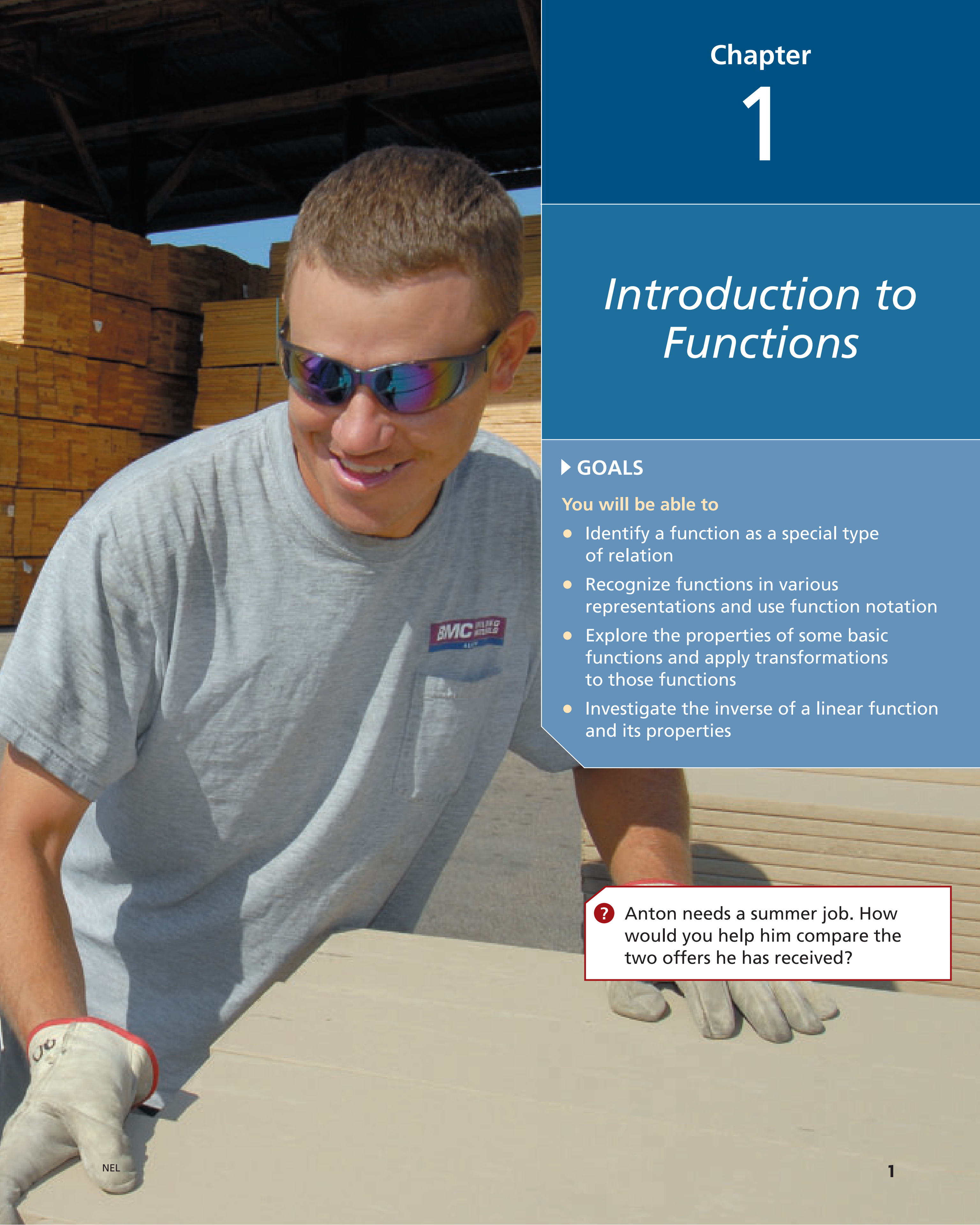

Help wanted:
Shipper at lumber store
$13.50/h
Potlatch
Help wanted:
Student-operated house painting business is hiring students now.
$650 per house

Chapter

1

Introduction to Functions



GOALS

You will be able to

- Identify a function as a special type of relation
- Recognize functions in various representations and use function notation
- Explore the properties of some basic functions and apply transformations to those functions
- Investigate the inverse of a linear function and its properties

? Anton needs a summer job. How would you help him compare the two offers he has received?

