15

b) \$1000 d) $V = 1000(1.06)^n$, \$2396.56

6. a) The doubling period is 10 hours.

b) 2 represents the fact that the population is doubling in number (100% growth rate).

c) 500 is the initial population.

d) 1149 bacteria

e) 2639 bacteria

f) The population is 2000 at 8 a.m. the next day.

7. **(c)** and **(d)** have bases between 0 and 1.

8. **a)** 15361
b) During the 29th year the population will double.
c) 17 years ago
d) $\{n \in \mathbf{R}\}$; $\{P \in \mathbf{R} \mid P \geq 0\}$

9. **a)** 82 °C **b)** 35 °C **c)** after 25 min

10. **a)** $C = 100(0.99)^w$.
100 refers to the percent of the colour at the beginning.
99 refers to the fact that 1% of the colour is lost during every wash.
w refers to the number of washes.
b) $P = 2500(1.005)^t$
2500 refers to the initial population.
1.005 refers to the fact that the population grows 0.5% every year.
t refers to the number of years after 1990.
c) $P = P_0(2)^t$
2 refers to the fact that the population doubles in one day.
t refers to the number of days.

11. **a)** 100% **b)** $P = 80(2^t)$ **c)** 5120 **d)** 226 **e)** 13.6 h **f)** $\{t \in \mathbf{R} \mid t \geq 0\}$; $\{P \in \mathbf{R} \mid p \geq 80\}$

12. **a)** $V = 5(1.06^t)$ **b)** \$0.36 **c)** \$0.91

13. **a)** $I = 100(0.91^d)$ **b)** 49.3%

14. **a)** $P = 100(0.01^a)$ **b)** 6 applications

15. approximately 2.7%

16. **a)** $P = 200(1.75^{\frac{t}{3}})$
b) 200 refers to the initial count of yeast cells.
1.75 refers to the fact that the cells grow by 75% every 3 h.
$\frac{t}{3}$ refers to the fact that the cells grow every 3 h.

17. **a)** It could be a model of exponential growth.
b) $y = 4.25^x$ may model the situation.
c) There are too few pieces of data to make a model and the number of girls may not be the same every year.

18. **a)** exponential decay **b)** 32.3%

Chapter Review, pp. 267–269

1. **a)** $x^2 > x^{-2}$, if $x > 1$; If $x > 1$, then $x^{-2} = \frac{1}{x^2}$ will be less than one and x^2 will be greater than one.
b) $x^{-2} > x^2$, if $-1 < x < 1$ and $x \neq 0$, then $\frac{1}{x^2}$ will be greater than one and x^2 will be less than one.

2. **a)** $(7)^{-1} = -\frac{1}{7}$ **b)** $(-2)^5 = -32$ **c)** $5^0 = 1$ **d)** $4^4 = 256$ **e)** $11^2 = 121$ **f)** $(-3)^3 = -27$

3. **a)** $x^{\frac{7}{3}}$ **b)** $\sqrt[5]{y^8}$ **c)** $p^{\frac{11}{2}}$ **d)** $\sqrt[4]{m^5}$

4. **a)** $\frac{125}{8}$ **b)** $\frac{15}{4}$ **c)** $-\frac{1}{15}$ **d)** 81 **e)** -64 **f)** 2

5. **a)** 1 **b)** b **c)** $\frac{1}{c^{\frac{1}{6}}}$ **d)** $d^{\frac{13}{2}}$ **e)** e^{14} **f)** $\frac{1}{f^{\frac{1}{5}}}$

6. Let $a = 9$ and $b = 16$; then $\sqrt{9 + 16} = \sqrt{25} = 5$ but $\sqrt{9} + \sqrt{16} = 7$, and $5 \neq 7$ so $\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$

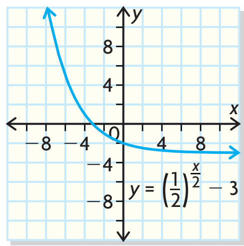
7. **a)** $200x^5 = -6400$ **b)** $\frac{64}{m^2}$ **c)** $\frac{3}{2w^3} = \frac{-1}{18}$ **d)** $3y^5 = -96$ **e)** $\frac{x^{12}}{6} = \frac{2048}{3}$ **f)** $\frac{16}{3x} = \frac{32}{3}$

8. **a)** $3xy^3$ **b)** $\frac{b}{a}$ **c)** $\frac{1}{m^2n^{\frac{1}{2}}}$ **d)** x^9 **e)** $-\frac{1}{x^{1.8}}$ **f)** $\frac{x^9}{y}$

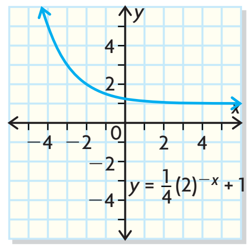
9. **a)** quadratic **b)** linear **c)** exponential **d)** exponential **e)** exponential **f)** exponential

10. **a)** exponential **b)** quadratic **c)** exponential

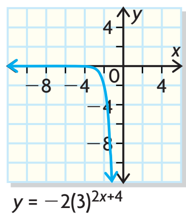
11. **a)** $y = \frac{1}{2}^x$; horizontal stretch by a factor of 2 and vertical translation of 3 down



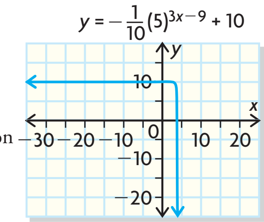
b) $y = 2^x$; vertical stretch of $\frac{1}{4}$, reflection in the y-axis, and vertical translation of 1 unit up



c) $y = 3^x$; reflection in the x-axis, vertical stretch by a factor of 2, horizontal compression by a factor of 2, and horizontal translation of 2 left



d) $y = 5^x$; reflection in the x-axis, vertical compression of $\frac{1}{10}$, horizontal compression by a factor of 3, horizontal translation of 3 units right, and vertical translation of 10 units up

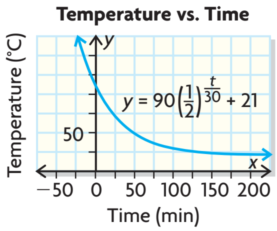


12. y-intercept $= 2$
asymptote: $y = 1$
equation: $y = 2^x + 1$

13.

	Function	Growth/ Decay	y-int	Growth/ Decay Rate
a)	$V(t) = 100(1.08)^t$	growth	100	8%
b)	$P(n) = 32(0.95)^n$	decay	32	−5%
c)	$A(x) = 5(3)^x$	growth	5	200%
d)	$Q(n) = 600\left(\frac{5}{8}\right)^n$	decay	600	−37.5%

14. **a)** The base of the exponent is less than 1.
b) 90 °C
c)



d) 44 °C
e) The 30 in the exponent would be a lesser number.
f) There would be a horizontal compression of the graph; that is, the graph would increase more quickly.

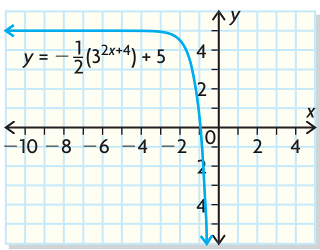
15. **a)** \$28 000 **c)** \$18 758 **e)** \$3500
b) 12.5% **d)** \$20 053 **f)** \$2052

16. **a)** $P = \frac{1}{3}(1.1)^n$
$\frac{1}{3}$ refers to the fact that the pond is $\frac{1}{3}$ covered by lilies.
1.1 refers to the 10% increase in coverage each week.
n refers to the number of weeks.
b) $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{4.5 \times 10^9}}$
A_0 refers to the initial amount of U_{238}.
$\frac{1}{2}$ refers to the half-life of the isotope.
t refers to the number of years.
c) $I = 100(0.96)^n$
0.96 refers to the 4% decrease in intensity per gel.
n refers to the number of gels.

17. **a)** $P = 45\,000(1.03)^n$ **c)** during 2014
b) 74 378 **d)** 7.2%

Chapter Self-Test, p. 270

1. **a)** There is a variable in the exponent part of the equation, so it's an exponential equation.
b) You can tell by the second differences.
c) reflection in the x-axis, vertical compression of $\frac{1}{2}$, horizontal compression by a factor of 2, and translations of 4 left and 5 up



2. **a)** $-\frac{1}{125}$
b) 9

3. **a)** $-243y^5$
b) $\frac{1}{3125a^3b}$
c) $2x$
d) $\frac{1}{4xy^4}$

4. **a)** $I = 100(0.964)^n$
b) 89.6%
c) As the number of gels increases the intensity decreases exponentially.

5. **a)** $P = 2(1.04)^n$, where P is population in millions and n is the number of years since 1990
b) 18 years after 1990 or in 2008

6. (d)

7. $n \neq 0$; n must be odd because you cannot take even roots of negative numbers.

Chapter 5

Getting Started, p. 274

1. **a)** $c = 13$ m
b) $f = \sqrt{57}$ m

2. **a)** $\sin A = \frac{5}{13}$, $\cos A = \frac{12}{13}$, $\tan A = \frac{5}{12}$
b) $\sin D = \frac{8}{11}$, $\cos D = \frac{\sqrt{57}}{11}$, $\tan D = \frac{8\sqrt{57}}{57}$

3. **a)** 67°
b) 43°

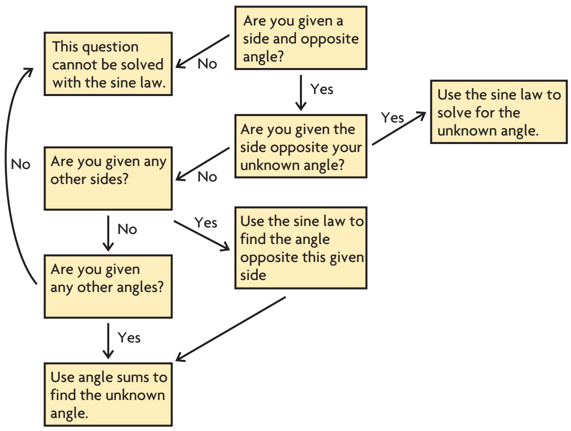
4. **a)** 0.515
b) 0.342

5. **a)** 71°
b) 45°
c) 48°

6. 61 m

7. 25.4 m

8. Answers may vary. For example,



Lesson 5.1, pp. 280–282

1. $\sin A = \frac{5}{13}$, $\cos A = \frac{12}{13}$, $\tan A = \frac{5}{12}$,
$\csc A = \frac{13}{5}$, $\sec A = \frac{13}{12}$, $\cot A = \frac{12}{5}$

2. $\csc\theta = \frac{17}{8}$, $\sec\theta = \frac{17}{15}$, $\cot\theta = \frac{15}{8}$

3. **a)** $\csc\theta = 2$ **b)** $\sec\theta = \frac{4}{3}$ **c)** $\cot\theta = \frac{2}{3}$ **d)** $\cot\theta = 4$

4. **a)** 0.83 **b)** 1.02 **c)** 0.27 **d)** 1.41

5. a) i) $\csc\theta = \frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = \frac{4}{3}$

ii) $\csc\theta = \frac{12}{8.5}$, $\sec\theta = \frac{12}{8.5}$, $\cot\theta = 1$

iii) $\csc\theta = \frac{3.6}{3}$, $\sec\theta = \frac{3.6}{2}$, $\cot\theta = \frac{2}{3}$

iv) $\csc\theta = \frac{17}{8}$, $\sec\theta = \frac{17}{15}$, $\cot\theta = \frac{15}{8}$

b) i) 37° ii) 56° iii) 45° iv) 28°

6. a) 17° b) 52° c) 46° d) 60°

7. a) 5.2 m b) 6.4 m

8. a) 1.2 cm b) 8.0 km

9. a) For any right triangle with acute angle θ, $\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$.

Case 1: If neither the adjacent side nor the opposite side is zero, the hypotenuse is always greater than either side and $\csc\theta > 1$.
Case 2: If the adjacent side is reduced to zero, each time you calculate $\csc\theta$, you get a smaller and smaller value until $\csc\theta = 1$.
Case 3: If the opposite side is reduced to zero, each time you calculate $\csc\theta$, you get a greater and greater value until you reach infinity.
So for all possible cases in a right triangle, cosecant is always greater than or equal to 1.

b) For any right triangle with acute angle θ, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.

Case 1: If neither the adjacent side nor the opposite side is zero, the hypotenuse is always greater than either side and $\cos\theta < 1$.
Case 2: If the opposite side is reduced to zero, each time you calculate $\cos\theta$, you get a greater and greater value until $\cos\theta = 1$.
Case 3: If the adjacent side is reduced to zero, each time you calculate $\cos\theta$, you get a smaller and smaller value until $\cos\theta = 0$.
So for all possible cases in a right triangle, cosine is always less than or equal to 1.

10. $\theta = 45°$ and adjacent side = opposite side

11. a) and b) 13.1 m

12. 7.36 m

13. (b) a right triangle with two 45° angles would have the greatest area, at an angle of 41°, (b) is closest to 45° and will therefore have the greatest area of those triangles.

14. 4.5 m

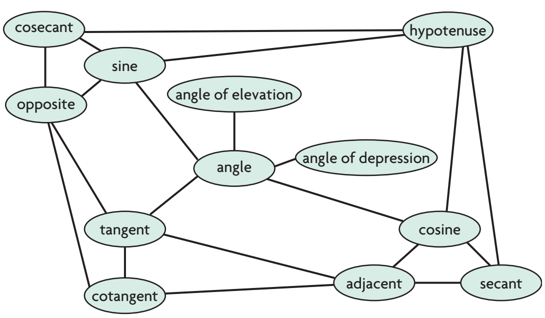
15. 8.15 m

16. a) Answers will vary. For example, 10° b) 7°

c) $\sin\theta = \frac{3}{\sqrt{634}}$, $\cos\theta = \frac{25}{\sqrt{634}}$, $\tan\theta = \frac{3}{25}$,

$\csc\theta = \frac{\sqrt{634}}{3}$, $\sec\theta = \frac{\sqrt{634}}{25}$, $\cot\theta = \frac{25}{3}$

17. Answers will vary. For example,



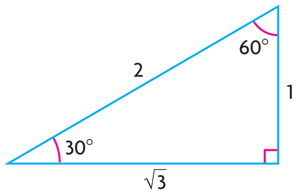
18. $p = 53$ cm, $q = 104$ cm, $\angle P = 27°$, $\angle Q = 63°$

19. Since $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$, the adjacent side must be the smallest side.

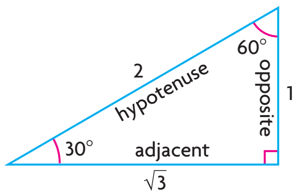
20. (csc and cot) 0°, (sec) 90°

Lesson 5.2, pp. 286–288

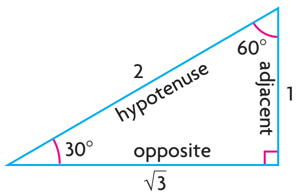
1. a)



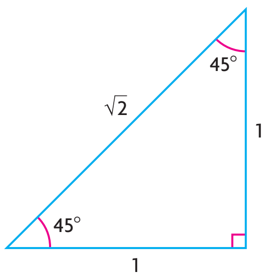
b)



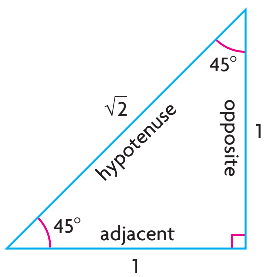
c)



2. a)



b)



3. a) $\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{3}}{2}$ c) 1 d) $\frac{\sqrt{2}}{2}$

4. a) 0 b) 1 c) $-\frac{1}{6}$ d) 0

5. a) $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$ c) $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$

b) $\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1$

6. **a)** $\dfrac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \dfrac{\sqrt{3}}{3}$ **b)** $\dfrac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = 1$ **c)** $\dfrac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \sqrt{3}$

7. **a)** 60° **b)** 30° **c)** 45° **d)** 30°

8. $\dfrac{5\sqrt{3}}{2}$ m, assuming that the wall is perpendicular to the floor

9. $\dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{3}} = \dfrac{1}{\frac{1}{2} \times \frac{\sqrt{3}}{2}} = \dfrac{4\sqrt{3}}{3}$

10. **a)** Use the proportions of the special triangle 45° − 45° − 90°, given that the two smaller sides are 27.4 m.
b) 38.7 m

11. **a)** $3(6 + 6\sqrt{3})$ square units
b) $\dfrac{169}{8}(3 + \sqrt{3})$ square units

12. **a)** 2.595 **b)** $\dfrac{2\sqrt{2} - \sqrt{6} + 10}{4}$
c) Megan didn't use a calculator. Her answer is exact, not rounded off.

13. $\dfrac{2\sqrt{(3 - 3}}{4}$

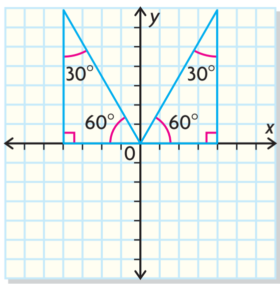
14. $\dfrac{1}{4}$

15. **a)** $1 + \left(\dfrac{3}{\sqrt{3}}\right)^2 = (2)^2$ **c)** $1 + \left(\dfrac{3}{\sqrt{3}}\right)^2 = \left(\dfrac{2}{\sqrt{3}}\right)^2$
b) $1 + (1)^2 = \left(\dfrac{2}{\sqrt{2}}\right)^2$

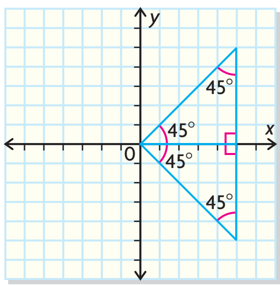
Lesson 5.3, p. 292

1. **a)** 135° **c)** 210°
b) 120°, 240° **d)** 45°, 225°

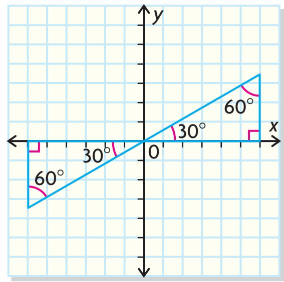
2. **a)**



b)



c)



3. **a)** 45°
b) $\tan\theta = 1$, $\cos\theta = -\dfrac{\sqrt{2}}{2}$, $\sin\theta = -\dfrac{\sqrt{2}}{2}$

4.

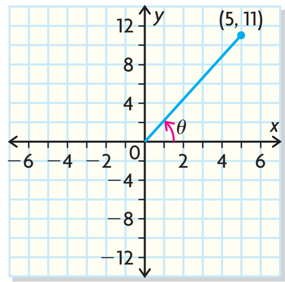
Trigonometric Ratio	Quadrant			
	1	2	3	4
sine	+	+	−	−
cosine	+	−	−	+
tangent	+	−	+	−

Lesson 5.4, pp. 299–301

1.

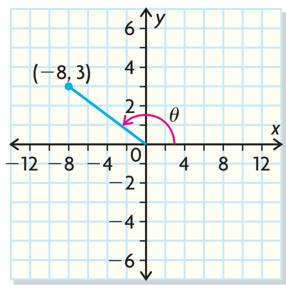
	quadrant	β	sign
a)	4	45°	−
b)	2	70°	−
c)	4	75°	+
d)	3	45°	+

2. **a)** **i)**



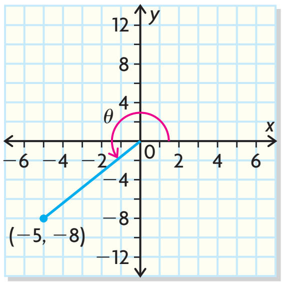
ii) $r = 12.1$
iii) $\sin\theta = \dfrac{11}{12.1}$, $\cos\theta = \dfrac{5}{12.1}$, $\tan\theta = \dfrac{11}{5}$
iv) $\theta = 66°$

b) **i)**



ii) $r = 8.5$
iii) $\sin\theta = \dfrac{3}{8.5}$, $\cos\theta = \dfrac{-8}{8.5}$, $\tan\theta = \dfrac{3}{-8}$
iv) $\theta = 159°$

c) i)

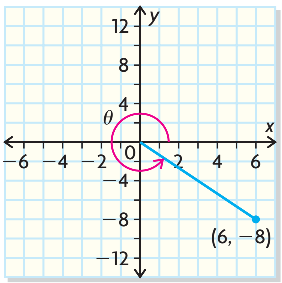


ii) $r = 9.4$

iii) $\sin\theta = \frac{-8}{9.4}$, $\cos\theta = \frac{-5}{9.4}$, $\tan\theta = \frac{8}{5}$

iv) $\theta = 238°$

d) i)



ii) $r = 10$

iii) $\sin\theta = \frac{-8}{10}$ or $\frac{-4}{5}$, $\cos\theta = \frac{6}{10}$ or $\frac{3}{5}$, $\tan\theta = \frac{-8}{6}$ or $\frac{-4}{3}$

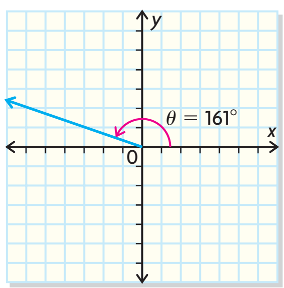
iv) $\theta = 307°$

3. **a)** $\sin 180° = 0$, $\cos 180° = -1$, $\tan 180° = 0$
b) $\sin 270° = -1$, $\cos 270° = 0$, $\tan 270°$ is undefined
c) $\sin 360° = 0$, $\cos 360° = 1$, $\tan 360° = 0$

4. **a)** $\sin 20°$ **b)** $\cos 60°$ **c)** $\tan 290°$ **d)** $\sin 190°$

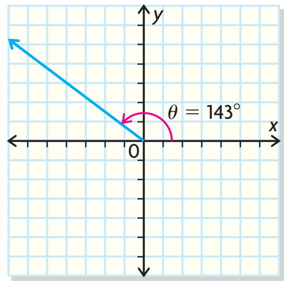
5. **a) i)** $\sin\theta$
ii) $\sin 165° = 0.26$, $\cos 165° = -0.97$, $\tan 165° = -0.27$
b) i) $\tan\theta$
ii) $\sin(–125°) = -0.82$, $\cos(–125°) = -0.57$, $\tan(–125°) = 1.43$
c) i) $\sin\theta$
ii) $\sin(-251°) = 0.95$, $\cos(-251°) = -0.33$, $\tan(-251°) = -2.90$
d) i) $\cos\theta$
ii) $\sin 332° = -0.47$, $\cos 332° = 0.88$, $\tan 332° = -0.53$

6. **a) i)** $x = -2\sqrt{2}$, $y = 1$, $r = 3$
ii)



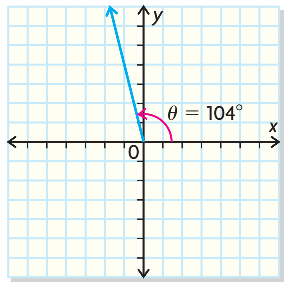
iii) $\theta = 161°$, $\beta = 19°$

b) i) $x = -4$, $y = 3$, $r = 5$
ii)



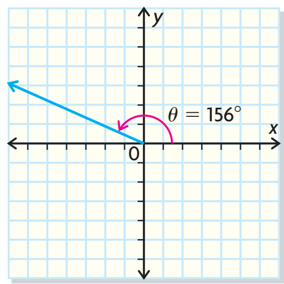
iii) $\theta = 143°$, $\beta = 37°$

c) i) $x = -1$, $y = \sqrt{15}$, $r = 4$
ii)



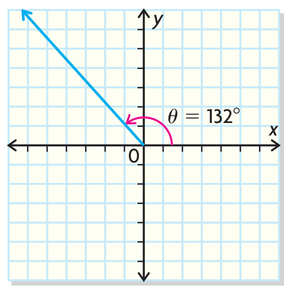
iii) $\theta = 104°$, $\beta = 76°$

d) i) $x = -\sqrt{21}$, $y = 2$, $r = 5$
ii)



iii) $\theta = 156°$, $\beta = 24°$

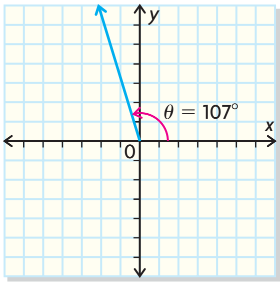
e) i) $x = -10$, $y = 11$, $r = \sqrt{221}$
ii)



iii) $\theta = 132°$, $\beta = 48°$

f) i) $x = -2, y = 3\sqrt{5}, r = 7$

ii)

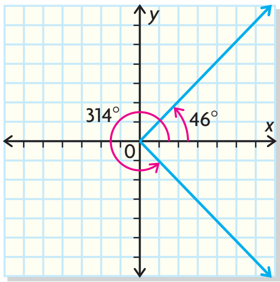


iii) $\theta = 107°, \beta = 73°$

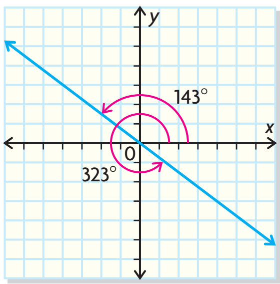
7. **a)** $-199°$ **b)** $-217°$ **c)** $-256°$ **d)** $-204°$ **e)** $-228°$ **f)** $-253°$

8. **a)** 29°, 151°, **b)** 171°, 351° **c)** 151°, 209° **d)** 7°, 187° **e)** 205°, 335° **f)** not possible

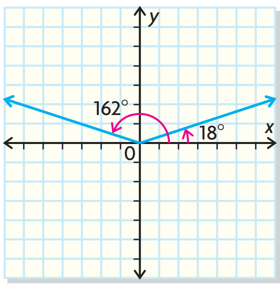
9. **a)** 46°, 314°



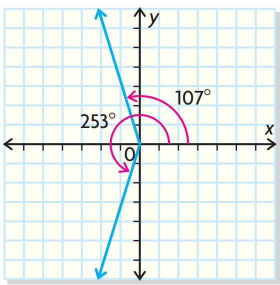
b) 143°, 323°



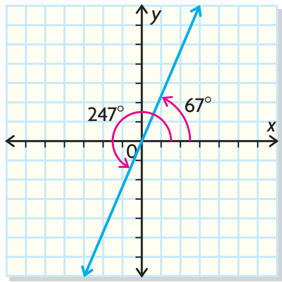
c) 18°, 162°



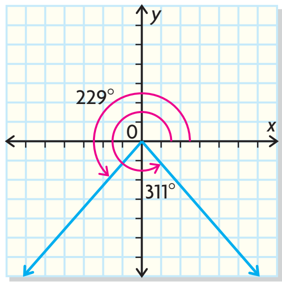
d) 107°, 253°



e) 67°, 247°



f) 229°, 311°



10. **a)** **i)** $225°, -135°$

ii) $\sin\theta = \frac{-\sqrt{2}}{2}$, $\cos\theta = \frac{-\sqrt{2}}{2}$, $\tan\theta = 1$

b) **i)** $270°, -90°$

ii) $\sin\theta = -1$, $\cos\theta = 0$, $\tan\theta$ is undefined

c) **i)** $180°, -180°$

ii) $\sin\theta = 0$, $\cos\theta = -1$, $\tan\theta = 0$

d) **i)** $0°, -360°$

ii) $\sin\theta = 0$, $\cos\theta = 1$, $\tan\theta = 0$

11. You can't draw a right triangle if $\theta \geq 90°$.

12. **a)** quadrant 2 or 3

b) Quadrant 2: $\sin\theta = \frac{\sqrt{119}}{12}$, $\cos\theta = \frac{-5}{12}$, $\tan\theta = \frac{\sqrt{119}}{-5}$

Quadrant 3: $\sin\theta = \frac{-\sqrt{119}}{12}$, $\cos\theta = \frac{-5}{12}$, $\tan\theta = \frac{\sqrt{119}}{5}$

c) 115°, 245°

13. $\alpha = 180°$

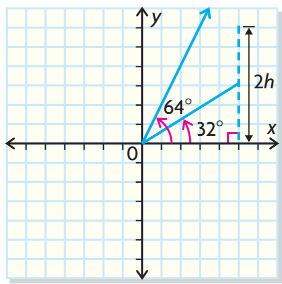
14. Answers may vary. For example, given $P(x, y)$ on the terminal arm of angle θ, $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $\tan\theta = \frac{y}{x}$.

15. **a)** 25°, 155°, 205°, 335° **b)** 148°, 352° **c)** 16°, 106°, 196°, 286°

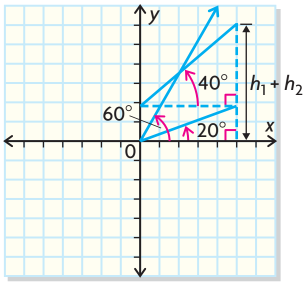
16. **a)** θ could lie in quadrant 3 or 4. $\theta = 233°$ or $307°$

b) θ could lie in quadrant 2 or 3. $\theta = 139°$ or $221°$

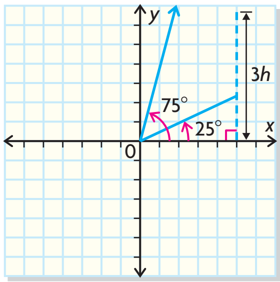
17. **a)**



b)



c)



Mid-Chapter Review, p. 304

1. **a)** 2.9238 **b)** 3.8637 **c)** 5.6713 **d)** 1.0125

2. **a)** 49° **b)** 76° **c)** 38° **d)** 66°

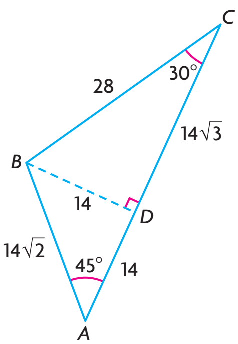
3. tan 54°, csc 46°, sec 44°, or cot 36°

4. 16.6 m

5. $45° < \theta < 90°$

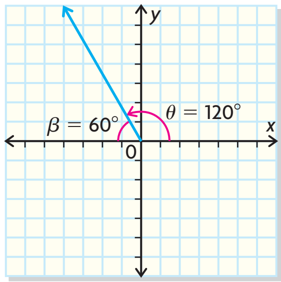
6. **a)** $\frac{\sqrt{3}}{2}$ **b)** 1 **c)** 2 **d)** $\sqrt{2}$

7. **a)**

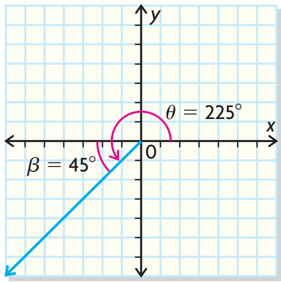


b) $\sin A = \frac{\sqrt{2}}{2}$, $\cos A = \frac{\sqrt{2}}{2}$, $\tan A = 1$, $\sin DBC = \frac{\sqrt{3}}{2}$, $\cos DBC = \frac{1}{2}$, $\tan DBC = \sqrt{3}$

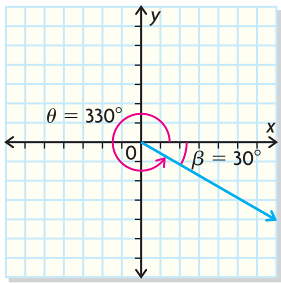
8. **a) i)** $\frac{\sqrt{3}}{2}$ **ii)** 60°



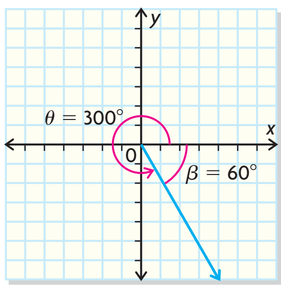
b) i) $\frac{-\sqrt{2}}{2}$ **ii)** 135°



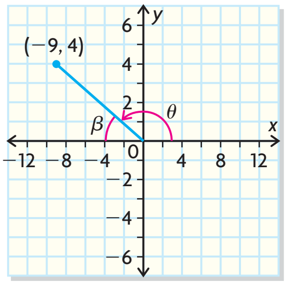
c) i) $\frac{-\sqrt{3}}{3}$ **ii)** 150°



d) i) $\frac{1}{2}$ **ii)** 60°



9. a)



b) 24°

c) 156°

10. No, the only two possible angles within the given range are 37° and 323°.

11. a) $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{-8}{17}$, $\csc\theta = \frac{17}{15}$, $\sec\theta = \frac{17}{-8}$, $\cot\theta = \frac{-8}{15}$

b) 118°

12. 235°, 305°

13. a), b), c), e), and f) must be false.

a) $-1 \le \cos\theta \le 1$ **b)** $\tan\theta < 0$ **c)** $\sec\theta < 0$

e) $\cot\theta < 0$ **f)** $-1 \le \sin\theta \le 1$

Lesson 5.5, pp. 310–311

1. a) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$\begin{aligned}\text{L.S.} &= \cot\theta\\ &= \frac{x}{y}\\ \text{R.S.} &= \frac{\cos\theta}{\sin\theta}\\ &= \frac{x}{r} \div \frac{y}{r}\\ &= \frac{x}{r} \times \frac{r}{y}\\ &= \frac{x}{y}\end{aligned}$$

$\therefore \cot\theta = \frac{\cos\theta}{\sin\theta}$ for all angles θ where $0° \le \theta \le 360°$ except 0°, 180°, and 360°.

b) $\tan\theta\cos\theta = \sin\theta$

$$\begin{aligned}\text{L.S.} &= \tan\theta\cos\theta\\ &= \frac{y}{x} \times \frac{x}{r}\\ &= \frac{y}{r}\\ \text{R.S.} &= \sin\theta\\ &= \frac{y}{r}\end{aligned}$$

$\therefore \tan\theta\cos\theta = \sin\theta$ for all angles θ where $0° \le \theta \le 360°$.

c) $\csc\theta = \frac{1}{\sin\theta}$

$$\begin{aligned}\text{L.S.} &= \csc\theta\\ &= \frac{r}{y}\\ \text{R.S.} &= \frac{1}{\sin\theta}\\ &= 1 \div \frac{y}{r}\\ &= 1 \times \frac{r}{y}\\ &= \frac{r}{y}\end{aligned}$$

$\therefore \csc\theta = \frac{1}{\sin\theta}$ for all angles θ where $0° \le \theta \le 360°$ except 0°, 180°, and 360°.

d) $\cos\theta\sec\theta = 1$

$$\begin{aligned}\text{L.S.} &= \cos\theta\sec\theta\\ &= \frac{x}{r} \times \frac{r}{x}\\ &= 1\\ \text{R.S.} &= 1\end{aligned}$$

$\therefore \cos\theta\sec\theta = 1$ for all angles θ where $0° \le \theta \le 360°$.

2. a) $\cos^2\alpha$ **b)** $\sec\alpha$ or $\frac{1}{\cos\alpha}$ **c)** 1 **d)** $\cos\alpha$

3. a) $(1 - \cos\theta)(1 + \cos\theta)$ **c)** $(\sin\theta - 1)^2$

b) $(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)$ **d)** $\cos\theta(1 - \cos\theta)$

4. $\frac{\cos^2\theta}{1 - \sin\theta} = 1 + \sin\theta$

$$\begin{aligned}\cos^2\theta &= (1 + \sin\theta) \times (1 - \sin\theta)\\ \cos^2\theta &= 1 + \sin\theta - \sin\theta - \sin^2\theta\\ \cos^2\theta &= 1 - \sin^2\theta\\ \cos^2\theta &= \cos^2\theta\end{aligned}$$

5. a) $\frac{\sin x}{\tan x} = \cos x$

$$\begin{aligned}\text{L.S.} &= \frac{\sin x}{\tan x}\\ &= \sin x \div \frac{\sin x}{\cos x}\\ &= \sin x \times \frac{\cos x}{\sin x}\\ &= \cos x\end{aligned}$$

= R.S., for all angles x where $0° \le x \le 360°$ except 0°, 90°, 180°, 270°, and 360°.

b) $\frac{\tan\theta}{\cos\theta} = \frac{\sin\theta}{1 - \sin^2\theta}$

$$\begin{aligned}\text{L.S.} &= \frac{\tan\theta}{\cos\theta}\\ &= \frac{\sin\theta}{\cos\theta} \div \cos\theta\\ &= \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}\\ &= \frac{\sin\theta}{\cos^2\theta}\\ &= \frac{\sin\theta}{1 - \sin^2\theta}\end{aligned}$$

= R.S., for all angles θ where $0° \le \theta \le 360°$ except 90° and 270°.

c) $\frac{1}{\cos\alpha} + \tan\alpha = \frac{1 + \sin\alpha}{\cos\alpha}$

$$\begin{aligned}\text{L.S.} &= \frac{1}{\cos\alpha} + \tan\alpha\\ &= \frac{1}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha}\\ &= \frac{1 + \sin\alpha}{\cos\alpha}\end{aligned}$$

= R.S., for all angles α where $0° \le \alpha \le 360°$ except 90° and 270°.

d) $\sin\theta\cos\theta\tan\theta = 1 - \cos^2\theta$

$$\begin{aligned}
\text{L.S.} &= \sin\theta\cos\theta\tan\theta \\
&= \frac{y}{r}\times\frac{x}{r}\times\frac{y}{x} \\
&= \frac{y^2}{r^2} \\
&= \sin^2\theta \\
&= 1-\cos^2\theta \\
&= \text{R.S., for all angles } \theta \text{ where } 0° \le \theta \le 360° \text{ except } 90° \text{ and } 270°.
\end{aligned}$$

6. You need to prove that the equation is true for *all* angles specified, not just one.

7. **a)** $\cos\theta(1-\sin\theta)$ **b)** $-\sin^2\theta$ **c)** 1 **d)** $\dfrac{\csc\theta - 2}{\csc\theta + 1}$, where $\csc\theta \ne 1$

8. **a)** $\dfrac{\sin^2\theta}{1-\cos\theta} = 1+\cos\theta$

$$\begin{aligned}
\text{L.S.} &= \frac{\sin^2\theta}{1-\cos\theta} \\
&= \frac{1-\cos^2\theta}{1-\cos\theta} \\
&= \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta} \\
&= 1+\cos\theta \\
&= \text{R.S., where } \cos\theta \ne 1
\end{aligned}$$

b) $\dfrac{\tan^2\alpha}{1+\tan^2\alpha} = \sin^2\alpha$

$$\begin{aligned}
\text{L.S.} &= \frac{\tan^2\alpha}{1+\tan^2\alpha} \\
&= \frac{\tan^2\alpha}{\sec^2\alpha} \\
&= \frac{\sin^2\alpha}{\cos^2\alpha}\div\frac{1}{\cos^2\alpha} \\
&= \frac{\sin^2\alpha}{\cos^2\alpha}\times\cos^2\alpha \\
&= \sin^2\alpha \\
&= \text{R.S., where } \tan\alpha \ne -1
\end{aligned}$$

c) $\cos^2 x = (1-\sin x)(1+\sin x)$

$$\begin{aligned}
\text{R.S.} &= (1-\sin x)(1+\sin x) \\
&= 1-\sin x+\sin x-\sin^2 x \\
&= 1-\sin^2 x \\
&= \cos^2 x \\
&= \text{L.S.}
\end{aligned}$$

d) $\sin^2\theta + 2\cos^2\theta - 1 = \cos^2\theta$

$$\begin{aligned}
\text{L.S.} &= \sin^2\theta + 2\cos^2\theta - 1 \\
&= \sin^2\theta + \cos^2\theta + \cos^2\theta - 1 \\
&= 1 + \cos^2\theta - 1 \\
&= \cos^2\theta \\
&= \text{R.S.}
\end{aligned}$$

e) $\sin^4\alpha - \cos^4\alpha = \sin^2\alpha - \cos^2\alpha$

$$\begin{aligned}
\text{L.S.} &= \sin^4\alpha - \cos^4\alpha \\
&= (\sin^2\alpha - \cos^2\alpha)(\sin^2\alpha + \cos^2\alpha) \\
&= (\sin^2\alpha - \cos^2\alpha)\times 1 \\
&= \sin^2\alpha - \cos^2\alpha \\
&= \text{R.S.}
\end{aligned}$$

f) $\tan\theta + \dfrac{1}{\tan\theta} = \dfrac{1}{\sin\theta\cos\theta}$

$$\begin{aligned}
\text{L.S.} &= \tan\theta + \frac{1}{\tan\theta} \\
&= \frac{\sin\theta}{\cos\theta} + \left(1\div\frac{\sin\theta}{\cos\theta}\right) \\
&= \frac{\sin\theta}{\cos\theta} + \left(1\times\frac{\cos\theta}{\sin\theta}\right) \\
&= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \\
&= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\cos\theta\sin\theta} \\
&= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} \\
&= \frac{1}{\sin\theta\cos\theta} \\
&= \text{R.S., where } \tan\theta \ne 0,\ \sin\theta \ne 0, \text{ and } \cos\theta \ne 0.
\end{aligned}$$

9. **a)** Farah's method only works for equations that don't have a trigonometric ratio in the denominator.

b) If an equation has a trigonometric ratio in the denominator that can't equal zero, Farah's method doesn't work.

10. not an identity; $\csc^2 45 + \sec^2 45 = 4$ is not an identity

11. $\sin^2 x\left(1+\dfrac{1}{\tan^2 x}\right) = 1$

$$\begin{aligned}
\text{L.S.} &= \sin^2 x\left(1+\frac{1}{\tan^2 x}\right) \\
&= \sin^2 x\left(1+1\div\frac{\sin^2 x}{\cos^2 x}\right) \\
&= \left(\sin^2 x+\sin^2 x\div\frac{\sin^2 x}{\cos^2 x}\right) \\
&= \left(\sin^2 x+\sin^2 x\times\frac{\cos^2 x}{\sin^2 x}\right) \\
&= \sin^2 x+\cos^2 x \\
&= 1 \\
&= \text{R.S.}
\end{aligned}$$

12. **a)** $\dfrac{\sin^2\theta + 2\cos\theta - 1}{\sin^2\theta + 3\cos\theta - 3} = \dfrac{\cos^2\theta + \cos\theta}{-\sin^2\theta}$

$$\begin{aligned}
\text{L.S.} &= \frac{\sin^2\theta + 2\cos\theta - 1}{\sin^2\theta + 3\cos\theta - 3} \\
&= \frac{(1-\cos^2\theta) + 2\cos\theta - 1}{(1-\cos^2\theta) + 3\cos\theta - 3} \\
&= \frac{-\cos^2\theta + 2\cos\theta}{-\cos^2\theta + 3\cos\theta - 2} \\
&= \frac{\cos\theta\times(2-\cos\theta)}{(2-\cos\theta)(\cos\theta-1)} \\
&= \frac{\cos\theta}{\cos\theta-1}
\end{aligned}$$

$$\begin{aligned}
\text{R.S.} &= \frac{\cos^2\theta+\cos\theta}{-\sin^2\theta} \\
&= \frac{\cos^2\theta+\cos\theta}{\cos^2\theta-1} \\
&= \frac{\cos\theta\times(\cos\theta+1)}{(\cos\theta+1)(\cos\theta-1)} \\
&= \frac{\cos\theta}{\cos\theta-1} \\
&= \text{L.S., where } \sin\theta \ne 0,\ \cos\theta \ne 1
\end{aligned}$$

b) $\sin^2\alpha - \cos^2\alpha - \tan^2\alpha = \dfrac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1 - \sin^2\alpha}$

$$\begin{aligned}
\text{L.S.} &= \sin^2\alpha - \cos^2\alpha - \tan^2\alpha \\
&= \sin^2\alpha - \cos^2\alpha - \frac{\sin^2\alpha}{\cos^2\alpha} \\
&= \frac{\cos^2\alpha \times \sin^2\alpha}{\cos^2\alpha} - \frac{\cos^4\alpha}{\cos^2\alpha} - \frac{\sin^2\alpha}{\cos^2\alpha} \\
&= \frac{\cos^2\alpha \times \sin^2\alpha - \cos^4\alpha - \sin^2\alpha}{1 - \sin^2\alpha} \\
&= \frac{\sin^2\alpha \times (1 - \sin^2\alpha) - (1 - \sin^2\alpha)^2 - \sin^2\alpha}{1 - \sin^2\alpha} \\
&= \frac{-\sin^4\alpha - (1 - 2\sin^2\alpha + \sin^4\alpha)}{1 - \sin^2\alpha} \\
&= \frac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1 - \sin^2\alpha} \\
&= \text{R.S., where } \sin\theta \neq 1
\end{aligned}$$

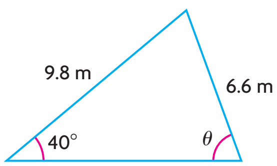
13. Answers may vary. For example, $\dfrac{\sin^3\theta}{\cos\theta} + \sin\theta\cos\theta = \tan\theta$ by multiplying by $\dfrac{\sin\theta}{\cos\theta}$

14. **a)** (iii)
b) **i)** $\sin^2 x \neq 1$
iv) $\sin\beta \neq \cos\beta$
v) $\sin\beta \neq 0$, $\cos\beta \neq -1$
vi) $\cos x \neq 1$

Lesson 5.6, pp. 318–320

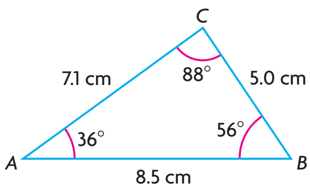
1. **a)** 47° **b)** 128°

2. **a)**



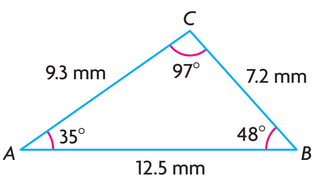
b) $h = 6.3$ m, h is less than either given side
c) two lengths (9.5 m or 5.6 m)

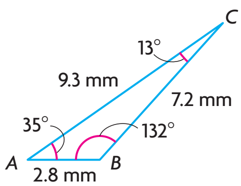
3. **a)** no triangle exists
b) no triangle exists
c)



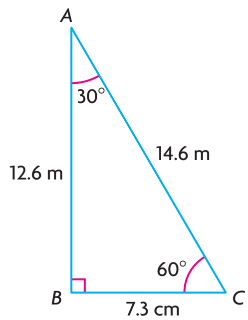
4. **a)** 40° **b)** 68° or 23°

5. **a)**

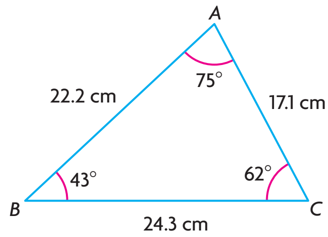




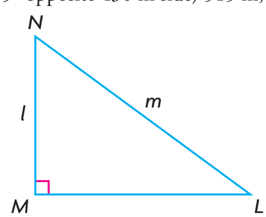
b)



c) no triangle exists
d)