1 Getting Started

SKILLS AND CONCEPTS You Need

Study Aid

For help, see the Essential Skills Appendix.

Question	Appendix
1	A-8
2	A-7
4	A-5
5	A-15
6	A-12
7	A-14
8	A-9, A-10

1. Simplify each expression.

a) $3(x + y) - 5(x - y)$

b) $(4x - y)(4x + y)$

c) $\frac{1}{2}(x^2 + 1) - \frac{3}{2}(x^2 - 1)$

d) $4x(x + 2) - 2x(x - 4)$

2. Evaluate each expression in question 1 when $x = 3$ and $y = -5$.

3. Solve each linear equation.

a) $5x - 8 = 7$

b) $-2(x - 3) = 2(1 - 2x)$

c) $\frac{5}{6}y - \frac{3}{4}y = -3$

d) $\frac{x - 2}{4} = \frac{2x + 1}{3}$

4. Graph each **linear relation**.

a) $y = 2x - 3$

b) $3x + 4y = 12$

5. Graph each circle.

a) $x^2 + y^2 = 9$

b) $3x^2 + 3y^2 = 12$

6. Graph each **parabola**, labelling the **vertex** and the **axis of symmetry**.

a) $y = x^2 - 6$

b) $y = (x - 2)^2 - 1$

c) $y = -3(x + 4)^2 + 2$

d) $y = -x^2 + 6x$

7. For each **quadratic relation**, list the **transformations** you need to apply to $y = x^2$ to graph the relation. Then sketch the graph.

a) $y = x^2 - 2$

b) $y = -4x^2 + 3$

c) $y = \frac{1}{2}(x - 1)^2 - 4$

d) $y = -2(x + 3)^2 + 5$

8. Solve each quadratic equation.

a) $x^2 - 5x + 6 = 0$

b) $3x^2 - 5 = 70$

9. Compare the properties of linear relations, circles, and quadratic relations. Begin by completing a table like the one shown. Then list similarities and differences among the three types of relations.

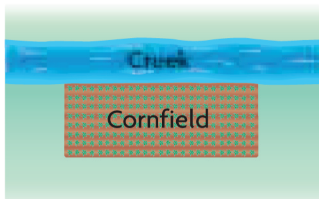
Property	Linear Relations	Circles	Quadratic Relations
Equation(s)			
Shape of graph			
Number of quadrants graph enters			
Descriptive features of graph			
Types of problems modelled by the relation			

APPLYING *What You Know*

Fencing a Cornfield

YOU WILL NEED
- graph paper

Rebecca has 600 m of fencing for her cornfield. The creek that goes through her farmland will form one side of the rectangular boundary. Rebecca considers different widths to maximize the area enclosed.



? How are the length and area of the field related to its width?

A. What are the minimum and maximum values of the width of the field?

B. What equations describe each?
- **i)** the relationship between the length and width of the field
- **ii)** the relationship between the area and width of the field

C. Copy and complete this table of values for widths that go from the least to the greatest possible values in intervals of 50 m.

Width (m)	Length (m)	Area (m^2)

D. Graph the data you wrote in the first two columns. Use width as the **independent variable**. Describe the graph. What type of relationship is this?

E. Now graph the data you wrote in the first and third columns. Use width as the independent variable again. Describe the graph. What type of relationship is this?

F. How could you have used the table of values to determine the types of relationships you reported in parts D and E?

G. How could you have used the equations from part B to determine the types of relationships you reported in parts D and E?

1.1 Relations and Functions

YOU WILL NEED

- graphing calculator or graph paper

GOAL

Recognize functions in various representations.

INVESTIGATE the Math

Ang recorded the heights and shoe sizes of students in his class.