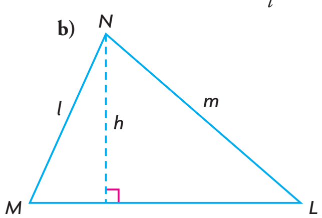


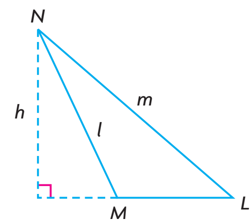
6. 34° **7.** 257.0 m **8.** 7499 m
9. 299.8 m **10.** 25 m
11. Carol only on same side is 66°. This is 11 m.
a) 28 m **b)** 31 m **c)** 52 m;
Carol only on same side as 66°. Other distance is 11 m.
a) 6 m **b)** 6 m **c)** 2 m;
All on same side. Distance to 66° is 11 m.
a) 28 m **b)** 37 m **c)** 16 m;
All on same side. Distance to 35° is 11 m.
a) 19 m **b)** 24 m **c)** 7 m
12. 481 m
13. (35° opposite 430 m side) 515 m, 8°, and 137°
14. **a)**



(right triangle) $l < m$, $\dfrac{\sin L}{l} = \dfrac{1}{m}$, (height) $h = l = m\sin L$

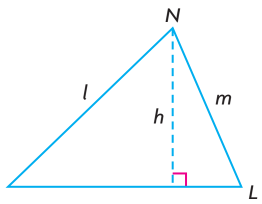
b)





$h < l < m$, (height) $h = m\sin L$

c)

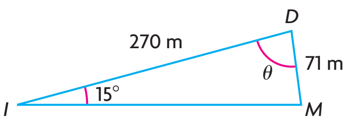


(acute triangle) $l > m$, (height) $h = m\sin L$

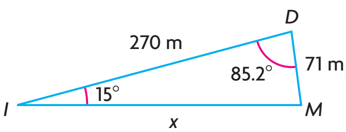
15. a) (lighthouse A) 3 km, (lighthouse B) 13 km b) 5.4 km or 5 km
16. a) 366 m b) no
17. (lower guy wire) 264 m, (upper guy wire) 408 m

Lesson 5.7, pp. 325–327

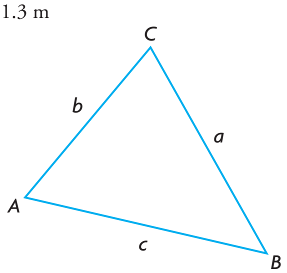
1. a) 6.2 b) 18.7
2. a) 35° b) 40°
3. a) 8.0 b) 104° c) 100° d) 69.4
4. a) $m \doteq 15.0$ cm, $\angle L \doteq 46°$, $\angle N \doteq 29°$
   b) $\angle R = 32°$, $t \doteq 13.9$ cm, $r \doteq 15.7$ cm
   c) $\angle A \doteq 98°$, $\angle B \doteq 30°$, $\angle C \doteq 52°$
   d) $\angle X = 124°$, $y \doteq 8.1$ cm, $z \doteq 12.9$ cm
5. 11°
6. 138 m
7. 1.4
8. (tower A) 31.5 km, (tower B) 22.3 km
9. a) Answers may vary. For example, Mike is standing on the other road and is 71 m from Darryl. From Darryl's position, what angle, to the nearest degree, separates the intersection from Mike?



   b) Answers may vary. For example, How far, to the nearest metre, is Mike from the intersection?



   (Answer: 424 m)
10. 101.3 m
11.



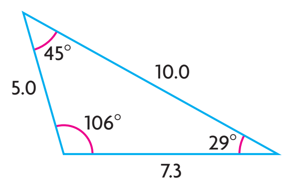
   a) a, b, and c or b, c, and $\angle A$

   $a^2 = b^2 + c^2 - 2bc\cos A$

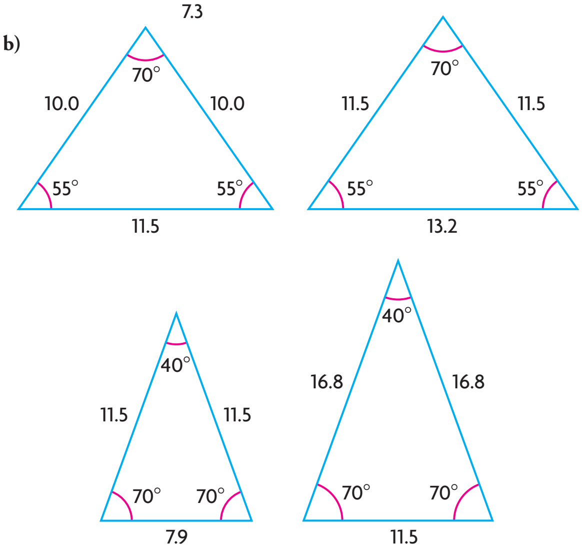
   b) a, b, and $\angle A$

   $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
12. 35 cm

13. a)



   b)



14. a) 2.4 km b) A is higher by 0.3 km.

Lesson 5.8, pp. 332–335

1. Answers may vary. For example, use primary trigonometric ratios to calculate the hypotenuse of each right triangle. Add the results together to get the length of line needed.
2. a) Answers may vary. For example, if you use the sine law, you don't have to solve a quadratic equation.
   b) Answers may vary. For example, use a right triangle with acute angles 40° and 50°. Then, solve $\cos 50° = \frac{2.5}{x}$.
3. a) 15 cm b) 37.9 cm c) 17 cm d) 93°
4. a) 520.5 m
   b) Use the sine law, then trigonometric ratios.
5. Yes, the distance is about 7127 m.
6. 258 m 7. 24 m 8. 736 m 9. 47°
10. 4.5 m, 2.0 m, 6.0 m piece fits in 2.6 m × 2.1 m × 6.0 m vehicle. Other two pieces fit in 2.5 m × 2.1 m × 4.0 m vehicle.
11. Yes, the height is about 23 m.
12. a) You can't solve the problem.
    b) You need the altitude of the balloon and angle of depression from the balloon to Bill and Chris.
13. a) 39 km b) 34 km
14. 524 m
15. 148.4 km
16. a) 84.4 m b) 64.2 m

Chapter Review, pp. 338–339

1. **a) i)** $\csc \theta = \frac{\sqrt{233}}{13}$, $\sec \theta = \frac{\sqrt{233}}{8}$, $\cot \theta = \frac{8}{13}$

ii) 58°

b) i) $\csc \theta = \frac{37}{12}$, $\sec \theta = \frac{37}{35}$, $\cot \theta = \frac{35}{12}$

ii) 19°

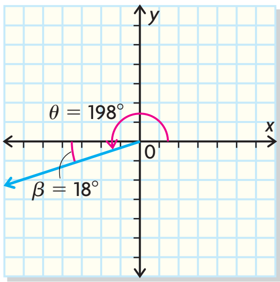
c) i) $\csc \theta = \frac{39}{23}$, $\sec \theta = \frac{39}{4\sqrt{62}}$, $\cot \theta = \frac{4\sqrt{62}}{23}$

ii) 36°

2. **a)** $\frac{3}{4}$ **b)** 0 **c)** $\frac{2\sqrt{3} + 3\sqrt{2}}{6}$

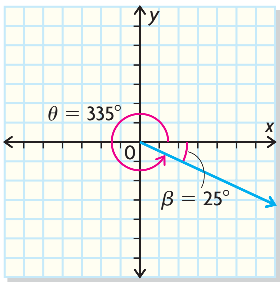
3. **a) i)** +; $\tan 18° = 0.3249$

ii) $\theta = 18°$ is the principal and related angle.



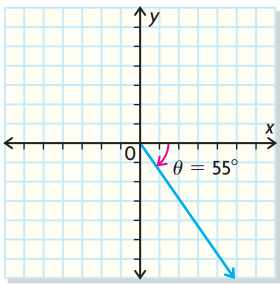
b) i) −; $\sin 205° = -0.4226$

ii) principal angle is $\theta = 205°$, $\beta = 25°$ is the related angle



c) i) +; $\cos(-55°) = 0.5736$

ii) principal angle is $\theta = 305°$, $\beta = 55°$ is the related angle



4. **a)** $\sin \theta = \frac{5\sqrt{29}}{29}$, $\cos \theta = \frac{-2\sqrt{29}}{29}$, $\tan \theta = \frac{5}{-2}$

b) $\sin \theta = \frac{-\sqrt{2}}{2}$, $\cos \theta = \frac{\sqrt{2}}{2}$, $\tan \theta = -1$

c) $\sin \theta = \frac{-5\sqrt{41}}{41}$, $\cos \theta = \frac{-4\sqrt{41}}{41}$, $\tan \theta = \frac{5}{4}$

5. **a)** quadrant 2 or 3

b) quadrant 2: $\sin \phi = \frac{2}{\sqrt{53}}$, $\tan \phi = \frac{2}{-7}$, $\csc \phi = \frac{\sqrt{53}}{2}$, $\sec \phi = \frac{\sqrt{53}}{-7}$, $\cot \phi = \frac{-7}{2}$; quadrant 3: $\sin \phi = \frac{-2}{\sqrt{53}}$, $\tan \phi = \frac{2}{7}$, $\csc \phi = \frac{\sqrt{53}}{-2}$, $\sec \phi = \frac{\sqrt{53}}{-7}$, $\cot \phi = \frac{7}{2}$

c) quadrant 2: 164°, quadrant 3: 196°

6. The equation is an identity.

7. **a)**
$$\begin{aligned}\tan \alpha \cos \alpha &= \left(\frac{\sin \alpha}{\cos \alpha}\right)(\cos \alpha)\\ &= \sin \alpha\end{aligned}$$

b)
$$\begin{aligned}\frac{1}{\cot \phi} &= \tan \phi\\ &= \frac{\sin \phi}{\cos \phi}\\ &= \sin \phi\left(\frac{1}{\cos \phi}\right)\\ &= \sin \phi \sec \phi\end{aligned}$$

ϕ cannot be equal to 90° or 270°.

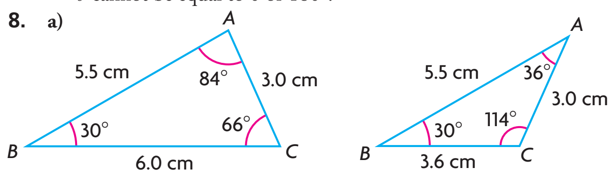
c)
$$\begin{aligned}1 - \cos^2 x &= \sin^2 x\\ &= \sin^2 x\left(\frac{\cos x}{\cos x}\right)\\ &= \sin x \ \sin x\left(\frac{\cos x}{\cos x}\right)\\ &= \sin x \ \cos x\left(\frac{\sin x}{\cos x}\right)\\ &= \sin x \ \cos x \ \tan x\\ &= \sin x \ \cos x\left(\frac{1}{\cot x}\right)\\ &= \frac{\sin x \cos x}{\cot x}\end{aligned}$$

x cannot be equal to 0 or 180°.

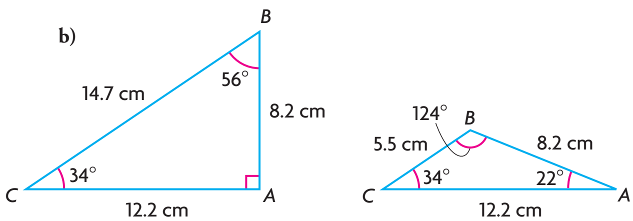
d)
$$\begin{aligned}&\sec \theta \cos \theta + \sec \theta \sin \theta\\ &= \left(\frac{1}{\cos \theta}\right)\cos \theta + \left(\frac{1}{\cos \theta}\right)\sin \theta\\ &= 1 + \frac{\sin \theta}{\cos \theta}\\ &= 1 + \tan \theta\end{aligned}$$

θ cannot be equal to 0 or 180°.

8. **a)**



b)



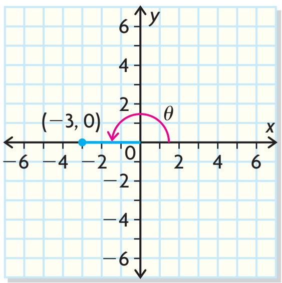
c) no triangle exists

9. 30.5 km

10. **a)** 15.5
b) 8.4
c) 5.2
11. 4.4 m
12. 13 m
13. 18°

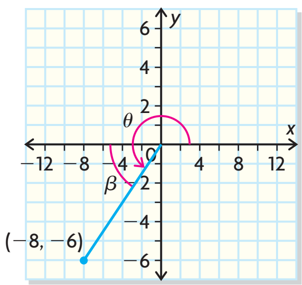
Chapter Self-Test, p. 340

1. **a) i)**



$\sin\theta = 0$ $\quad$ $\csc\theta$ is undefined
$\cos\theta = -1$ $\quad$ $\sec\theta = -1$
$\tan\theta = 0$ $\quad$ $\cot\theta$ is undefined
ii) $\theta = 180°$ is the principal angle; related angle is 0°

b) i)



$\sin\theta = \frac{-3}{5}$ $\quad$ $\csc\theta = \frac{5}{-3}$

$\cos\theta = \frac{-4}{5}$ $\quad$ $\sec\theta = \frac{5}{-4}$

$\tan\theta = \frac{3}{4}$ $\quad$ $\cot\theta = \frac{4}{3}$

ii) $\theta = 217°$ is the principal angle, $\beta = 37°$ is the related angle

2. **a)** 210°, 330° $\quad$ **c)** 135°, 315°
b) 30°, 330° $\quad$ **d)** 120°, 240°
3. **a)** $\frac{-60}{169}$ $\quad$ **b)** 1
4. **i) a)** $\sin^2\theta + \cos^2\theta = 1$

$$\frac{1}{\cos^2\theta}(\sin^2\theta + \cos^2\theta) = \left(\frac{1}{\cos^2\theta}\right)(1)$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

b) $\sin^2\theta + \cos^2\theta = 1$

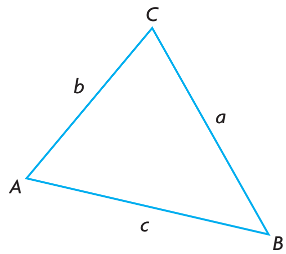
$$\frac{1}{\sin^2\theta}(\sin^2\theta + \cos^2\theta) = \left(\frac{1}{\sin^2\theta}\right)(1)$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2 = \csc^2\theta$$

ii) These identities are derived from $\sin^2\Phi + \cos^2\Phi = 1$

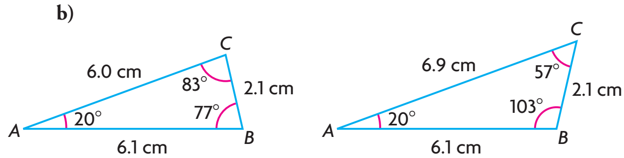
5. **a)**



b) $a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$

c) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

6. **a)** 97.4 m $\quad$ **b)** 1.6 m
7. **a)** no triangle exists
b)

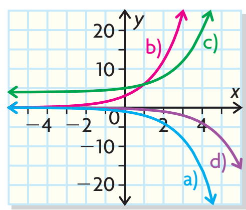


8. 22 m

Chapter 6

Getting Started, p. 344

1. **a)** x represents the number of times the price is reduced by \$2. The factor $(30 - 2x)$ represents the price of one T-shirt in terms of the number of times the price is reduced; the factor $(100 + 20x)$ represents the total number of T-shirts sold in terms of the number of times the price is reduced.
b) 15 times $\quad$ **d)** \$4000 $\quad$ **f)** 200 T-shirts
c) 5 times $\quad$ **e)** \$20
2. **a)** 360 cm $\quad$ **b)** 0.25 s $\quad$ **c)** 720 cm/s
d) domain: $\{t \in \mathbf{R} \mid 0 \le t \le 0.5\}$;
range: $\{d \in \mathbf{R} \mid 0 \le d \le 180\}$
3. **a)** 32° $\quad$ **b)** 154°
4. 3.2 m
5.



6. 25 m
7. Answers will vary and may include the following:
- Vertical translation $y = x^2 + c$
  $y = x^2 - c$
- Horizontal translation $(x + d)^2$
  $(x - d)^2$
- Vertical stretch ≠ compression $y = ax^2$

Lesson 6.1, pp. 352–356

1. **a)** is periodic because the cycle repeats;
b) is not periodic because the cycle does not repeat;
c) is periodic because the cycle repeats;
d) is not periodic because the cycle does not repeat

2. range: $\{y \in \mathbf{R} \mid 2 \leq y \leq 10\}$; period: 4; axis: $y = 6$; amplitude: 4

3. **a)** 1 s
b) 1.5 cm
c) range: $\{d \in \mathbf{R} \mid 0 \leq d \leq 1.5\}$
d) domain: $\{t \in \mathbf{R} \mid 0 \leq t \leq 5\}$
e) $y = 0.75$
f) 0.75 cm
g) horizontal component of graph: device is not in motion, it remains fixed at 1.5 cm. Component of the graph with negative slope: the device is approaching the appliance and simultaneously attaching the bolt. Component of the graph with the positive slope; the device has finished attaching the bolt and is moving away from the house appliance.

4. **a)** period: 6
b) period: 2
c) not periodic
d) not periodic
e) period: 5.5
f) period: 20

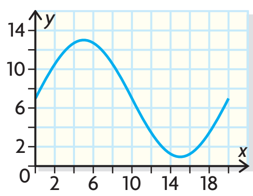
5. (a), (b), and (f)

6. (b) repeating cycle

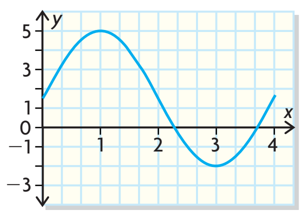
7. **a)** yes
b) 10 cm
c) 2 min
d) 8 min
e) range: $\{d \in \mathbf{R} \mid 4 \leq d \leq 10\}$
f) 7 cm
g) At $t = 6$ min and every 8 min from that time
h) 10 cm

8. **a)** 8 s; one rotation of Ferris wheel
b) $h = 4$
c) 3 m
d) $\{h \in \mathbf{R} \mid 1 \leq h \leq 7\}$
e) 32 s
f) 4 s, 12 s, 20 s, 28 s
g) at $t = 26$ s and $t = 30$ s

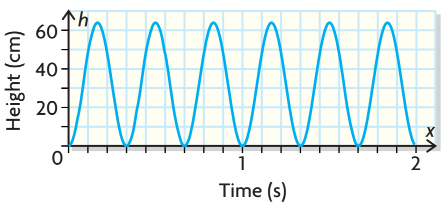
9.



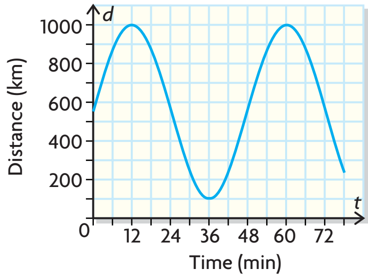
10.



11.

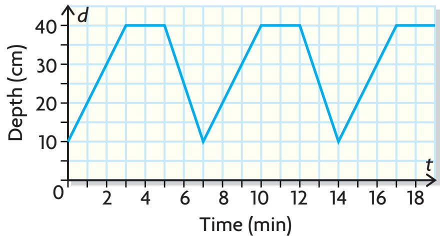


12. **a)**



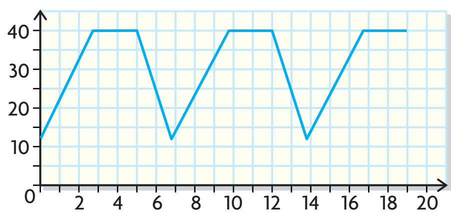
b) yes
c) 48 min: time to complete one orbit
d) approximately 900 km
e) At $t = 12$ min and every 48 min after that time
f) $\{t \in \mathbf{R} \mid 0 \leq t \leq 288\}$

13. **a)**

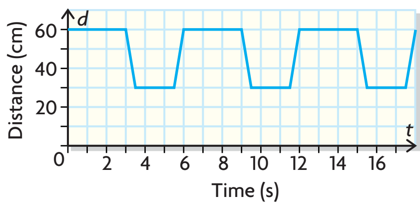


b) yes
c) period: 7; axis: $d = 25$; amplitude: 15
d) 10 cm/min
e) 15 cm/min
f) no, never intersects the t-axis

14. A periodic function is a function that produces a graph that has a regular repeating pattern over a constant interval. It describes something that happens in a cycle, repeating in the same way over and over.
Example:



15. **a)**



b) 6 s
c) range: $\{d \in \mathbf{R} \mid 30 \leq d \leq 60\}$; domain: $\{t \in 0 \leq t \leq 18\}$

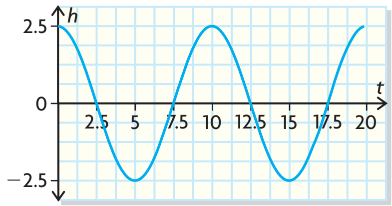
16. At time $t = 0$, the paddle is 40 cm in front of the CBR and doesn't move for 1 s. At 1 s, the paddle moves 30 cm away from the CBR and then returns to its original position of 40 cm in front of the CBR at 1.5 s. For 1 s, the paddle doesn't move. At $t = 2.5$ s, the paddle moves 30 cm away from the CBR and then returns to its original position of 40 cm in front of the CBR at $t = 3$ s where it remains for 1 s until 4 s.

Lesson 6.2, pp. 363–364

1. **a)** amplitude: 3; period: 180°; axis: $y = 1$
b) amplitude: 4; period: 720°; $y = -2$

2. **a)** -0.18 **b)** 90°, 270°

3. **a)**



b) 10 s **c)** -1 m **d)** 4 s

4. (1.29, 1.53)

5. **a)** periodic and sinusoidal **b)** neither **c)** periodic **d)** neither **e)** periodic and sinusoidal **f)** neither

6. not necessarily periodic or sinusoidal, answers may vary

7. **a)** $g(90) = 1$; when $x = 90$, $y = 1$, or the sine of (y-coordinate of a point on the unit circle) at 90 is 1.
b) $h(90) = 0$; when $x = 90$, $y = 0$, or the cosine of (x-coordinate of a point on the unit circle) at 90 is 0.

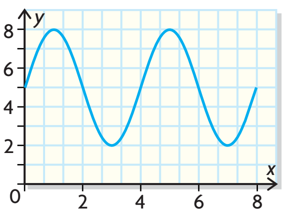
8. **a)** amplitude: 2; period: 360; increasing interval: 0 to 90, 270 to 360; decreasing interval: 90 to 270; axis: $y = 3$
b) amplitude: 3; period: 360; increasing interval: 0 to 90, 270 to 360; decreasing interval: 90 to 270; axis: $y = 1$
c) amplitude: 1; period: 720; increasing interval: 0 to 180, 540 to 720; decreasing interval: 180 to 540; axis: $y = 2$
d) amplitude: 1; period: 180; increasing interval: 0 to 45, 135 to 180; decreasing interval: 45 to 135; axis: $y = -1$
e) amplitude: 2; period: 1440; increasing interval: 0 to 360, 1080 to 1440; decreasing interval: 360 to 1080; axis: $y = 0$
f) amplitude: 3; period: 720; increasing interval: 0 to 180, 540 to 720; decreasing interval: 180 to 540; axis: $y = 2$

9. **a)** 0.82 **b)** 0.34 **c)** 1.5 **d)** 180° **e)** 270°

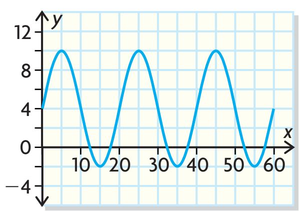
10. $x = -315°, -135°, 45°, 225°$

11. **a)** (0.91, 0.42) **b)** (0.87, 4.92) **c)** $(-2, 3.46)$ **d)** $(-1.93, -2.30)$

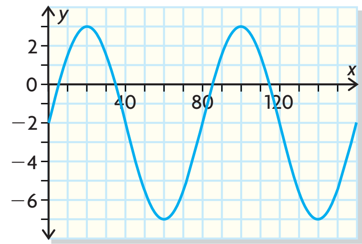
12. **a)**



b)



c)



13. **a)** Where Jim is on the Ferris wheel at 10 s, $h(10) = -5$. Ferris wheel is at lowest point.
b) Now $h(10) = 0$, Jim is at the midpoint.

14. Same period, amplitude, and axis. Different starting point for each circle.

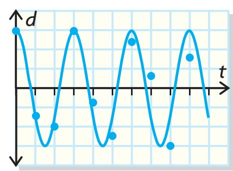
15. $y = \sin x + 0.5$

16. **a)**

t(s)	*d*(*t*) (cm)
0	0.5
0.5	0.25
2	−0.25
1.5	−0.5
2	−0.25
2.5	0.25
3	0.5
3.5	0.25
4	−0.25
4.5	−0.5

t(s)	*d*(*t*) (cm)
5	−0.25
5.5	0.25
6	0.5
6.5	0.25
7	−0.25
7.5	−0.5
8	−0.25
8.5	0.25
9	0.5

b)



c) The function repeats itself every 3 s.
d) The amplitude and the displacement from rest are the same.

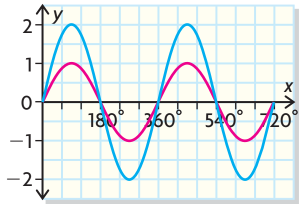
Lesson 6.3, pp. 370–373

1. **a)** $y = 8$; resting position of the swing
b) 6 m
c) 4 s; time to complete one full swing
d) 2 m
e) No, 2 s is not long enough to run safely.
f) Amplitude would increase with each swing. It would not be sinusoidal because the amplitude is changing.

2.

	Period	Amplitude	Axis	Maximum	Minimum	Speed (m/s)
A	12 s	3 m	$y = 2$	5	−1	1.57
B	16 s	4 m	$y = 3$	7	−1	1.57

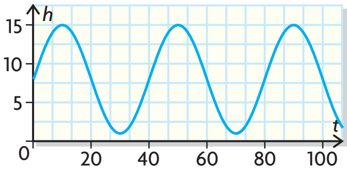
3. Answers may vary. For example,



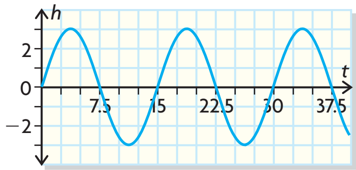
4. a) 1 in.
b) 0.04 s, how long it takes the blade to make a full rotation
c) 4 in.; radius of saw blade
d) 628 in./s

5. a) about 0.035 s b) $y = 0$ amperes c) 4.5 amperes

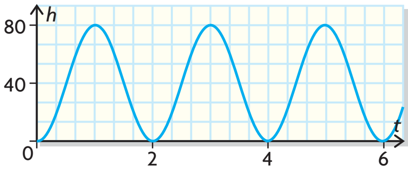
6. a)



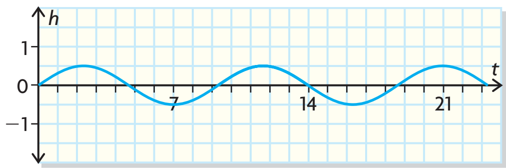
b)



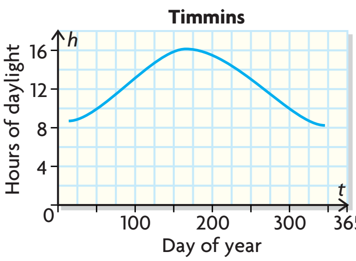
c)

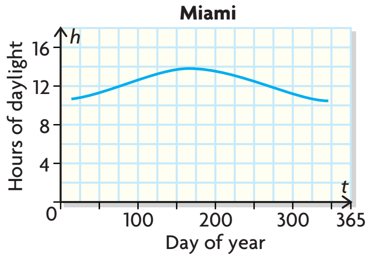


d)



7. a)

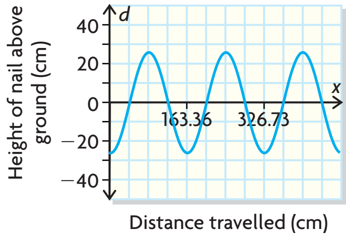




b) Timmins: period: 365; amplitude: 3.9 h; axis: $h = 12.2$ h
Miami: period: 365; amplitude: 1.7 h; axis: $h = 12.2$ h

c) The farther north one goes, more extreme differences occur in the hours of daylight throughout the year.

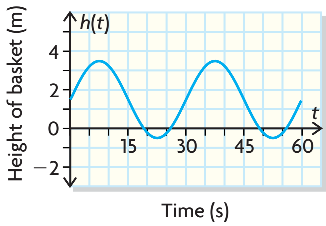
8. a)



b) approximately 25 cm
c) approximately 687 cm
d) The driver doesn't spin the wheels.

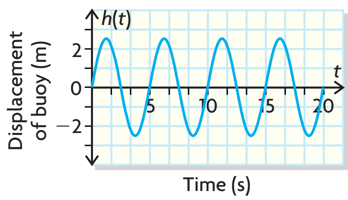
9. Have the same period $\left(\frac{1}{3}\text{ s}\right)$ and equation of the axis ($d = 0$); have different amplitudes (3 and 2). Conjecture: lower the wind speed and you decrease the distance the post shakes back and forth

10. a)

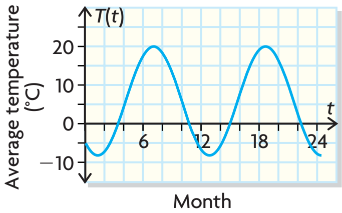


b) 30 s; period, one revolution
c) 2 m; amplitude, radius
d) $y = 1.5$ m; equation of axis
e) 3.232; at $t = 10$ s, the basket is at a height of 3.232 m.

11. a)



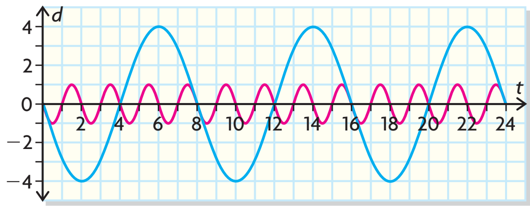
b) 5 s; period
c) 12 waves; graph completes 12 cycles in 1 min
d) 5 m; vertical distance between the maximum and minimum values of h

12. a)


b) The period represents 12 months, or 1 year.
c) between an average high of 20.1 °C and an average low of −8.3 °C
d) 5.9 °C
e) 17.4; on the 30th month, the average monthly temperature is 17.4 °C.

13. a) A: period: 8 s; B: period: 6 s; time for the wrecking ball to complete a swing back and forth
b) A and B: equation of axis $d = 0$; resting position of each wrecking ball
c) A: amplitude: 4 m; B: amplitude: 3 m; maximum distance balls swing back and forth from the resting position
d) A: $\{y \in \mathbf{R} \mid -4 \leq y \leq 4\}$; B: $\{y \in \mathbf{R} \mid -3 \leq y \leq 3\}$
e) The wrecking ball modelled using the red curve sways at a faster rate but doesn't swing as far.

14. You need the amplitude, where it starts on the graph, axis, and period.

15. a) clockwise
b) 8 s
c)


d) 0.5 m
e) 0.81 m
f) 0 m

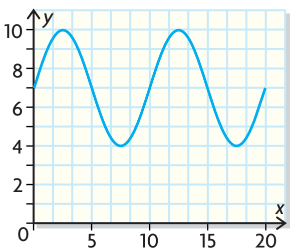
Time (s)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
d small gear	0	−1	0	1	0	−1	0	1	0	−1	0	1

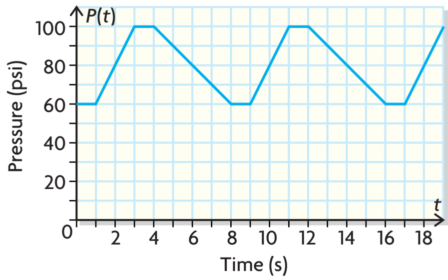
Time (s)	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0
d small gear	0	−1	0	1	0	−1	0	1	0	−1	0	1	0

Time (s)	0	2.0	4.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.0	24.0
d large gear	0	−4	0	4	0	−4	0	4	0	−4	0	4	0

Mid-Chapter Review, p. 376

1. Answers may vary. For example,



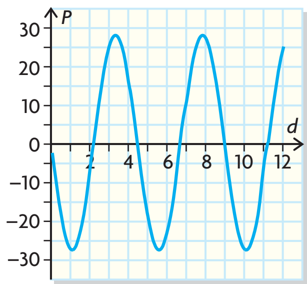
2. a)


b) cycle repeats
c) 8 s
d) $P = 80$
e) 20
f) 20 psi/s
g) 10 psi/s
h) No, because its lowest pressure value is 60 psi.

3. a) period: 180; axis: $g = 7$; amplitude: 5; range: $\{g \in \mathbf{R} \mid 2 \leq g \leq 12\}$
b) smooth, repeating waves
c) 5.3
d) 0°, 180°, 360°

4. (3.1, 6.3)

5. a) Both have a period of 0.25; the time for the tire to complete one revolution.
b) Both have same equation of the axis, $h = 30$; the height of the axle.
c) 1: amplitude: 30; 2: amplitude: 20; distance from white mark to the centre of the wheel
d) 1: $\{h \in \mathbf{R} \mid 0 \leq h \leq 60\}$; 2: $\{h \in \mathbf{R} \mid 10 \leq h \leq 50\}$
e) 1: 754 cm/s; 2: 502 cm/s
f) This graph would be periodic in nature and have a smaller amplitude than the graph of Mark 2 (the red graph).

6. a)


b) period: 365 days or 1 year
c) axis: $P = 0$; the position is 0° with respect to due west
d) amplitude: 28; maximum number of degrees north or south of due west the Sun can be at sunset for this particular latitude
e) $\{P \in \mathbf{R} \mid -28 \leq P \leq 28\}$
f) −16.3°

Lesson 6.4, p. 379

1. a) vertical stretch of 3
b) horizontal translation of 50°
c) reflection in the x-axis
d) horizontal compression of $\frac{1}{5}$
e) vertical translation of −6
f) horizontal translation of −20°

2. a) axis: $y = 2$
b) amplitude: 4
c) period: 45°
d) horizontal translation of −30°; period: 180°
e) amplitude: 0.25
f) period: 720°

3. (a), (e)

Lesson 6.5, pp. 383–385

1. **a)** horizontal compression: $\frac{1}{4}$, vertical translation: 2

b) horizontal translation: 20, vertical compression: $\frac{1}{4}$

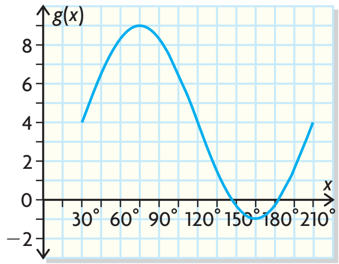
c) horizontal stretch: 2; reflection in x-axis

d) horizontal compression: $\frac{1}{18}$; vertical stretch: 12; vertical translation: 3

e) horizontal stretch: 3; horizontal translation: 40; vertical stretch: 20; reflection in x-axis

2. period: 120°; amplitude: 4; axis: $y = 6$; domain: $\{x \in \mathbf{R} \mid 0° \le x \le 240°\}$; range: $\{y \in \mathbf{R} \mid 2 \le y \le 10\}$

3.



4. **a)** horizontal translation: −10; vertical stretch: 2; reflection in x-axis

b) horizontal compression: $\frac{1}{5}$; vertical translation: 7

c) horizontal compression: $\frac{1}{2}$; horizontal translation: −6; vertical stretch: 9; vertical translation: −5

d) horizontal translation: 15; vertical compression: $\frac{1}{5}$; vertical translation: 1

e) horizontal stretch: 4; horizontal translation: −37; reflection in x-axis; vertical translation: −2

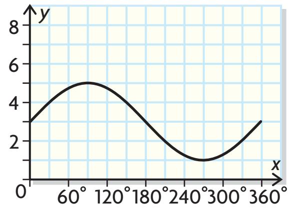
f) horizontal compression: $\frac{1}{3}$; vertical stretch: 6; reflection in x-axis; vertical translation: 22

5. **a)** (ii) **b)** (iii) **c)** (i)

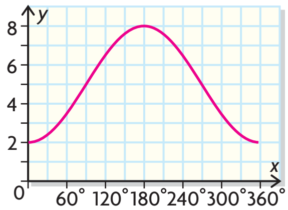
6.

	Period	Amplitude	Equation of the Axis	Domain	Range
a)	360°	3	$y = 2$	$\{x \in \mathrm{R} \mid 0° \le x \le 1080°\}$	$\{y \in \mathrm{R} \mid -1 \le y \le 5\}$
b)	180°	4	$g = 7$	$\{x \in \mathrm{R} \mid 0° \le x \le 540°\}$	$\{g \in \mathrm{R} \mid 3 \le g \le 11\}$
c)	360°	$\frac{1}{2}$	$h = -5$	$\{t \in \mathrm{R} \mid 0° \le t \le 1080°\}$	$\{h \in \mathrm{R} \mid -5.5 \le h \le -4.5\}$
d)	90°	1	$h = -9$	$\{x \in \mathrm{R} \mid 0° \le x \le 270°\}$	$\{h \in \mathrm{R} \mid -10 \le h \le -8\}$
e)	2°	10	$d = -30$	$\{t \in \mathrm{R} \mid 0° \le t \le 6°\}$	$\{d \in \mathrm{R} \mid -40 \le d \le -20\}$
f)	180°	$\frac{1}{2}$	$j = 0$	$\{x \in \mathrm{R} \mid 0° \le x \le 540°\}$	$\{j \in \mathrm{R} \mid -0.5 \le j \le 0.5\}$

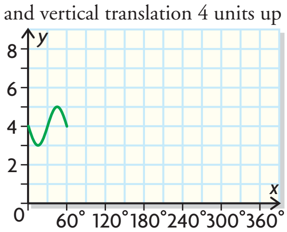
7. **a)** vertical stretch by a factor of 2 and vertical translation 3 units up



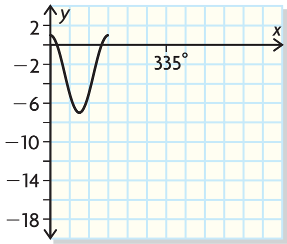
b) vertical stretch by a factor of 3, reflection in the x-axis, and vertical translation 5 units up



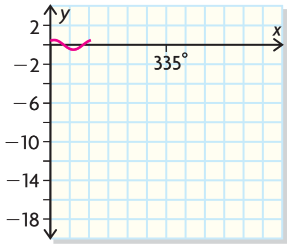
c) horizontal compression by a factor of $\frac{1}{6}$, reflection in the x-axis, and vertical translation 4 units up



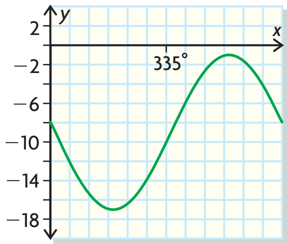
d) vertical stretch by a factor of 4, horizontal compression by a factor of $\frac{1}{2}$, and vertical translation 3 units down



e) vertical compression by a factor of $\frac{1}{2}$, horizontal compression by a factor of $\frac{1}{3}$, and horizontal translation 40° to the right



f) vertical stretch by a factor of 8, reflection in the *x*-axis, horizontal stretch by a factor of 2, horizontal translation 50° to the left, and vertical translation 9 units down



8.

	X min	*X* max	*Y* min	*Y* max
a)	0°	180°	5	5
b)	0°	720°	15	25
c)	0°	40°	75	89
d)	0°	1°	−26.5	−27.5

9. **a)** period: 1.2 s; one heart beat
b) $\{P \in \mathbf{R} \mid 80 \leq P \leq 120\}$, maximum blood pressure of 120, minimum blood pressure of 80

10. **a)** $y = 4 \sin\left(\frac{1}{2}x\right) + 3$
b) $y = 4 \cos\left(\frac{1}{2}x - 90\right) + 3$

11. Reflection, amplitude of $\frac{1}{2}$, vertical translation 30 upward, horizontal compression of 120 resulting in a period of 3.

12. horizontal translation: $-45°$

13. **a)** The number of hours of daylight increases to a maximum and decreases to a minimum in a regular cycle as Earth revolves around the Sun.
b) Mar. 21: 12 h; Sept. 21: 12 h, spring and fall equinoxes
c) June 21: 16 h; Dec. 21: 8 h, longest and shortest days of year; summer and winter solstices
d) 12 is the axis of the curve representing half the distance between the maximum and minimum hours of daylight.

Lesson 6.6, pp. 391–393

1. **a)** $y = 2\cos(4x) + 6$
b) $y = \cos(x - 90°) + 2$
c) $y = 2\cos(3x) - 2$

2. $y = 2\cos(2x) + 7$

3. $y = 4\cos(3x) + 5$

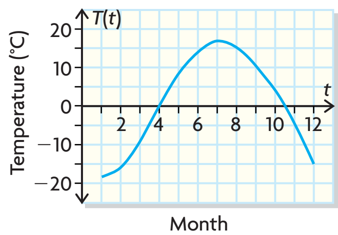
4. **a)** **i)** $y = 3\cos(60(x - 4°)) + 5$;
ii) $y = -0.5\cos(120x) + 1$;
iii) $y = \cos(90(x - 3°)) - 2$
b) **i)** $y = 5\cos(180(x - 1.5°)) + 25$;
ii) $y = 5\cos(120(x - 2°)) + 10$;
iii) $y = 10\cos(360x) - 5$

5. **a)** $y = \cos(3x) + 2$
b) $y = 4\cos\left(\left(\frac{1}{2}\right)(x - 180°)\right) + 17$
c) $y = 3\cos\left(\left(\frac{3}{2}\right)(x - 60°)\right) - 4$
d) $y = 3\cos(3(x - 10°)) + 2$

6. **a)** $y = 3\cos x + 11$
b) $y = 4\cos(2(x - 30°)) + 15$
c) $y = 2\cos(9(x - 7°))$
d) $y = 0.5\cos\left(\left(\frac{1}{2}\right)(x + 56°)\right) - 3$

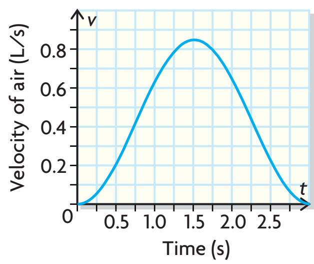
7. $y = -6\cos(8x) + 7$

8. **a)**



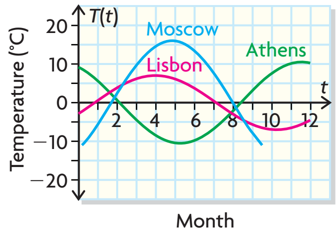
b) sinusoidal model because it changes with a cyclical pattern over time
c) $T(t) = -17.8 \cos 30t - 0.8$ **d)** 8.1 °C

9. **a)** The respiratory cycle is an example of a periodic function because we inhale, rest, exhale, rest, inhale, and so on in a cyclical pattern.
b) $v = -0.425\cos(120t)° + 0.425$



c) The equation is almost an exact fit on the scatter plot.
d) 0 L/s; period is 3; troughs occur at 0, 3, and 6 s
e) $t = 0.8$ s and 2.3 s

10. **a)**



b) Athens: $T(t) = -10.5 \cos 30t + 22.5$;
Lisbon: $T(t) = -7 \cos 30t + 20$;
Moscow: $T(t) = -16 \cos 30t + 7$

c) latitude affects amplitude and vertical translation

d) Athens and Lisbon are close to the same latitude; Moscow is farther north.

11. **a)** $y = 3 \cos(9000(x - 0.01))° + 8$
b) maximum equivalent stress
c) 6.64 MPa

12. Find the amplitude. Whatever the amplitude is, a in the equation $y = a\cos(k(x - d)) + c$ will be equal to it. Find the period. Whatever the period is, k in the equation $y = a\cos(k(x - d)) + c$ will be equal to 360 divided by it. Find the equation of the axis. Whatever the equation of the axis is, c in the equation $y = a\cos(k(x - d)) + c$ will be equal to it. Find the phase shift. Whatever the phase shift is, d in the equation $y = a\cos(k(x - d)) + c$ will be equal to it. Determine if the function is reflected in its axis. If it is, the sign of a will be negative; otherwise, it will be positive. Determine if the function is reflected in the y-axis. If it is, the sign of k will be negative; otherwise, it will be positive.

13. $y = -30 \cos(1.909\ 859x)° + 30$; 23.4 cm

14. $h = 7 \cos(15.8t)° + 8$, t in seconds, h in metres

Lesson 6.7, pp. 398–401

1. **a)** $d = 1.5$ m, distance between tail lights and the curb if the trailer isn't swinging back and forth
b) amplitude: 0.5 m, distance the trailer swings to the left and right
c) period: 2 s, the time it takes for the trailer to swing back and forth
d) $d = -0.5 \cos(180t)° + 1.5$; $\{d \in \mathbf{R} \mid 1 \le d \le 2\}$
e) range is the distance the trailer swings back and forth; domain is time
f) 1.2 m

2. **a)** $h = 10$ m, axle height
b) amplitude: 7 m, length of blade
c) period: 20 s, time in seconds to complete revolution
d) domain: $\{t \in \mathbf{R} \mid 0 \le t \le 140\}$;
range: $\{h \in \mathbf{R} \mid 3 \le h \le 17\}$
e) $h = -7 \cos(18x)° + 10$
f) period would be larger

3. $d = 4 \cos(90(t - 1))° + 8$

4. **a)** same period (24), same horizontal translation (12), different amplitude (2.5 and 10), different equations of the axis ($T = 17.5$ and $T = -20$). The top one is probably the interior temperature (higher, with less fluctuation).
b) domain (for both): $\{t \in \mathbf{R} \mid 0 \le t \le 48\}$;
range (top): $\{T \in \mathbf{R} \mid 15 \le T \le 20\}$;
range: (bottom): $\{T \in \mathbf{R} \mid -30 \le T \le -10\}$
c) blue: $T = 2.5 \cos(15(h - 12))° + 17.5$;
red: $T = 10 \cos(15(h - 12))° - 20$

5. **a)** $d = -30 \cos(18t)°$
b) $d = 9 \cos(36(t - 12))°$

6. **a)** $d = -0.3 \cos(144t)° + 1.8$
b) amplitude: 0.3, height of crest relative to normal water level
c) 2 m
d) 16
e) $d = -0.3 \cos(120t)° + 1.8$

7. $C = 4.5 \cos(21\ 600t)°$

8. **a)** $h = 8 \cos(450(t - 0.2))° + 12$
b) domain: $\{t \in \mathbf{R}\}$; range: $\{h \in \mathbf{R} \mid 4 \le h \le 20\}$
c) $h = 12$cm, resting position of the spring
d) 6.3 cm

9. **a)** $h = -30 \cos[(0.025)x]° + 40$
b) domain: $\{d \in \mathbf{R} \mid 0 \le d \le 400\pi\}$;
range: $\{h \in \mathbf{R} \mid 10 \le h \le 70\}$
c) 69.7 cm

10. The periods and the horizontal translations are the same. As the rabbit population goes down, so does the fox population. As the rabbit population goes up, so does the fox population. The amplitudes differ (rabbits are higher). The axes are different (rabbit times 10 that of fox).