Shoe Size	Height (cm)
10	158
11.5	175
10	173
9	164
9	167
10	170
11	172
8	160
8	174
11	175
8	166
7.5	153
10	171
11	181
11	171
10	170

Shoe Size	Height (cm)
8	156
7.5	161
12	179
11	178
10.5	173
8.5	177
8	165
12	182
13	177
13	192
7.5	157
8.5	163
12	183
10	168
11	180

? Can you predict a person's height from his or her shoe size?

A. Plot the data, using shoe size as the **independent variable**. Describe the relationship shown in the scatter plot.

B. Use your plot to predict the height of a person with each shoe size.
i) 8 **ii)** 10 **iii)** 13

C. Use your plot to predict what shoe size corresponds to each height.
i) 153 cm **ii)** 173 cm **iii)** 177 cm

D. Draw a **line of good fit** on your plot. Write the equation of your line, and use it to determine the heights corresponding to the shoe sizes in part B. How are your results different from those in part B?

Tech *Support*

For help drawing a line of best fit on a graphing calculator, see Technical Appendix, B-11.

E. Describe the domain and range of the relationship between shoe size and height in Ang's class.

F. Explain why the relation plotted in part A is not a function.

G. Is the relation drawn in part D a function? Explain.

H. Which of the relations in parts A and D could be used to predict a single height for a given shoe size? Explain.

domain
the set of all values of the independent variable of a relation

range
the set of all values of the dependent variable of a relation

relation
a set of ordered pairs; values of the independent variable are paired with values of the dependent variable

function
a relation where each value of the independent variable corresponds with only one value of the dependent variable

Reflecting

I. How did the numbers in the table of values show that the relation was not a function?

J. How did the graph of the linear function you drew in part D differ from the graph of the relation you plotted in part A?

K. Explain why it is easier to use the linear function than the scatter plot of the actual data to predict height.

APPLY the Math

EXAMPLE 1 Representing functions in different ways

The ages and soccer practice days of four students are listed.

Student	Age	Soccer Practice Day
Craig	15	Tuesday
Magda	16	Tuesday
Stefani	15	Thursday
Amit	17	Saturday

Communication *Tip*

Use braces to list the values, or elements, in a set.

For example, the set of the first five even numbers is {2, 4, 6, 8, 10}.

For each of the given relations, state the domain and range and then determine whether or not the relations are functions.

a) students and the day for soccer practice

b) ages and the day for soccer practice

Jenny's Solution: Using Set Notation

a) {(Craig, Tuesday),
(Magda, Tuesday),
(Stefani, Thursday),
(Amit, Saturday)}

Domain = {Craig, Magda, Stefanie, Amit}

Range = {Tuesday, Thursday, Saturday}

I wrote the relation as a set of ordered pairs, (student's name, day for practice). I wrote the domain by listing the students' names—the independent variable, or first elements, in each ordered pair.

I listed the day for practice—the dependent variable, or second elements—to write the range.

Each element of the domain corresponds with only one element in the range, so the relation between students and their soccer practice day is a function. The first elements appear only once in the list of ordered pairs. No name is repeated.

Each student has only one practice day, so the relation is a function. In this case, if I know the student's name, I can predict his or her practice day.

b) {(15, Tuesday), (16, Tuesday), (15, Thursday), (17, Saturday)}

I noticed that one 15-year-old practiced on Tuesday, but another practiced on Thursday, so I can't predict a practice day just by knowing the age. This is not a function.

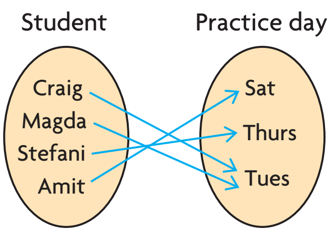
Domain = {15, 16, 17}

Range = {Tuesday, Thursday, Saturday}

15 in the domain corresponds with two different days in the range, so this relation is not a function.

Olivier's Solution: Using a Mapping Diagram

a)



I drew a diagram of the relation between students and soccer practice days by listing the student names in an oval and the days in another oval. Then I drew arrows to match the students with their practice days. The diagram is called a mapping diagram, since it *maps* the elements of the domain onto the elements of the range.