11. The period and amplitude, as well as where it starts on the x-axis and the position on the y-axis when it started

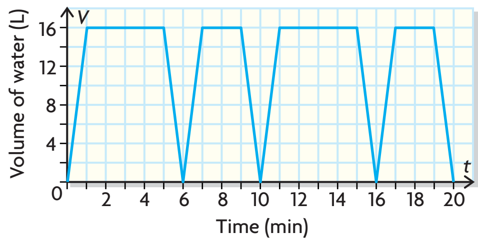
12. $h = -6\cos(0.5x)° + 13$

13. **a)** $f(x) = -3 \cos x - 1$ **b)** -3.8 **c)** (i) **d)** (iv)

14. 0°, 180°, 360°

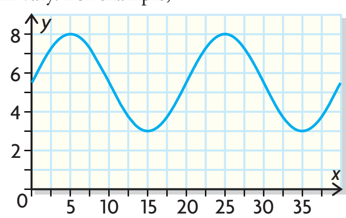
Chapter Review, pp. 404–405

1. **a)**

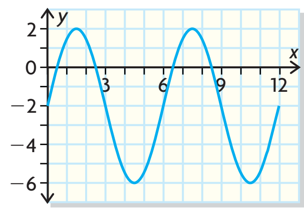


b) yes
c) period: 10 min, how long it takes for the dishwasher to complete one cycle
d) $y = 8$ L
e) 8 L
f) $\{V \in \mathbf{R} \mid 0 \le V \le 16\}$

2. Answers will vary. For example,



3. Answers will vary. For example,



4. **a)** 80 s, time to complete one revolution
b) $y = 16$ m, height of axle above the ground
c) 9 m, radius of wheel
d) yes, graph started at maximum height opposed to boarding height
e) 0.71 m/s
f) $\{h \in \mathbf{R} \mid 7 \le h \le 25\}$
g) boarding height: 1 m

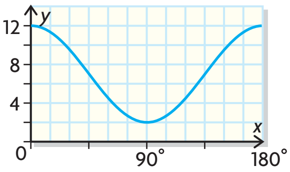
5. **a)** period: 120°; axis: $h = 9$; amplitude: 4; $\{h \in \mathbf{R} \mid 5 \le h \le 13\}$
b) yes
c) 6.2
d) 60°, 180°, 300°

6. **a)** 12 s, time between each wave
b) 5 m
c) 5.5 m
d) $\{d \in \mathbf{R} \mid 3 \le d \le 7\}$
e) 9 s

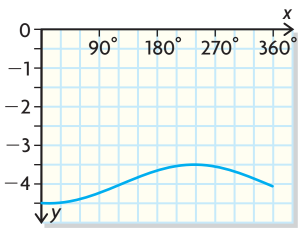
7. (3.63, 1.69)

8. **a)** axis $y = -3$
b) period: 90°
c) amplitude: 7
d) none

9. **a)**

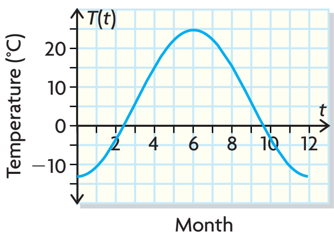


b)



10. **a)** $\{y \in \mathbf{R} \mid -1 \le y \le 5\}$
b) $\{y \in \mathbf{R} \mid -0.5 \le y \le 0.5\}$

11. **a)**



b) On a yearly basis, the average temperature of each month will be roughly the same. It fluctuates in a cyclical pattern over a specific period of time.
c) max: 24.7 °C; min: −13.1 °C
d) 12; the curve repeats after 12 months, representing one year
e) $T = 5.8°$
f) 6 units right
g) $T(t) = -18.9 \cos 30t + 5.8$
h) −3.7 °C; month 38 is February, and the table shows a temperature close to that value.

12. **a)** $y = \sin(0.5(\theta - 180)) + 2.5$
b) $y = 2 \sin[2(\theta + 90)] + 4$

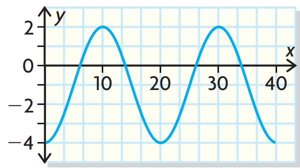
13. **a)** period: 1.5 s, time to rock back and forth
b) 26 cm
c) $\{t \in \mathbf{R} \mid 0 \le t \le 60\}$
d) $\{d \in \mathbf{R} \mid 18 \le d \le 34\}$
e) 8 cm, maximum distance the chair rocks to the front or back from its resting position
f) $y = 8 \cos(240(t - 1.75))° + 26$
g) 30 cm

14. To determine the equation of a sinusoidal function, first calculate the period, amplitude, and equation of the axis. This information will help you determine the values of k, a, and c, respectively, in the equations $g(x) = a \sin(k(x - d)) + c$ and $h(x) = a \cos(k(x - d)) + c$.

Chapter Self-Test, p. 406

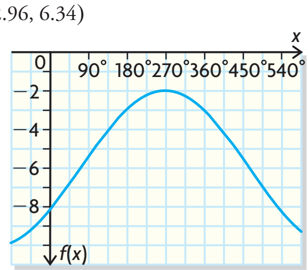
1. **a)** 40 s, time for stair to return to its initial position
b) $h = 2$m
c) the height at the top of the escalator
d) $\{h \in \mathbf{R} \mid -1 \le h \le 5\}$
e) $\{t \in \mathbf{R} \mid 0 \le t \le 400\}$
f) No, 300 is not a multiple of 40. Since it started at the ground, it would need to be for this to be true.

2.



3. (2.96, 6.34)

4. **a)**



b) amplitude: 4; period: 720°; axis: $y = -6$
c) −4.47
d) $\{y \in \mathbf{R} \mid -10 \le y \le -2\}$

5. **a)** minimum distance between the tip of the metre stick and the edge of the plywood
b) periods are the same; even though you are tracking different ends of the metre stick, the ends do belong to the same metre stick
c) 180 cm
d) the amplitudes are 30cm and 70cm, distance from nail to the ends of the metre stick
e) range 1: $\{d \in \mathbf{R} \mid 150 \le d \le 210\}$;
range 2: $\{d \in \mathbf{R} \mid 110 \le d \le 250\}$
f) $\{t \in \mathbf{R} \mid 0 \le t \le 25\}$
g) $d = 30 \cos(72t)° + 180$; $d = -70 \cos(72t)° + 180$
h) 189.3 cm

Chapters 4–6 Cumulative Review, pp. 408–411

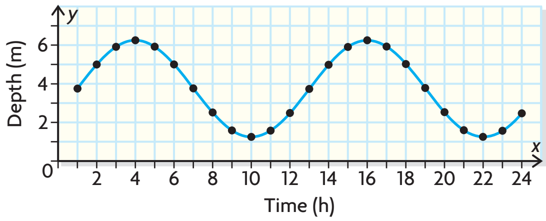
1. (b)	**7.** (b)	**13.** (a)	**19.** (b)	**25.** (c)
2. (a), (d)	**8.** (c)	**14.** (d)	**20.** (d)	**26.** (c)
3. (c)	**9.** (a)	**15.** (d)	**21.** (c)	**27.** (a)
4. (a)	**10.** (c)	**16.** (a)	**22.** (a)	**28.** (c)
5. (a)	**11.** (b)	**17.** (c)	**23.** (a)	
6. (c)	**12.** (a)	**18.** (a)	**24.** (b)	

29. 23 folds

30.

Angle (degrees)	Length of *BC* (cm)
105	80.7
125	62.6
145	43.5
165	26.1
175	20.8

31. **a)**



$f(h) = 2.5\cos(30(h - 4))^\circ + 3.75$ or $f(h) = 2.5\sin(30(h - 1))^\circ + 3.75$

b) 6.25 m

c) The minimum depth of the water at this location is 1.25 m. Therefore, since the hull of the boat must have a clearance of at least 1 m at all times, if the bottom of the hull is more than 0.25 m below the surface of the water, then this location is not suitable for the dock. However, if the bottom of the hull is less than or equal to 0.25 m below the surface of the water, then this location is suitable for the dock.

Chapter 7

Getting Started, p. 414

1. **a)** $y = -\frac{2}{5}x + 8$
b) $y = -9x + 49$
c) $y = \frac{7}{5}x - 7$
d) $y = -2x - 7$

2. **a)** 6 **b)** $\frac{13}{10}$ or 1.3 **c)** 0 **d)** 4

3. **a)** linear **b)** neither **c)** quadratic

4. **a)** $x = 5$ **b)** $x = -5$ **c)** $x = \frac{33}{16}$ **d)** $x = 1.53$

5. about 2.2 g

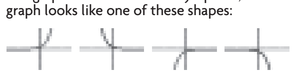
6. 51.2%

7.

Exponential Functions

Definition:
A function of the form $f(x) = a \times b^x$, where a and b are constants.
Constant changes in the independent variable result in the dependent variable being multiplied by a constant.

Rules/Method:
The graph has a horizontal asymptote, and the graph looks like one of these shapes:



In a table of values, look at the 1st ratios. If they are constant, the function is an exponential.

Examples:
$f(x) = 9 \times 5^x$
$f(x) = \frac{2}{3} \times \left(\frac{5}{11}\right)^x$

Non-examples:
$y = \frac{2}{3}x - 7$ (linear function)
$y = x^3$ (cubic function)
7×2^x (exponential expression)

Lesson 7.1, pp. 424–425

1. **a)** arithmetic, $d = 4$
b) not arithmetic
c) not arithmetic
d) arithmetic, $d = -11$

2. **a)** General term: $t_n = 14n + 14$
Recursive formula: $t_1 = 28$, $t_n = t_{n-1} + 14$, where $n > 1$
b) General term: $t_n = 57 - 4n$
Recursive formula: $t_1 = 53$, $t_n = t_{n-1} - 4$, where $n > 1$
c) General term: $t_n = 109 - 110n$
Recursive formula: $t_1 = -1$, $t_n = t_{n-1} - 110$, where $n > 1$

3. $t_{12} = 53$

4. $t_{15} = 323$

5.

	i)	ii)
a)	arithmetic	General term: $t_n = 3n + 5$ Recursive formula: $t_1 = 8$, $t_n = t_{n-1} + 3$, where $n > 1$
b)	not arithmetic	—
c)	not arithmetic	—
d)	not arithmetic	—
e)	arithmetic	General term: $t_n = 11n + 12$ Recursive formula: $t_1 = 23$, $t_n = t_{n-1} + 11$, where $n > 1$
f)	arithmetic	General term: $t_n = \left(\frac{1}{6}\right)n$ Recursive formula: $t_1 = \frac{1}{6}$, $t_n = t_{n-1} + \frac{1}{6}$, where $n > 1$

6. **a)** Recursive formula: $t_1 = 19$, $t_n = t_{n-1} + 8$, where $n > 1$
General term: $t_n = 8n + 11$
b) Recursive formula: $t_1 = 4$, $t_n = t_{n-1} - 5$, where $n > 1$
General term: $t_n = 9 - 5n$
c) Recursive formula: $t_1 = 21$, $t_n = t_{n-1} + 5$, where $n > 1$
General term: $t_n = 5n + 16$
d) Recursive formula: $t_1 = 71$, $t_n = t_{n-1} - 12$, where $n > 1$
General term: $t_n = 83 - 12n$

7.

	i)	ii)
a)	arithmetic	13, 27, 41, 55, 69; $d = 14$
b)	not arithmetic	—
c)	not arithmetic	—
d)	arithmetic	1, 2, 3, 4, 5; $d = 1$

8.

	i)	ii)	iii)
a)	$t_n = 5n + 30$	$t_1 = 35$, $t_n = t_{n-1} + 5$, where $n > 1$	$t_{11} = 85$
b)	$t_n = 42 - 11n$	$t_1 = 31$, $t_n = t_{n-1} - 11$, where $n > 1$	$t_{11} = -79$
c)	$t_n = -17 - 12n$	$t_1 = -29$, $t_n = t_{n-1} - 12$, where $n > 1$	$t_{11} = -149$
d)	$t_n = 11$	$t_1 = 11$, $t_n = t_{n-1}$, where $n > 1$	$t_{11} = 11$
e)	$t_n = \left(\frac{1}{5}\right)n + \frac{4}{5}$	$t_1 = 1$, $t_n = t_{n-1} + \frac{1}{5}$, where $n > 1$	$t_{11} = 3$
f)	$t_n = 0.17n + 0.23$	$t_1 = 0.4$, $t_n = t_{n-1} + 0.17$, where $n > 1$	$t_{11} = 2.1$

9.

	i)	ii)
a)	arithmetic	6, 4, 2, 0, −2; $d = -2$
b)	not arithmetic	—
c)	arithmetic	$\frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}$; $d = \frac{1}{4}$
d)	not arithmetic	—

10. **a)** 90 seats **b)** 23 rows

11. 63rd month

12. 16 years

13. **a)** 29 **b)** 38 **c)** 15 **d)** 14 **e)** 18 **f)** 9

14. $t_{100} = (t_8 - t_4) \times 23 + t_8$
The 4th and 8th terms differ by $4d$. The 8th and 100th term differ by $92d = 23 \times 4d$.

15. $t_n = 7n - 112$, where $n \in \mathbf{N}$

16. **a)** Answers will vary. For example,
20, 50, 80, … with $a = 20$ and $d = 30$
50, 20, −10, … with $a = 50$ and $d = -30$
5, 20, 35, 50, … with $a = 5$ and $d = 15$
b) The common difference must divide 30 (50 − 20) or −30 (20 − 50) evenly. The first term must be an integer multiple of the common difference away from 20 and 50.

17. $t_{100} = 112, 211, 310, 409, 607$, or 1201

18. yes

Lesson 7.2, pp. 430–432

1. **a)** not geometric **b)** geometric, $r = 3$ **c)** not geometric **d)** geometric, $r = \frac{1}{2}$

2. **a)** General term: $t_n = 9 \times 4^{n-1}$
Recursive formula: $t_1 = 9$, $t_n = 4t_{n-1}$, where $n > 1$
b) General term: $t_n = 625 \times 2^{n-1}$
Recursive formula: $t_1 = 625$, $t_n = 2t_{n-1}$, where $n > 1$
c) General term: $t_n = 10\,125 \times \left(\frac{2}{3}\right)^{n-1}$
Recursive formula: $t_1 = 10\,125$, $t_n = \left(\frac{2}{3}\right)t_{n-1}$, where $n > 1$

3. $t_{33} = 9963$

4. $t_{10} = 180$

5.

	i)	ii)
a)	geometric	General term: $t_n = 12 \times 2^{n-1}$ or $t_n = 3 \times 2^{n+1}$ Recursive formula: $t_1 = 12$, $t_n = 2t_{n-1}$, where $n > 1$
b)	not geometric	—
c)	not geometric	—
d)	geometric	General term: $t_n = 5 \times (-3)^{n-1}$ Recursive formula: $t_1 = 5$, $t_n = -3t_{n-1}$, where $n > 1$
e)	not geometric	—
f)	geometric	General term: $t_n = 125 \times \left(\frac{2}{5}\right)^{n-1}$ Recursive formula: $t_1 = 125$, $t_n = \left(\frac{2}{5}\right)t_{n-1}$, where $n > 1$

6.

	i)	ii)	iii)
a)	$t_n = 4 \times 5^{n-1}$	$t_1 = 4$, $t_n = 5t_{n-1}$, where $n > 1$	$t_6 = 12\,500$
b)	$t_n = -11 \times 2^{n-1}$	$t_1 = -11$, $t_n = 2t_{n-1}$, where $n > 1$	$t_6 = -352$
c)	$t_n = 15 \times (-4)^{n-1}$	$t_1 = 15$, $t_n = -4t_{n-1}$, where $n > 1$	$t_6 = -15\,360$
d)	$t_n = 896 \times \left(\frac{1}{2}\right)^{n-1}$ or $t_n = 7 \times 2^{8-n}$	$t_1 = 896$, $t_n = \left(\frac{1}{2}\right)t_{n-1}$, where $n > 1$	$t_6 = 28$
e)	$t_n = 6 \times \left(\frac{1}{3}\right)^{n-1}$ or $t_n = 2 \times 3^{2-n}$	$t_1 = 6$, $t_n = \left(\frac{1}{3}\right)t_{n-1}$, where $n > 1$	$t_6 = \frac{2}{81}$
f)	$t_n = 0.2^{n-1}$	$t_1 = 1$, $t_n = 0.2t_{n-1}$, where $n > 1$	$t_6 = 0.000\,32$

7.

	i)	ii)
a)	arithmetic	$t_n = 4n + 5$
b)	geometric	$t_n = 7 \times (-3)^{n-1}$
c)	geometric	$t_n = 18 \times (-1)^{n-1}$
d)	neither	—
e)	arithmetic	$t_n = 39 - 10n$
f)	geometric	$t_n = 128 \times \left(\frac{3}{4}\right)^{n-1}$

8. **a)** Recursive formula: $t_1 = 19$, $t_n = 5t_{n-1}$, where $n > 1$
General term: $t_n = 19 \times 5^{n-1}$
b) Recursive formula: $t_1 = -9$, $t_n = -4t_{n-1}$, where $n > 1$
General term: $t_n = -9 \times (-4)^{n-1}$
c) Recursive formula: $t_1 = 144$, $t_n = \left(\frac{1}{4}\right)t_{n-1}$, where $n > 1$
General term: $t_n = 144 \times \left(\frac{1}{4}\right)^{n-1}$ or $t_n = 9 \times 4^{3-n}$
d) Recursive formula: $t_1 = 900$, $t_n = \left(\frac{1}{6}\right)t_{n-1}$, where $n > 1$
General term: $t_n = 900 \times \left(\frac{1}{6}\right)^{n-1}$ or $t_n = 150 \times 6^{2-n}$

9.

	i)	ii)
a)	not geometric	—
b)	geometric	$-8, 24, -72, 216, -648; r = -3$
c)	geometric	$123, 41, \frac{41}{3}, \frac{41}{9}, \frac{41}{27}; r = \frac{1}{3}$
d)	geometric	$10, 20, 40, 80, 160; r = 2$

10.

	i)	ii)
a)	geometric	$4, 16, 64, 256, 1024; r = 4$
b)	not geometric	—
c)	not geometric	—
d)	geometric	$-\frac{7}{125}, \frac{7}{25}, -\frac{7}{5}, 7, -35; r = -5$
e)	not geometric	—
f)	geometric	$\frac{11}{13}, \frac{11}{169}, \frac{11}{2197}, \frac{11}{28\,561}, \frac{11}{371\,293}; r = \frac{1}{13}$

11. 1 474 560

12. 131 220 bacteria

13. $10 794.62

14. **a)** 65.61% **b)** 29 dosages

15. Yes, $t_{29} = t_7 \times \left(\frac{t_7}{t_5}\right)^{11}$. Use the formula for the general term to write t_{29} in terms of a and r. Then write the equation for t_{29} using the laws of exponents. Evaluate $\frac{t_7}{t_5}$ and then rewrite t_{29} in terms of t_5 and t_7.

16. **a)** 243 shaded triangles **b)** about 0.338 cm^2

17. Both sequences are recursive, so the recursive formulas look similar, except that you add with an arithmetic sequence but multiply with a geometric sequence. The general terms also look similar, except that with an arithmetic sequence, you add a multiple of the common difference to the first term but with a geometric sequence, you multiply a power of the common ratio with the first term.

18. $1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}$; The sum gets closer to 2.

19. $t_{10} = 10\,752$, general term: $t_n = 2^{n-1}(2n + 1)$

20. Yes, only for the sequence $a, a, a, \ldots$, where $d = 0$ and $r = 1$.

21. Answers will vary. For example, **1**, **2**, 3, **4**, 5, 6, 7, **8**, … (arithmetic sequence). To form the geometric sequence (shown in red), the previous term determines which term you select.

22. 35.44 cm^2

Lesson 7.3, pp. 439–440

1. Yes, each term depends only on the two previous terms (difference), so the sequence will repeat. Check Sam's formula with other terms to see that it works.
2. $t_n = \frac{n}{n+1}$
3. a) $t_n = 2n + 1$ c) $t_n = 2n^2 + 2n$
   b) $t_n = 3n + 1$ d) (triangles) $t_4 = 2(4) + 1 = 9$ toothpicks
   (squares) $t_4 = 2(4)^2 + 2(4) = 40$ toothpicks
4. a) $t_n = -\left(\frac{n-1}{2}\right)$ if n is odd
   $= \frac{n}{2}$ if n is even
   b) Another rule is $t_1 = 0$, $t_n = \frac{n}{2}$ for even n, and $t_n = -t_{n-1}$ for odd n. The other rule is better because each term can be calculated directly, instead of having to calculate terms for even n before calculating terms for odd n.
   c) $t_{12345} = -6172$
5. $nx + \frac{1}{y^n}$
6. $t_n = \frac{N_n}{D_n}$, where $N_n = 3 \times 7^{n-1}$ and D_n has n fives or
   $t_n = \frac{3 \times 7^{n-1}}{\frac{5}{9}(10^n - 1)}$
7. a) 159, 319, 639 c) 34, 55, 89 e) $-216, 343, -512$
   b) 85, 79, 72 d) 54, 108, 110 f) 111, 223, 447
8. 9900 comparisons
9. 169 271, 846 354, 4 231 771
10. $t_{1000} = 20$
11. Answers will vary. For example, 2, 3, 6, 11, 18, 27, ... The 1st differences form a sequence of odd numbers. So to generate new terms, work backward from the 1st differences.

Lesson 7.4, p. 443

1. The ratio of consecutive terms tends toward the same value as the Fibonacci and Lucas sequences, and the sequences have similar identities.
2. The ratio of $\frac{F_n}{F_{n-1}}$ and $\frac{L_n}{L_{n-1}}$ get close to $r = \frac{1+\sqrt{5}}{2}$.
3. a) 1, 5, 7, 17, 31, 65, 127, 257, 511, and 1025
   b) 5, 1.4, 2.43, 1.82, 2.10, 1.95, 2.02, 1.99, and 2.00. Ratios get close to 2.
   c) $t_n = 2^n + (-1)^n$

Mid-Chapter Review, p. 447

1.

	i)	ii)	iii)
a)	$t_1 = 29$, $t_n = t_{n-1} - 8$, where $n > 1$	$t_n = 37-8n$	$t_{10} = -43$
b)	$t_1 = -8$, $t_n = t_{n-1} - 8$, where $n > 1$	$t_n = -8n$	$t_{10} = -80$
c)	$t_1 = -17$, $t_n = t_{n-1} + 8$, where $n > 1$	$t_n = 8n - 25$	$t_{10} = 55$
d)	$t_1 = 3.25$, $t_n = t_{n-1} + 6.25$, where $n > 1$	$t_n = 6.25n - 3$	$t_{10} = 59.5$
e)	$t_1 = \frac{1}{2}$, $t_n = t_{n-1} + \frac{1}{6}$, where $n > 1$	$t_n = \frac{1}{6}n + \frac{1}{3}$	$t_{10} = 2$
f)	$t_1 = x$, $t_n = t_{n-1} + 2x + 3y$, where $n > 1$	$t_n = (2n-1) \times x + 3(n-1)y$	$t_{10} = 19x + 27y$

2. a) Recursive formula: $t_1 = 17$, $t_n = t_{n-1} + 11$, where $n > 1$
   General term: $t_n = 11n + 6$
   b) Recursive formula: $t_1 = 38$, $t_n = t_{n-1} - 7$, where $n > 1$
   General term: $t_n = 45 - 7n$
   c) Recursive formula: $t_1 = 55$, $t_n = t_{n-1} + 18$, where $n > 1$
   General term: $t_n = 18n + 37$
   d) Recursive formula: $t_1 = 42$, $t_n = t_{n-1} - 38$, where $n > 1$
   General term: $t_n = 80 - 38n$
   e) Recursive formula: $t_1 = 159$, $t_n = t_{n-1} - 17$, where $n > 1$
   General term: $t_n = 176 - 17n$
3. 315 seats
4.

	i)	ii)
a)	arithmetic	General term: $t_n = 15n$ Recursive formula: $t_1 = 15$, $t_n = t_{n-1} + 15$, where $n > 1$ $t_6 = 90$
b)	geometric	General term: $t_n = 640 \times \left(\frac{1}{2}\right)^{n-1}$ Recursive formula: $t_1 = 640$, $t_n = \left(\frac{1}{2}\right)t_{n-1}$, where $n > 1$ $t_6 = 20$
c)	geometric	General term: $t_n = 23 \times (-2)^{n-1}$ Recursive formula: $t_1 = 23$, $t_n = -2t_{n-1}$, where $n > 1$ $t_6 = -736$
d)	geometric	General term: $t_n = 3000 \times (0.3)^{n-1}$ Recursive formula: $t_1 = 3000$, $t_n = 0.3t_{n-1}$, where $n > 1$ $t_6 = 7.29$
e)	arithmetic	General term: $t_n = 1.2n + 2.6$ Recursive formula: $t_1 = 3.8$, $t_n = t_{n-1} + 1.2$, where $n > 1$ $t_6 = 9.8$
f)	geometric	General term: $t_n = \left(\frac{1}{2}\right) \times \left(\frac{2}{3}\right)^{n-1}$ Recursive formula: $t_1 = \left(\frac{1}{2}\right) \times \left(\frac{2}{3}\right)t_{n-1}$, where $n > 1$ $t_6 = \frac{16}{243}$

5.

	i)	ii)
a)	geometric	5, 25, 125, 625, 3125
b)	geometric	$\frac{3}{7}, \frac{3}{19}, \frac{3}{67}, \frac{3}{259}, \frac{3}{1027}$
c)	arithmetic	$5, -7, -19, -31, -43$
d)	geometric	$-2, 4, -8, 16, -32$
e)	arithmetic	8, 11, 14, 17, 20

6. 45 weeks
7. 349, 519, 737
   The 3rd differences are constant, so use them to determine terms.
8. $t_n = x^n + ny$
9. a) 1, 8, 27 c) $t_n = n^3$
   b) 64, 125, 216 d) 3375
10. a) $t_{15} = 1453$
    b) $t_n = t_{n-2} + t_{n-1}$

Lesson 7.5, pp. 452–453

1. a) $S_{10} = 815$ c) $S_{10} = -1345$
   b) $S_{10} = -50$ d) $S_{10} = 210$
2. $S_{20} = 2450$
3. 670 bricks

Answers

4.
	i)	ii)
a)	arithmetic	$S_{25} = 1675$
b)	not arithmetic	—
c)	not arithmetic	—
d)	arithmetic	$S_{25} = 1650$
e)	arithmetic	$S_{25} = -1925$
f)	not arithmetic	—

5. a) $t_{12} = 81, S_{12} = 708$
b) $t_{12} = -134, S_{12} = -882$
c) $t_{12} = 48, S_{12} = 180$
d) $t_{12} = \frac{57}{10}$ or 5.7, $S_{12} = \frac{177}{5}$ or 35.4
e) $t_{12} = 15.51, S_{12} = 112.2$
f) $t_{12} = 12p + 22q, S_{12} = 78p + 132q$

6. a) $S_{20} = 1110$
b) $S_{20} = 1400$
c) $S_{20} = -1980$
d) $S_{20} = 2570$
e) $S_{20} = 2410$
f) $S_{20} = 1630$

7. a) $S_{20} = 970$
b) $S_{26} = 4849$
c) $S_{24} = -168$
d) $S_{711} = 760\,770$
e) $S_{16} = -1336$
f) $S_{22} = 0$

8. a) $D_n = \dfrac{n(n-3)}{2}$ b) 14 diagonals

9. \$5630

10. 1102.5 m

11. 2170 toys

12. 3050 s or 50 min 50 s

13. 700 km

14. Two copies of the first representation fit together to form a rectangle $t_1 + t_n$ by n, yielding the formula $S_n = \dfrac{n(t_1 + t_n)}{2}$.
Two copies of the second representation fit together to form a rectangle. You can see a and d and get the formula $S_n = \dfrac{n[2a + (n-1)d]}{2}$.

15. $t_{25} = 79$

16. 26 terms

Lesson 7.6, p. 459–461

1. a) $S_7 = 6558$
b) $S_7 = \dfrac{3175}{16}$ or 198.4375
c) $S_7 = 4376$
d) $S_7 = \dfrac{127}{192}$

2. $S_6 = 15\,015$

3. a) $t_6 = 18\,750, S_6 = 23\,436$
b) $t_6 = -2673, S_6 = -4004$
c) $t_6 = 6720, S_6 = 26\,248\,320$
d) $t_6 = \dfrac{128}{1215}, S_6 = \dfrac{532}{243}$
e) $t_6 = -138.859, S_6 = -92.969$
f) $t_6 = 243x^{10}, S_6 = \dfrac{729x^{12} - 1}{3x^2 - 1}$

4.
	i)	ii)
a)	arithmetic	—
b)	geometric	$S_8 = 22\,960$
c)	geometric	$S_8 = \frac{13\,107}{8}$ or 1638.375
d)	neither	—
e)	geometric	$S_8 \doteq 12.579$
f)	arithmetic	—

5. a) $S_7 = 253\,903$
b) $S_7 = 1397$
c) $S_7 = \dfrac{163\,830}{1024}$ or about 159.990
d) $S_7 = 2186$
e) $S_7 = 49\,416$
f) $S_7 = \dfrac{645}{48}$ or 13.4375

6. a) $S_8 = 335\,923$
b) $S_7 = 1905$
c) $S_{10} = -250\,954$
d) $S_6 = 28\,234.9725$
e) $S_8 = 78\,642$
f) $S_{13} = \dfrac{8191}{1024}$

7. about 10.8 m

8. If $r = 1$, all the terms are the same, $a + a + a + \ldots$. So the sum of n terms would be $S_n = na$.

9. 1 048 575 line segments

10. 12.25 m^2

11. 5465 employees

12. 14 559 864

13. Answers will vary. For example, the first prize is \$1829, each prize is 3 times the previous one, and there are 7 prizes altogether. The total value of the prizes is \$1 999 097.

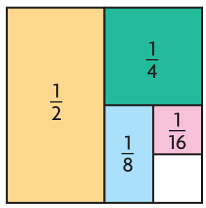
14.

Arithmetic	Geometric	Similarities	Differences
$S_n = \dfrac{n(t_1 + t_n)}{2}$ Write the terms of S_n twice, once forward, then once backward above each other. Then add pairs of terms. The sum of the pairs is constant.	$S_n = \dfrac{t_{n+1} - t_1}{r - 1}$, where $r \neq 1$ Write the terms of S_n and rS_n above each other. Then subtract pairs of terms. The difference of all middle pairs is zero.	Both general formulas involve two "end" terms of the series.	You add with formula for arithmetic series, but subtract for geometric series. You divide by 2 for arithmetic series, but by $r - 1$ for geometric series.

15. $t_5 = 15\,552$

16. 13 terms

17. $x^{15} - 1 = (x - 1)(x^2 + x + 1)(x^4 + x^3 + x^2 + x + 1)(x^8 - x^7 + x^5 - x^4 + x^3 - x + 1)$

18. a)


b) The formula for the sum of geometric series gives the sum of the first n terms.
c) Yes. Consecutive terms of this series are getting smaller and smaller, so the sum is getting closer and closer to 1.

Lesson 7.7, p. 466

1. 1, 13, 78, and 286

2. a) $(x + 2)^5 = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$
b) $(x - 1)^6 = x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$
c) $(2x - 3)^3 = 8x^3 - 36x^2 + 54x - 27$

3. a) $(x + 5)^{10} = x^{10} + 50x^9 + 1125x^8 + \ldots$
b) $(x - 2)^8 = x^8 - 16x^7 + 112x^6 - \ldots$
c) $(2x - 7)^9 = 512x^9 - 16\,128x^8 + 225\,792x^7 - \ldots$

4. a) $(k + 3)^4 = k^4 + 12k^3 + 54k^2 + 108k + 81$
b) $(y - 5)^6 = y^6 - 30y^5 + 375y^4 - 2500y^3 + 9375y^2 - 18\,750y + 15\,625$
c) $(3q - 4)^4 = 81q^4 - 432q^3 + 864q^2 - 768q + 256$
d) $(2x + 7y)^3 = 8x^3 + 84x^2y + 294xy^2 + 343y^3$
e) $(\sqrt{2}x + \sqrt{3})^6 = 8x^6 + 24\sqrt{6}x^5 + 180x^4 + 120\sqrt{6}x^3 + \sqrt{270}x^2 + 54\sqrt{6}x + 27$
f) $(2z^3 - 3y^2)^5 = 32z^{15} - 240z^{12}y^2 + 720z^9y^4 - 1080z^6y^6 + 810z^3y^8 - 243y^{10}$

5. a) $(x - 2)^{13} = x^{13} - 26x^{12} + 312x^{11} - \ldots$
b) $(3y + 5)^9 = 19\,683y^9 + 295\,245y^8 + 1\,968\,300y^7 + \ldots$
c) $(z^5 - z^3)^{11} = z^{55} - 11z^{53} + 55z^{51} - \ldots$
d) $(\sqrt{a} + \sqrt{5})^{10} = a^5 + 10\sqrt{5a}\,a^4 + 225a^4 + \ldots$

e) $\left(3b^2 - \frac{2}{b}\right)^{14} = 4\,782\,969b^{28} - 44\,641\,044b^{25} + 193\,444\,524b^{22} + \ldots$

f) $(5x^3 + 3y^2)^8 = 390\,625x^{24} + 1\,875\,000x^{21}y^2 + 3\,937\,500x^{18}y^4 + \ldots$

6. **a)** The sum of all the numbers in a row of Pascal's triangle is equal to a power of 2.

b) If you alternately subtract and add the numbers in a row of Pascal's triangle, the result is always zero.

7. 1, 1, 2, 3; These are terms in the Fibonacci sequence.

8. 252 ways

9. Write $(x + y + z)^{10} = [x + (y + z)]^{10}$ and use the pattern for expanding a binomial twice.

10. $(3x - 5y)^6 = 729x^6 - 7290x^5y + 30\,375x^4y^2 - 67\,500x^3y^3 + 84\,375x^2y^4 - 56\,250xy^5 + 15\,625y^6$

11. To expand $(a + b)^n$, where $n \in \mathbf{N}$, write the numbers from the nth row of Pascal's triangle. Each term in the expansion is the product of a number from Pascal's triangle, a power of a, and a power of b. The exponents of a go down, term by term, to zero. The exponents of b start at zero and go up, term by term, to n.

12. The 1st differences of a cubic correspond to the differences between the $(x + 1)$th value and the xth value. The 1st differences of a cubic are quadratic. The 2nd differences of a cubic correspond to the 1st differences of a quadratic, and the 3rd differences of a cubic correspond to the 2nd differences of a quadratic. Since the 2nd differences of a quadratic are constant, the 3rd differences of a cubic are constant.

13. $\left(\frac{1}{2} + \frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{10} + 10\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right) + 45\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + \ldots$

The first three terms represent the probablility of getting heads 10, 9, and 8 times, respectively.

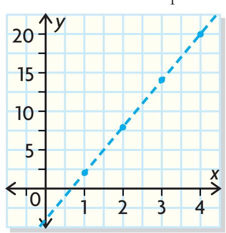
Chapter Review, pp. 468–469

1. **a)** Arithmetic sequence with first term 2, and each term afterward increases by 6.

b) General term: $t_n = 6n - 4$

Recursive formula: $t_1 = 2$, $t_n = t_{n-1} + 6$, where $n > 1$

c)



2. Check if the difference between consecutive terms is constant.

3.

	i)	ii)
a)	$t_n = 15n + 43$	$t_1 = 58$, $t_n = t_{n-1} + 15$, where $n > 1$
b)	$t_n = 9n - 58$	$t_1 = -49$, $t_n = t_{n-1} + 9$, where $n > 1$
c)	$t_n = 87 - 6n$	$t_1 = 81$, $t_n = t_{n-1} - 6$, where $n > 1$

4. $t_{100} = -3348$

5. 9 weeks

6. Check if the ratio of consecutive terms is constant.

7.

	i)	ii)
a)	geometric	$t_6 = 1215$
b)	neither	—
c)	geometric	$t_6 = 0.000\,09$
d)	arithmetic	$t_6 = 210$
e)	arithmetic	$t_6 = -26$
f)	geometric	$t_6 = 121.5$

8.

	i)	ii)	iii)
a)	$t_1 = 7$, $t_n = -3t_{n-1}$, where $n > 1$	$t_n = 7 \times (-3)^{n-1}$	7, −21, 63, −189, 567
b)	$t_1 = 12$, $t_n = \left(\frac{1}{2}\right)t_{n-1}$, where $n > 1$	$t_n = 12 \times \left(\frac{1}{2}\right)^{n-1}$	12, 6, 3, $\frac{3}{2}$, $\frac{3}{4}$
c)	$t_1 = 9$, $t_n = 4t_{n-1}$, where $n > 1$	$t_n = 9 \times 4^{n-1}$	9, 36, 144, 576, 2304

9.

	i)	ii)
a)	arithmetic	9, 13, 17, 21, 25
b)	neither	$\frac{1}{4}, \frac{1}{11}, \frac{1}{18}, \frac{1}{25}, \frac{1}{32}$
c)	neither	0, 3, 8, 15, 24
d)	neither	−17, −16, −14, −11, −7

10. **a)** 47 104 000 bacteria

b) No. The bacteria would eventually run out of food and space in the culture.

11. \$1933.52

12. $t_n = n^2 + 3n$

13. $t_{100} = \frac{100}{299}$

14. **a)** $S_{50} = 9850$ **b)** $S_{50} = -3850$ **c)** $S_{50} = 27\,275$ **d)** $S_{50} = -6575$ **e)** $S_{50} = 2590$ **f)** $S_{50} = 11\,750$

15. **a)** $S_{25} = 3900$ **b)** $S_{25} = 5812.5$ **c)** $S_{25} = -6000$ **d)** $S_{25} = -1700$ **e)** $S_{25} = 375$ **f)** $S_{25} = 950$

16. **a)** $S_{13} = 949$ **b)** $S_{124} = 252\,774$ **c)** $S_{50} = 25$

17. 325 m

18. **a)** $t_6 = 2673$, $S_6 = 4004$

b) $t_6 = 11\,111.1$, $S_6 \doteq 12\,345.654$

c) $t_6 = -192$, $S_6 = -126$

d) $t_6 = 4320$, $S_6 = 89\,775$

e) $t_6 = -\frac{4131}{32}$ or $-129.093\,75$, $S_6 = -\frac{2261}{32}$

f) $t_6 = \frac{243}{6250}$, $S_6 = \frac{3724}{3125}$

19. **a)** $S_8 = -131\,070$ **b)** $S_8 \doteq 3276.087\,34$ **c)** $S_8 \doteq 426.660$ **d)** $S_8 = 136\,718.4$

20. 61 425 orders

21. $S_{10} = 12\,276$

22. **a)** 7161

b) 1533

c) $\frac{25\,999}{64}$

d) 64 125

e) 18 882.14

23. **a)** $(a + 6)^4 = a^4 + 24a^3 + 216a^2 + 864a + 1296$

b) $(b - 3)^5 = b^5 - 15b^4 + 90b^3 - 270b^2 + 405b - 243$

c) $(2c + 5)^3 = 8c^3 + 60c^2 + 150c + 125$

d) $(4 - 3d)^6 = 4096 - 18\,432d + 34\,560d^2 - 34\,560d^3 + 19\,440d^4 - 5832d^5 + 729d^6$

e) $(5e - 2f)^4 = 625e^4 - 1000e^3f + 600e^2f^2 - 160ef^3 + 16f^4$

f) $\left(3f^2 - \frac{2}{f}\right)^4 = 81f^8 - 216f^5 + 216f^2 - \frac{96}{f} + \frac{16}{f^4}$

Answers

Chapter Self-Test, p. 470

1.

	i)	ii)
a)	45, 135, 405, 1215, 3645	geometric
b)	$\frac{5}{3}, \frac{8}{5}, \frac{11}{7}, \frac{14}{9}, \frac{17}{11}$	neither
c)	5, 10 , 15, 20, 25	arithmetic
d)	5, 35, 245, 1715, 12 005	geometric
e)	19, −18, 19, −18, 19	neither
f)	7, 13, 19, 25, 31	arithmetic

2.

	i)	ii)
a)	$t_n = (-9) \times (-11)^{n-1}$	$t_1 = -9, t_n = -11t_{n-1}$, where $n > 1$
b)	$t_n = 1281 - 579n$	$t_1 = 702, t_n = t_{n-1} - 579$, where $n > 1$

3. **a)** 26 terms
b) 8 terms

4. **a)** $(x - 5)^4 = x^4 - 20x^3 + 150x^2 - 500x + 625$
b) $(2x + 3y)^3 = 8x^3 + 36x^2y + 54xy^2 + 27y^3$

5. **a)** $S_{31} = 7099$
b) $S_{10} = 259\ 586.8211$

6. $t_{123} = \frac{3}{2}$

7. $17 850

8. **a)** 61, 99, 160
b) $p^6 + 6p, p^7 + 7p, p^8 + 8p$
c) $-\frac{5}{18}, -\frac{11}{21}, -\frac{17}{24}$

Chapter 8

Getting Started, p. 474

1.

	i)	ii)	iii)
a)	23, 27	$t_n = 3 + 4n$	$t_1 = 7, t_n = t_{n-1} + 4$, where $n > 1$
b)	−50, −77	$t_n = 85 - 27n$	$t_1 = 58, t_n = t_{n-1} - 27$, where $n > 1$
c)	1280, 5120	$t_n = 5 \times 4^{n-1}$	$t_1 = 5, t_n = 4t_{n-1}$, where $n > 1$
d)	−125, 62.5	$t_n = 1000 \times \left(\frac{-1}{2}\right)^{n-1}$	$t_1 = 1000$, $t_n = \left(\frac{-1}{2}\right)t_{n-1}$, where $n > 1$

2. **a)** $t_5 = 147$ **b)** $d = 101$ **c)** $a = -257$ **d)** $t_{100} = 9742$

3. **a)** geometric—There is a constant rate between the terms.
b) $t_1 = 8000, t_n = (1.05)t_{n-1}$, where $n > 1$
c) $t_n = 8000 \times (1.05)^{n-1}$
d) $t_{10} \doteq 12\ 410.6257$

4. **a)** $S_{10} = 120$ **c)** $S_{10} = -285$
b) $S_{10} = 0$ **d)** $S_{10} = 5\ 456\ 000$

5. **a)** (1st year) 210 000, (2nd year) 220 500, (3rd year) 231 525
b) about 325 779

6. $x = 12$

7. **a)** $x \doteq 19.93$ **c)** $x \doteq 11.26$
b) $x \doteq 3.48$ **d)** $x \doteq 8.72$

8.

Example:	**Visual representation:**
$f(x) = 5 \times 2^x$	The graph looks like one of these shapes: 
Definition in your own words:	**Personal association:**
A function of the form $f(x) = a \times b^x$, where a and b are constants. When x increases or decreases by a constant, y is multiplied by a constant.	half-life geometric sequences

Lesson 8.1, pp. 481–482

1. **a)** **i)** (1st year) $532, (2nd year) $564, (3rd year) $596
ii) $980
b) **i)** (1st year) $1301.25, (2nd year) $1352.50, (3rd year) $1403.75
ii) $2018.75
c) **i)** (1st year) $26 250, (2nd year) $27 500, (3rd year) $28 750
ii) $43 750
d) **i)** (1st year) $1739.10, (2nd year) $1778.20, (3rd year) $1817.30
ii) $2286.50

2. **a)** $P = \$2000$ **c)** $r = 6\%/a$
b) $I = \$600$ **d)** $A(t) = 2000 + 120t$

3. 3 years and 132 days

4. about 28%/a

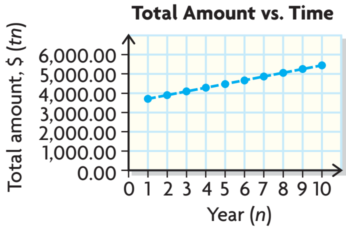
5. **a)** $I = \$192, A = \$692$ **d)** $I = \$9.60, A = \$137.60$
b) $I = \$3763.20, A = \$6963.20$ **e)** $I = \$3923.08, A = \$53\ 923.08$
c) $I = \$260, A = \$5260$ **f)** $I = \$147.95, A = \$4647.95$

6. about 7.84%/a

7. $47 619.05

8. **a)** $192.50
b) (1st year) $3692.50, (2nd year) $3885.00, (3rd year) $4077.50, (4th year) $4270.00, (5th year) $4462.50
c) $t_n = 3500 + 192.50n$
d)

Total Amount vs. Time



9. **a)** $3740 **b)** 27.2%/a

10. **a)** $1850
b) $t_n = 1850 + 231.25n$
c) 24 years and 157 days

11. 66 years and 8 months

12. $P = \$750$, $r = 3.7\%/a$ $A(t) = P + Prt; P = 750; Prt = 27.75t;$ $750rt = 27.75t; 750r = 27.75; r = 0.037$

13. $D = \frac{1}{r}$

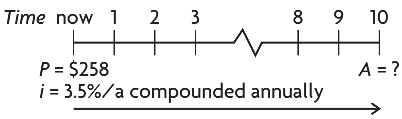
14. $23 400

Lesson 8.2, pp. 490–492

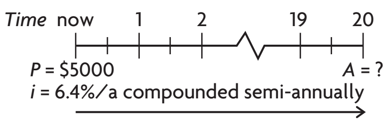
1.

	Interest Rate per Compounding Period, *i*	Number of Compounding Periods, *n*
a)	0.027	10
b)	0.003	36
c)	0.007 25	28
d)	0.000 5	43.3

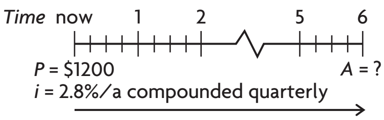
2. **a)** **i)** (1st year) \$10 720.00, (2nd year) \$11491.84, (3rd year) \$12 319.25, (4th year) \$13 206.24, (5th year) \$14 157.09
ii) $A(n) = 10\,000(1.072)^n$
b) **i)** (1st half-year) \$10 190.00, (2nd half-year) \$10 383.61, (3rd half-year) \$10 580.90, (4th half-year) \$10 781.94, (5th half-year) \$10 986.79
ii) $A(n) = 10\,000(1.019)^n$
c) **i)** (1st quarter) \$10 170.00, (2nd quarter) \$10 342.89, (3rd quarter) \$10 518.72, (4th quarter) \$10 697.54, (5th quarter) \$10 879.40
ii) $A(n) = 10\,000(1.017)^n$
d) **i)** (1st month) \$10 090.00, (2nd month) \$10 180.81, (3rd month) \$10 272.44, (4th month) \$10 364.89, (5th month) \$10 458.17
ii) $A(n) = 10\,000(1.009)^n$

3. **a)**


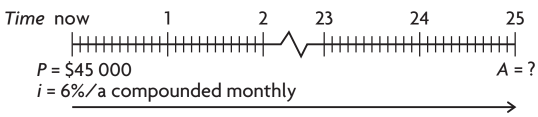
\$363.93

b)


\$17 626.17

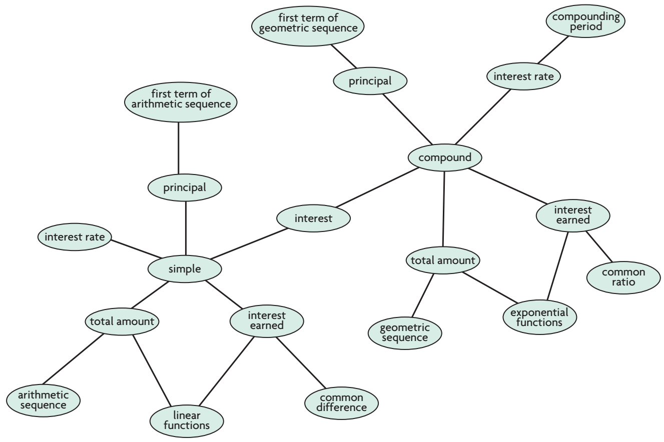
c)


\$1418.69

d)


\$200 923.64

4. **a)** $A = \$4502.04$, $I = \$502.04$
b) $A = \$10\,740.33$, $I = \$3240.33$
c) $A = \$16\,906.39$, $I = \$1906.39$
d) $A = \$48\,516.08$, $I = \$20\,316.08$
e) $A = \$881.60$, $I = \$31.60$
f) $A = \$2332.02$, $I = \$107.02$

5. **a)** 6% **b)** \$4000
6. about 13 years
7. \$3787.41
8. $P = \$5000$, $i = 9\%$/a compounded monthly
9. Plan B; Plan A = \$1399.99; Plan B = \$1049.25
10. \$1407.10
11. \$14 434.24
12. If he invests for 26 years or less, the first option is better.
13. Answers may vary. For example, How long will it take for both investments to be worth the same amount?
a) \$5000 at 5%/a compounded annually
b) \$3000 at 7%/a compounded annually
Answer: about 27 years
14. 6.5%/a compounded quarterly, 6.55%/a compounded semi-annually, 6.45%/a compounded monthly, 6.6%/a compounded annually
15. \$4514.38
16. \$4543.12
17. \$3427.08
18.


19. \$13 995.44
20. **a)** about 6.40% **b)** about 4.28% **c)** about 3.24%

Lesson 8.3, pp. 498–499

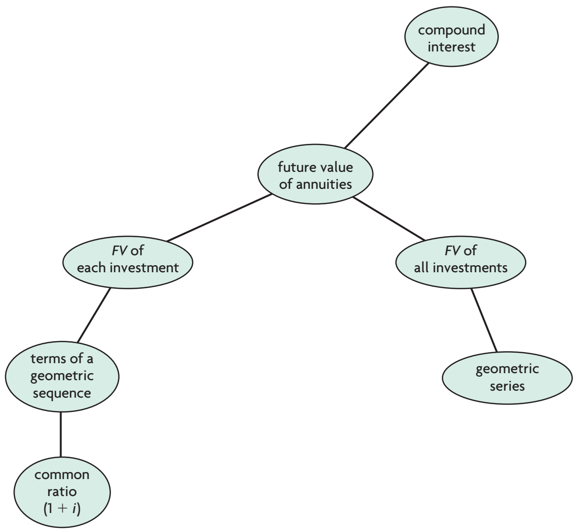
1. **a)** \$6755.64 **c)** \$10 596.47
b) \$73 690.81 **d)** \$3.46
2. Lui
3. **a)** $PV = \$7920.94$, $I = \$2079.06$
b) $PV = \$4871.78$, $I = \$1328.22$
c) $PV = \$8684.66$, $I = \$11\,315.34$
d) $PV = \$8776.74$, $I = \$4023.26$
4. \$8500.00
5. \$1900.00
6. \$5586.46
7. \$10 006.67
8. 8.85%
9. Franco, \$204.20
10. \$7200.00
11. **a)** 8-year guarantee **b)** \$8324.17
12. With present value, you are looking back in time. Present value is an exponential function with ratio $(1 + i)^{-1}$, so the amount is decreasing the further you go into the past, just like the amount of radioactive material decreases as time goes on.
13. about $16\frac{1}{2}$ years
14. 11.14%
15. about \$3047.98
16. $PV = \frac{A}{1 + in}$

Mid-Chapter Review, p. 503

1. **a)** $I = \$5427.00$, $A = \$10\ 827.00$
b) $I = \$51.20$, $A = \$451.20$
c) $I = \$3300.00$, $A = \$18\ 300.00$
d) $I = \$278.42$, $A = \$2778.42$
2. 3 years and 53 days
3. **a)** \$64.60 **b)** \$950.00 **c)** 6.8%/a
4. **a)** $A = \$8805.80$, $I = \$2505.80$
b) $A = \$34\ 581.08$, $I = \$20\ 581.08$
c) $A = \$822\ 971.19$, $I = \$702\ 971.19$
d) $A = \$418.17$, $I = \$120.17$
5. 11 years and 5 months
6. 18.85%/a
7. \$2572.63
8. \$350.00
9. **a)** 9.40%/a **b)** \$8324.65

Lesson 8.4, pp. 511–512

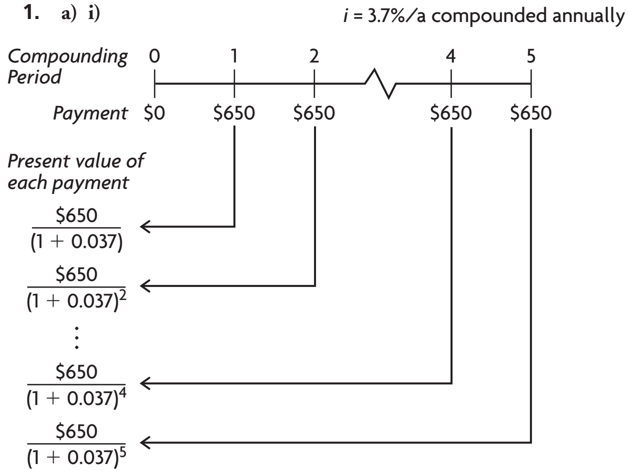
1. **a)** (1st investment) \$16 572.74, (2nd investment) \$15 316.76, (3rd investment) \$14 155.97, (4th investment) \$13 083.15
b) geometric
c) \$188 191.50
2. **a)** \$167 778.93 **c)** \$9920.91
b) \$146 757.35 **d)** \$49 152.84
3. \$59 837.37
4. \$4889.90
5. **a)** \$20 051.96 **c)** \$79 308.62
b) \$1569.14 **d)** \$57 347.07
6. **a)** \$148.77 **b)** \$638.38
7. Investment (a)—more compounding periods
8. 5 years and 9 months
9. **a)** \$198.25 **b)** \$926 980.31
10. 6.31%
11.



12. **a)** 3 years and 8 months **b)** \$918.30
13. \$924.32
14. 76 payments

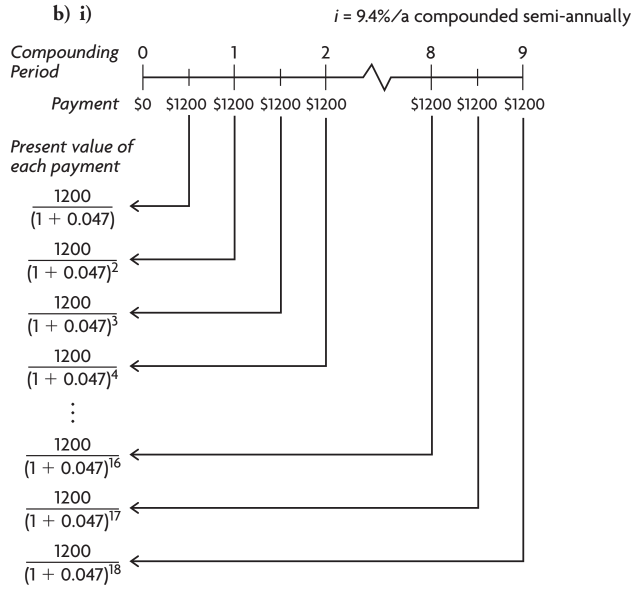
Lesson 8.5, pp. 520–522

1. **a)** **i)**



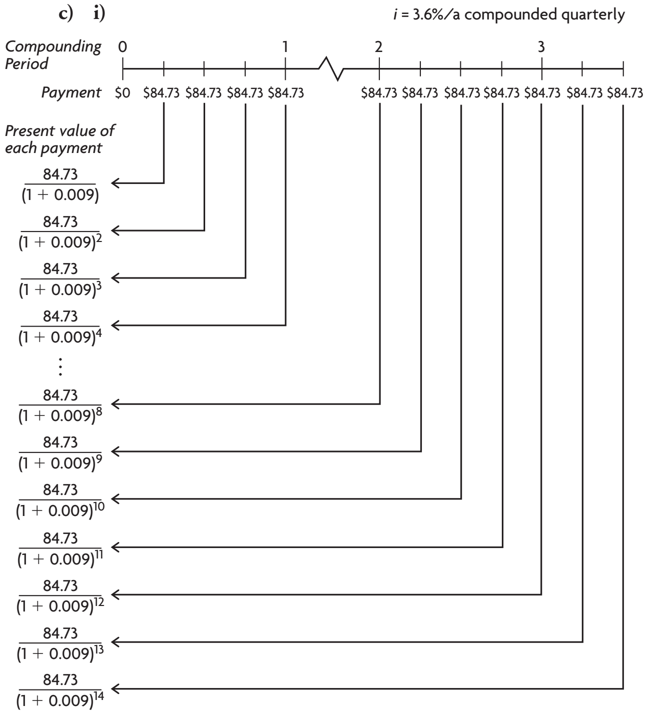
ii) $PV = 650(1.037)^{-1} + 650(1.037)^{-2} + 650(1.037)^{-3} + \ldots + 650(1.037)^{-5}$
iii) \$2918.23
iv) \$331.77

b) **i)**



ii) $PV = 1200(1.047)^{-1} + 1200(1.047)^{-2} + 1200(1.047)^{-3} + \ldots + 1200(1.047)^{-18}$
iii) \$14 362.17
iv) \$7237.83

c) i)

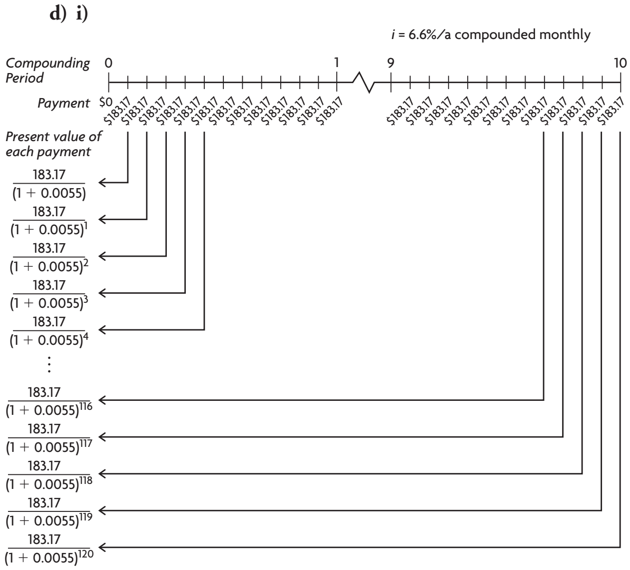


ii) $PV = 84.73(1.009)^{-1} + 84.73(1.009)^{-2} + 84.73(1.009)^{-3} + \ldots + 84.73(1.009)^{-14}$

iii) $1109.85

iv) $76.37

d) i)



ii) $PV = 183.17(1.0055)^{-1} + 183.17(1.0055)^{-2} + 183.17(1.0055)^{-3} + \ldots + 183.17(1.0055)^{-120}$

iii) $16 059.45

iv) $5920.95

2. **a) i)** $PV_1 = \$7339.45$, $PV_2 = \$6733.44$, $PV_3 = \$6177.47$, $PV_4 = \$5667.40$, $PV_5 = \$5199.45$, $PV_6 = \$4770.14$, $PV_7 = \$4376.27$

ii) $PV = 8000(1.09)^{-1} + 8000(1.09)^{-2} + 8000(1.09)^{-3} + \ldots + 8000(1.09)^{-7}$

iii) $40 263.62

b) i) $PV_1 = \$288.46$, $PV_2 = \$277.37$, $PV_3 = \$266.70$, $PV_4 = \$256.44$, $PV_5 = \$246.58$, $PV_6 = \$237.09$, $PV_7 = \$227.98$

ii) $PV = 300(1.04)^{-1} + 300(1.04)^{-2} + 300(1.04)^{-3} + \ldots + 300(1.04)^{-7}$

iii) $1800.62

c) i) $PV_1 = \$735.29$, $PV_2 = \$720.88$, $PV_3 = \$706.74$, $PV_4 = \$692.88$, $PV_5 = \$679.30$, $PV_6 = \$665.98$, $PV_7 = \$652.92$, $PV_8 = \$640.12$

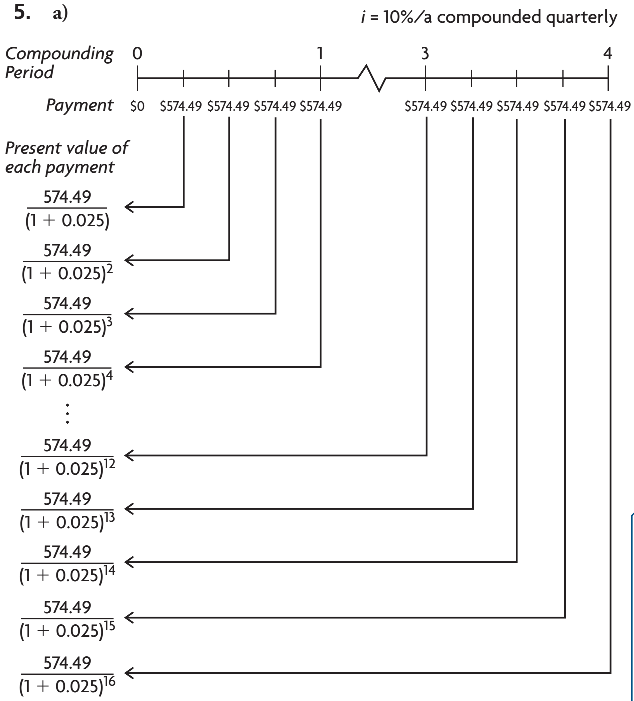
ii) $PV = 750(1.02)^{-1} + 750(1.02)^{-2} + 750(1.02)^{-3} + \ldots + 750(1.02)^{-8}$

iii) $5494.11

3. **a)** $20 391.67 **c)** $2425.49
b) $4521.04 **d)** $1093.73

4. $64.90

5. **a)**



b) $PV = 574.49(1.025)^{-1} + 574.49(1.025)^{-2} + 574.49(1.025)^{-3} + \ldots + 574.49(1.025)^{-16}$

c) $574.49

6. **a)** $418.89 **b)** $31.11

7. $971.03

8. **a)** (7-year term) $1029.70, (10-year term) $810.72
b) $10 791.60
c) Answers may vary. For example, how much they can afford per month, interest rates possibly dropping

9. Choose bank financing. TL: $33 990.60 B: $33 156.00

10. **a)** (5-year term) $716.39, (10-year term) $432.08, (15-year term) $342.61
b) (5-year term) $7983.40, (10-year term) $16 849.60, (15-year term) $26 669.80

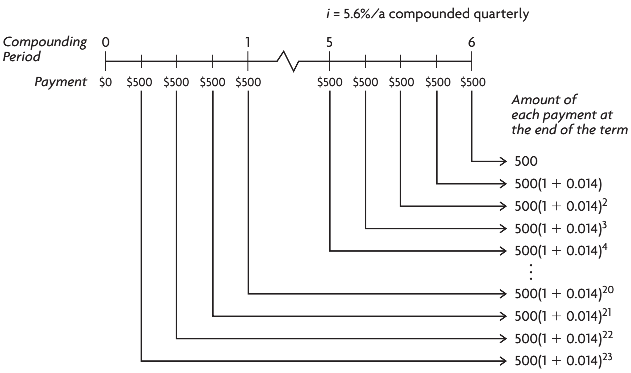
11. **a)** $316.84 **b)** $28.16

12. 19.00%
13. $1572.66
14. $219.85
15. **a)** Option B
b) If the person lives 29 or more years and/or the interest rate is 6.2% or less.
16. **a)** Answers may vary. For example, a lump sum is a one-time payment unlike an annuity, which has multiple payments. If a contest prize can be collected either as a lump sum or an annuity, the annuity will earn interest until the last payment is made. The lump sum will not earn any interest unless the contest winner deposits that amount into an account that earns interest.
b) Answers may vary. For example, future value is the value of an investment some time in the future unlike present value, which is the value of the investment now. If the parents of a child are planning for their child's education, they could deposit an amount (present value) into an account that earns interest. Once the child reaches the appropriate age, the balance in the account (future value) will be used for tuition and other fees.
17. **a)**

Year	Investment	Value of Each Investment at the End of 10 Years
1	$500.00	$1 024.77
2	$500.00	$946.23
3	$500.00	$873.71
4	$500.00	$806.75
5	$500.00	$744.92
6	$500.00	$687.83
7	$500.00	$635.12
8	$500.00	$586.44
9	$500.00	$541.50
10	$500.00	$500.00
		$7 347.29

$$PV = \frac{7347.29}{(1.083)^{10}}$$
$$= \$3310.11$$

b) Answers may vary. For example, How much would you need to invest now at 5.6%/a compounded quarterly to provide $500 for the next six years?
Answer:



Year	Investment	Value of Each Investment at the End of 10 Years
1	$500.00	$688.40
2	$500.00	$678.90
3	$500.00	$669.53
4	$500.00	$660.29
5	$500.00	$651.17
6	$500.00	$642.17
7	$500.00	$633.31
8	$500.00	$624.56
9	$500.00	$615.94
10	$500.00	$607.44
11	$500.00	$599.05
12	$500.00	$590.78
13	$500.00	$582.62
14	$500.00	$574.58
15	$500.00	$566.65
16	$500.00	$558.82
17	$500.00	$551.11
18	$500.00	$543.50
19	$500.00	$535.99
20	$500.00	$528.59
21	$500.00	$521.30
22	$500.00	$514.10
23	$500.00	$507.00
24	$500.00	$500.00
		$14 145.78

$$PV = \frac{14\,145.78}{(1.014)^{24}}$$
$$= \$10\,132.49$$

c) $PV = \dfrac{FV}{(1 + i)^n}$

18. 5 years and 3 months
19. $R = W \times \dfrac{1 - (1 + i)^{-n}}{(1 + i)^m - 1}$

Lesson 8.6, pp. 530–531

1. **a)** 12 years **b)** 7 years **c)** 19 years **d)** 8 years
2. **a)** 7.10% **b)** 5.80% **c)** 16.30% **d)** 22.19%
3. $99.86
4. **a)** $817.76 **b)** 5 years and 2 months **c)** $36 368.74
5. $3651.03
6. 8 years
7. **a)** $1143.52 **b)** 9 years
8. 12.36%
9. **a)** Answers may vary. For example, for a loan of $3500 at 6.6%/a compounded monthly and amortized over 2 years, the monthly payment will be $156.07. If the interest rate became 13.2%/a compounded monthly, to keep the same amortization period, *R* would have to be $166.73, which is not double the original *R*-value.
b) Answers may vary. For example, for a loan of $3500 at 6.6%/a compounded monthly and amortized over 2 years, the monthly payment will be $156.07. If the loan became $7000, to keep the same amortization period, *R* would have to be $312.14, which is double the original *R*-value.
10. 3 years and 3 months

11. E.g.,

Type of Technology	Advantages	Disadvantages
spreadsheet	• can set up the spreadsheet so that you only need to type equations once and just input the values in a question • can see how changing one or more values affects the rest of the calculation	• need a computer
graphing calculator	• may be more easily available	• have to make too many keystrokes • could mistype a number or equation if there are several operations to be performed • can't see how changing one value affects the rest of the values

12. $4697.58 or less

13. **a)** (amortization period) 19.58 years
b) (amortization period) 13.64 years
c) (amortization period) 13.57 years

14. 5 years and 2 months

Chapter Review, pp. 534–535

1. **a)** $I = \$2100.00$, $A = \$5600.00$
b) $I = \$4950.00$, $A = \$19\,950.00$
c) $I = \$25.39$, $A = \$305.39$
d) $I = \$474.04$, $A = \$1324.04$
e) $I = \$176.40$, $A = \$21\,176.40$

2. **a)** 5.40% **b)** $3445.00 **c)** 18 years and 6 months

3. **a)** $5000 **b)** 10% **c)** 30 years

4. about 11 years

5. **a)** $A = \$8631.11$, $I = \$4331.11$
b) $A = \$1604.47$, $I = \$1104.47$
c) $A = \$30\,245.76$, $I = \$5245.76$
d) $A = \$607.31$, $I = \$300.31$

6. **a)** 8.00%/a **b)** $5000

7. 1.73%

8. **a)** $5784.53 **c)** $64 089.29
b) $1032.07 **d)** $589.91

9. $667.33

10. 11.10%/a

11. **a)** $FV = \$46\,332.35$, $I = \$16\,332.35$
b) $FV = \$13\,306.97$, $I = \$3806.97$
c) $FV = \$31\,838.87$, $I = \$1838.87$

12. 12 years and 7 months

13. $263.14

14. **a)** $PV = \$2276.78$, $I = \$423.22$
b) $PV = \$17\,185.88$, $I = \$4189.12$
c) $PV = \$2069.70$, $I = \$530.52$
d) $PV = \$1635.15$, $I = \$259.71$

15. **a)** $1022.00 **b)** $109 280.00

16. 20.40%

17. $29.12

18. $182.34

19. 4 years

20. $1979.06

Chapter Self-Test, p. 536

1. **a)** $A = \$1309.00$, $I = \$459.00$
b) $A = \$15\,913.05$, $I = \$10\,453.05$
c) $A = \$21\,005.02$, $I = \$3065.02$

2. **a)** (Loan #1) simple interest − common difference, (Loan #2) compound interest − common ratio
b) (Loan #1) 4.00%, (Loan #2) 6.00%
c) (Loan #1) $3650.00, (Loan #2) $870.00
d) (Loan #1) $5110.00, (Loan #2) $1558.04

3. $12 075.91

4. $22 831.55

5. 5.88%/a compounded monthly

6. 5.98%

7. $205.30

Cumulative Review Chapters 7–8, pp. 538–539

1. (a)
2. (c)
3. (c)
4. (a) and (c)
5. (c)
6. (a)
7. (a)
8. (b)
9. (a)
10. (a)
11. (b)
12. (c)
13. (c)
14. (d)
15. (b)
16. (b)

17. **a)** $t_1 = 350$, $t_n = 0.32(t_{n-1}) + 350$
b) 514.7 mg
c) 54 h

18. **a)** $t_n = 25(1.005)^{n-1}$
b) 168 payments, $6557.62
c)

A	B	C	C
1	Payment number	Payment	Future Value of each payment
2	1	$25.00	=C2*(1.005)^167
3	=B2 + 1	$25.00	=C2*(1.005)^166
4			
168	=B167 + 1	$25.00	=C2*(1.005)^1
169	=B168 + 1	$25.00	=C2
			=SUM(C2:C169)

d) $13 115.24

Appendix A

A–1 Operations with Integers, p. 542

1. **a)** 3 **c)** −24 **e)** −6
b) 25 **d)** −10 **f)** 6

2. **a)** < **c)** >
b) > **d)** =

3. **a)** 55 **c)** −7 **e)** $\frac{15}{7}$
b) 60 **d)** 8 **f)** $\frac{1}{49}$

4. **a)** 5 **c)** −9 **e)** −12
b) 20 **d)** 76 **f)** −1

5. a) 3 c) −2
b) −1 d) 1

A–2 Operations with Rational Numbers, p. 544

1. a) $-\frac{1}{2}$ c) $-\frac{19}{12}$ e) $-\frac{41}{20}$
b) $\frac{7}{6}$ d) $-8\frac{7}{12}$ f) 1

2. a) $-\frac{16}{25}$ c) $\frac{2}{15}$ e) $-3\frac{2}{5}$
b) $-\frac{9}{5}$ d) $\frac{3}{2}$ f) $32\frac{7}{24}$

3. a) 2 c) $\frac{16}{9}$ e) $\frac{15}{2}$
b) $-4\frac{3}{4}$ d) $-\frac{9}{2}$ f) $\frac{2}{3}$

4. a) $\frac{1}{5}$ c) $\frac{1}{15}$ e) $\frac{36}{5}$
b) $\frac{3}{10}$ d) $-\frac{1}{18}$ f) $-\frac{3}{8}$

A–3 Exponent Laws, p. 546

1. a) 16 c) 9 e) −125
b) 1 d) −9 f) $\frac{1}{8}$

2. a) 2 c) 9 e) −16
b) 31 d) $\frac{1}{18}$ f) $\frac{13}{36}$

3. a) 9 c) 4 194 304
b) 50 d) $\frac{1}{27}$

4. a) x^8 c) y^7 e) x^6
b) m^9 d) a^{bc} f) $\frac{x^{12}}{y^9}$

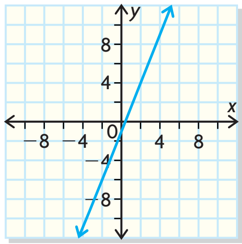
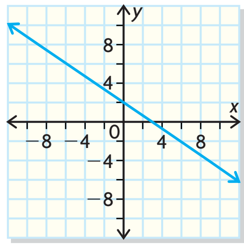
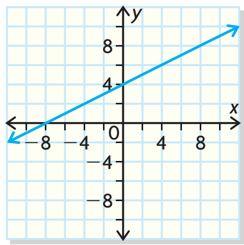
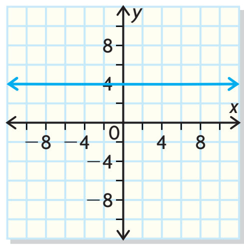
5. a) x^5y^6 c) $25x^4$
b) $108m^{12}$ d) $\frac{4u^2}{v^2}$

A–4 The Pythagorean Theorem, pp. 547–548

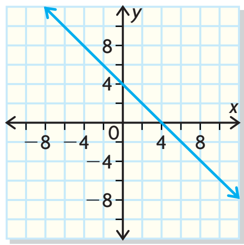
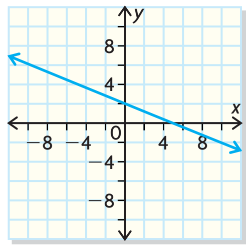
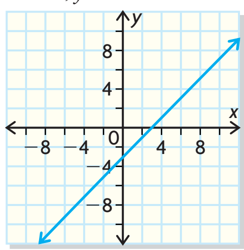
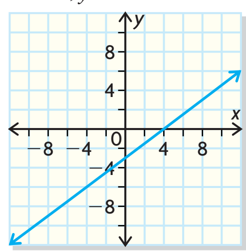
1. a) $x^2 = 6^2 + 8^2$ c) $9^2 = y^2 + 5^2$
b) $c^2 = 13^2 + 6^2$ d) $8.5^2 = a^2 + 3.2^2$
2. a) 10 cm c) 7.5 cm
b) 14.3 cm d) 7.9 cm
3. a) 13.93 c) 23.07
b) 6 d) 5.23
4. a) 11.2 m c) 7.4 cm
b) 6.7 cm d) 4.9 m
5. 10.6 cm
6. 69.4 m

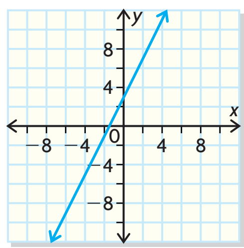
A–5 Graphing Linear Relationships, p. 550

1. a) $y = 2x + 3$ c) $y = -\frac{1}{2}x + 2$
b) $y = \frac{1}{2}x - 2$ d) $y = 5x + 9$

2. a)

c)

b)

d)


3. a) x-int: 10, y-int: 10 c) x-int: 5, y-int: 50
b) x-int: 8, y-int: 4 d) x-int: 2, y-int: 4

4. a) x-int: 4, y-int: 4

c) x-int: 5, y-int: 2

b) x-int: 3, y-int: −3

d) x-int: 4, y-int: −3


5. a)

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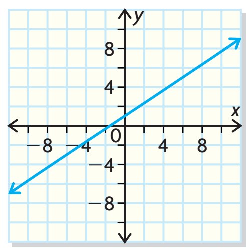
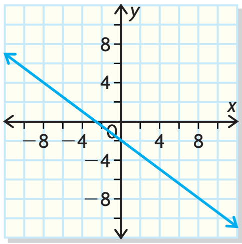
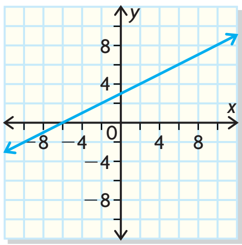


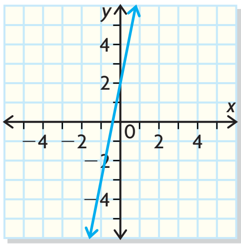
c)



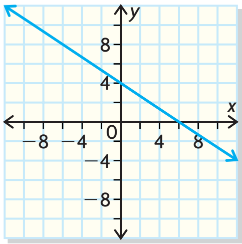
d)



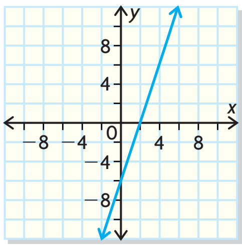
6. a)



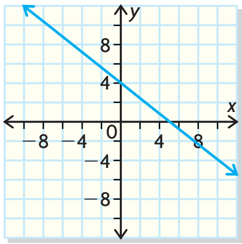
c)



b)

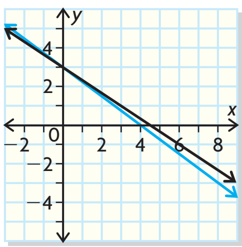


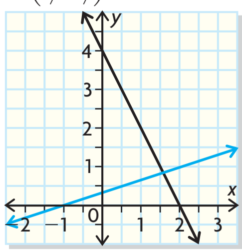
d)



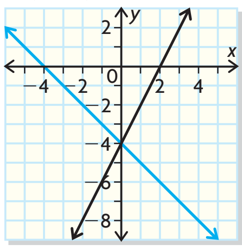
A–6 Solving Linear Systems, p. 553

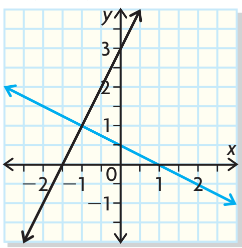
1. a) $(3, 2)$ b) $(5, -10)$
2. a) $(0, 3)$ c) $\left(\frac{11}{7}, \frac{6}{7}\right)$





b) $(0, -4)$ d) $(-1, 1)$





3. a) $(1, -1)$ d) $(2, 1)$
   b) $(6, 9)$ e) $\left(\frac{9}{11}, \frac{28}{11}\right)$
   c) $(6, -6)$ f) $\left(\frac{11}{7}, \frac{6}{7}\right)$

A–7 Evaluating Algebraic Expressions and Formulas, p. 554

1. a) 28 b) -17 c) 1 d) $\frac{9}{20}$
2. a) $\frac{1}{6}$ b) $\frac{5}{6}$ c) $\frac{-17}{6}$ d) $\frac{-7}{12}$
3. a) 82.35 cm^2 b) 58.09 m^2 c) 10 m d) 4849.05 cm^3

A–8 Expanding and Simplifying Algebraic Expressions, p. 555

1. a) $-2x - 5y$ c) $-9x - 10y$
   b) $11x^2 - 4x^3$ d) $4m^2n - p$
2. a) $6x + 15y - 6$ c) $3m^4 - 2m^2n$
   b) $5x^3 - 5x^2 + 5xy$ d) $4x^7y^7 - 2x^6y^8$
3. a) $8x^2 - 4x$ c) $-13m^5n - 22m^2n^2$
   b) $-34h^2 - 23h$ d) $-x^2y^3 - 12xy^4 - 7xy^3$
4. a) $12x^2 + 7x - 10$ c) $20x^2 - 23xy - 7y^2$
   b) $14 + 22y - 12y^2$ d) $15x^6 - 14x^3y^2 - 8y^4$

A–9 Factoring Algebraic Expressions, p. 556

1. a) $4(1 - 2x)$ c) $3m^2n^3(1 - 3mn)$
   b) $x(6x - 5)$ d) $14x(2x - y)$
2. a) $(x + 2)(x - 3)$ c) $(x - 5)(x - 4)$
   b) $(x + 2)(x + 5)$ d) $3(y + 4)(y + 2)$
3. a) $(3y - 2)(2y + 1)$ c) $(5ax - 3)(a + 2)$
   b) $(3x + 1)(4x - 1)$ d) $6(2x + 1)(x - 2)$

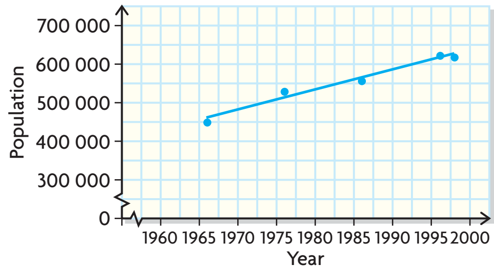
A–10 Solving Quadratic Equations Algebraically, p. 558

1. a) $x = 3, 2$ d) $x = \frac{3}{2}, \frac{4}{3}$
   b) $x = \frac{5}{2}, \frac{1}{3}$ e) $y = -\frac{5}{2}, \frac{7}{3}$
   c) $m = 4, 3$ f) $n = \frac{3}{5}, \frac{4}{3}$
2. a) $2, -1$ d) $-\frac{1}{2}, \frac{2}{3}$
   b) $4, -5$ e) $-\frac{4}{3}, \frac{1}{2}$
   c) $3, -5$ f) $3, -5$
3. a) $1 + \frac{\sqrt{3}}{2}, 1 - \frac{\sqrt{3}}{2}$ d) $-2, \frac{3}{5}$
   b) $\frac{3}{2}, -\frac{3}{2}$ e) $3, -2$
   c) $\frac{1}{2}, -\frac{1}{3}$ f) $7, 2$
4. at 6 s
5. in the year 2010

A–11 Creating Scatter Plots and Lines or Curves of Good Fit, p. 561

1. **a)** **i)**

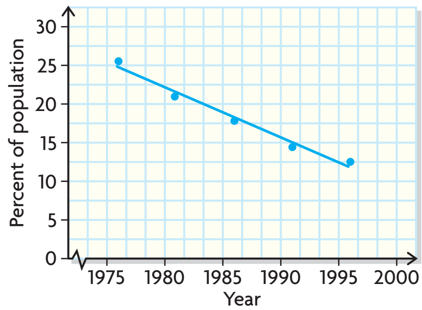
Population of the Hamilton–Wentworth, Ontario, Region



ii) The data displays a strong positive correlation.

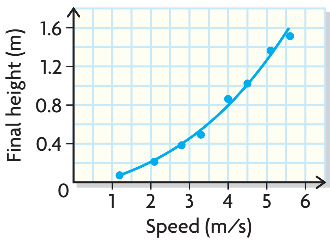
b) **i)**

Percent of Canadians with Less than Grade 9 Education



ii) The data displays a strong negative correlation.

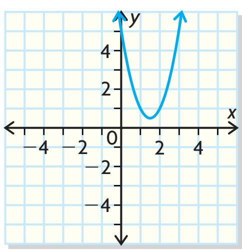
2. **a)**



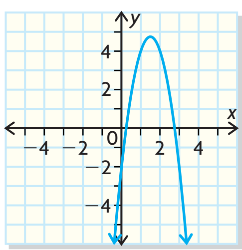
b) The motion sensor's measurements are consistent since the curve goes through several of the points.

A–12 Using Properties of Quadratic Relations to Sketch Their Graphs, p. 563

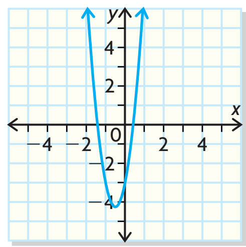
1. **a)**



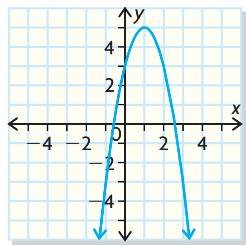
b)



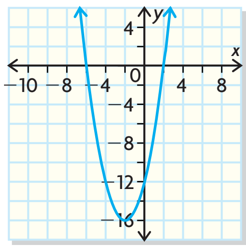
c)



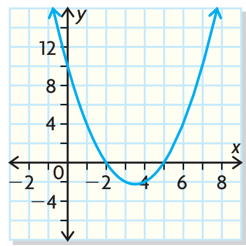
d)



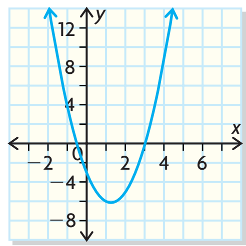
2. **a)**



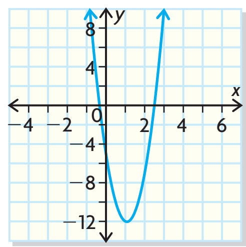
b)



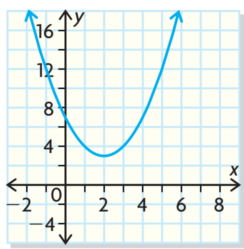
c)



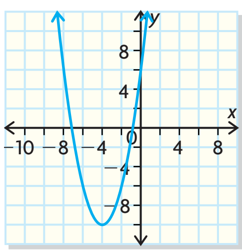
d)



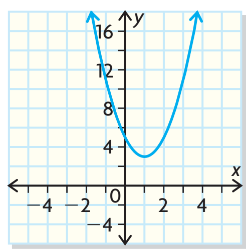
3. **a)**



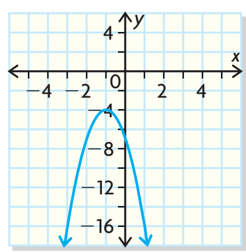
b)



c)



d)



A–13 Completing the Square to Convert to the Vertex Form of a Parabola, p. 564

1. **a)** $y = (x + 1)^2$ **c)** $y = (x + 3)^2$
b) $y = (x + 2)^2$ **d)** $y = (x + 5)^2$

2. **a)** $y = (x + 1)^2 + 1$ **c)** $y = (x - 6)^2 + 4$
b) $y = (x + 2)^2 + 2$ **d)** $y = (x - 9)^2 - 1$

3. **a)** $y = 2(x - 1)^2 + 5; x = 1; (1, 5)$
b) $y = 5(x + 1)^2 + 1; x = -1; (-1, 1)$
c) $y = -3(x + 2)^2 + 14; x = -2; (-2, 14)$
d) $y = -2(x - 1.5)^2 + 6.5; x = 1.5; (1.5, 6.5)$

4. **a)** 6 m **b)** 1 s

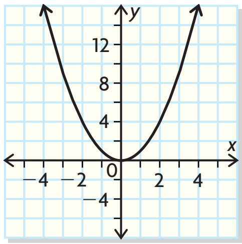
A–14 Transformations of Quadratic Relations, p.566

1. **a)** (2, 7) **b)** (−2, −1) **c)** (−5, 1) **d)** (7, 1) **e)** (3, −3) **f)** (2, 8)

2. **a)** (−4, 9) **b)** (3, −3) **c)** (−6, 7) **d)** (−5, −4)

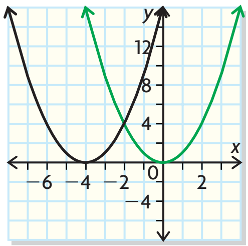
3. **a)** $y = x^2 + 2$ **b)** $y = 3x^2$ **c)** $y = (x - 4)^2$ **d)** $y = x^2 - 2$

4. **a)**

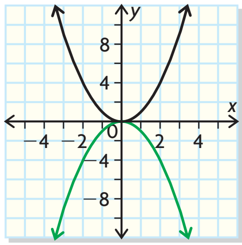


Graphs are identical.

d)

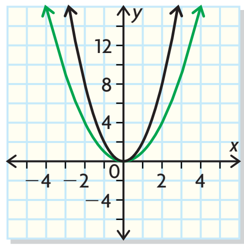


b)

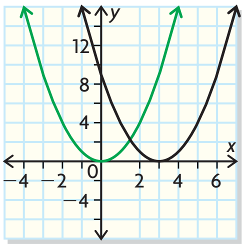


Graphs are identical.

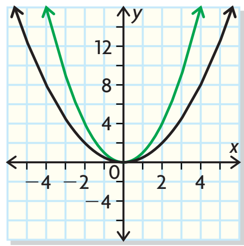
e)



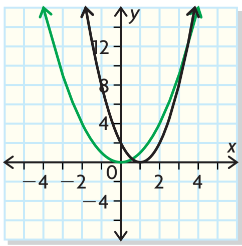
c)



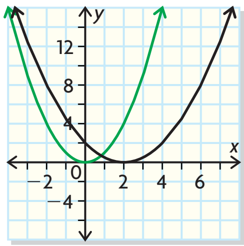
f)



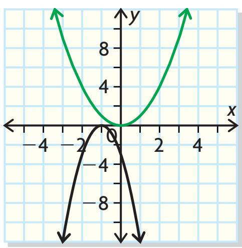
5. **a)**



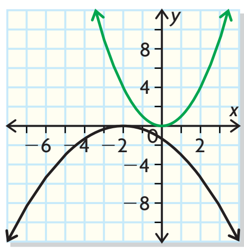
c)



b)



d)



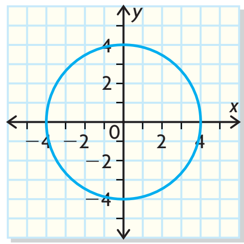
A–15 Equations of Circles Centred at the Origin, p. 568

1. **a)** $x^2 + y^2 = 9$ **b)** $x^2 + y^2 = 49$ **c)** $x^2 + y^2 = 64$ **d)** $x^2 + y^2 = 1$

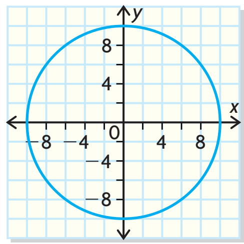
2. **a)** 3 **b)** 9 **c)** 3.87 **d)** 5.20 **e)** 2.50 **f)** 4.20

3. 5, −5

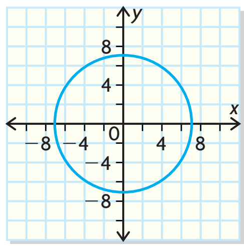
4. **a)**



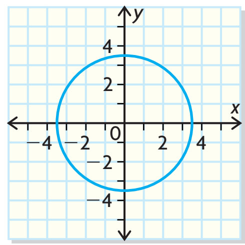
c)



b)



d)



A–16 Trigonometry of Right Triangles, p. 572

1. 27 m

2. **a)** $\sin A = \frac{12}{13}$, $\cos A = \frac{5}{13}$, $\tan A = \frac{12}{5}$

b) $\sin A = \frac{4}{5}$, $\cos A = \frac{3}{5}$, $\tan A = \frac{4}{3}$

c) $\sin A = \frac{8}{17}$, $\cos A = \frac{15}{17}$, $\tan A = \frac{8}{15}$

d) $\sin A = \frac{3}{5}$, $\cos A = \frac{4}{5}$, $\tan A = \frac{3}{4}$

3. **a)** 4.4 **b)** 6.8 **c)** 5.9 **d)** 26.9

4. **a)** 39° **b)** 54° **c)** 49° **d)** 25°

5. **a)** 12.5 cm **b)** 20.3 cm **c)** 19.7 cm **d)** 24°

6. **a)** 12.4 cm **b)** 5.7 cm **c)** 27° **d)** 46°

7. 8.7 m

8. 84.2 m

9. 195 m

A–17 Trigonometry of Acute Triangles: The Sine Law and the Cosine Law, p. 575

1. **a)** 10.3 **b)** 36.2° **c)** 85.1° **d)** 12.4 **e)** 47.3° **f)** 5.8

2. **a)** 16° **b)** 42.3° **c)** 23.4 **d)** 13.2 **e)** 33.1° **f)** 30.3

3. **a)** $t = 6.1$ cm, $\angle A = 72.8°$, $\angle C = 48.2°$

b) $\angle A = 33.8°$, $\angle B = 42.1°$, $\angle C = 104.1°$

c) $\angle F = 31.5°$, $\angle E = 109.5°$, $e = 25.8$ cm

4. 46.6

5. 1068.3 m

6. 12.2 m