Domain = {Craig, Magda, Stefanie, Amit}

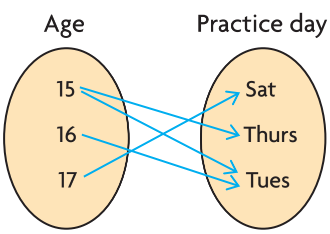
Range = {Tuesday, Thursday, Saturday}

The elements in the left oval are the values of the independent variable and make up the domain. The elements in the right oval are the values of the dependent variable and make up the range. I wrote the domain and range by listing what was in each oval.

Each element of the domain has only one corresponding element in the range, so the relation is a function.

The relation is a function because each student name has only one arrow leaving it.

b)



I drew another mapping diagram for the age and practice day relation. I matched the ages to the practice days.

Domain = {15, 16, 17}

Range = {Tuesday, Thursday, Saturday}

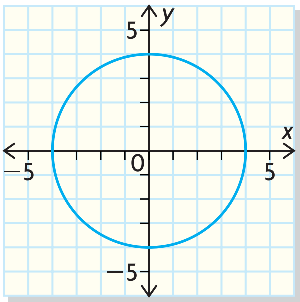
The value 15 of the independent variable, age, maps to two different values of the dependent variable, days. This relation is not a function.

Two arrows go from 15 to two different days. This cannot be a function. An element of the domain can't map to two elements in the range.

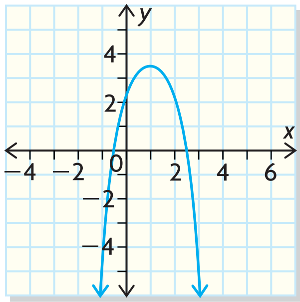
EXAMPLE 2 Selecting a strategy to recognize functions in graphs

Determine which of the following graphs are functions.

a)

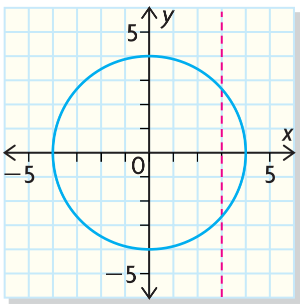


b)



Ken's Solution

a)



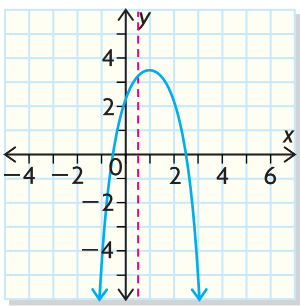
I used the **vertical-line test** to see how many points on the graph there were for each value of *x*.

An easy way to do this was to use a ruler to represent a vertical line and move it across the graph.

The ruler crossed the graph in two places everywhere except at the leftmost and rightmost ends of the circle. This showed that there are *x*-values in the domain of this relation that correspond to two *y*-values in the range.

At least one vertical line drawn on the graph intersects the graph at two points. This is not the graph of a function.

b)



I used the vertical-line test again. Wherever I placed my ruler, the vertical line intersected the graph in only one place. This showed that each *x*-value in the domain corresponds with only one *y*-value in the range.

Any vertical line drawn on the graph intersects the graph at only one point. This is the graph of a function.

vertical-line test

if any vertical line intersects the graph of a relation more than once, then the relation is not a function

EXAMPLE 3 | Using reasoning to recognize a function from an equation

Determine which equations represent functions.

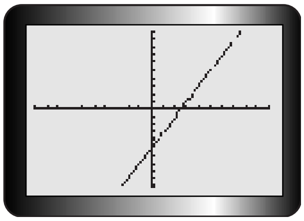
a) $y = 2x - 5$ **b)** $x^2 + y^2 = 9$ **c)** $y = 2x^2 - 3x + 1$

Keith's Solution: Using the Graph Defined by its Equation

a) This equation defines the graph of a linear function with a positive slope. Its graph is a straight line that increases from left to right.

I used my graphing calculator and entered $y = 2x - 5$. I graphed the function and checked it with the vertical-line test.

This graph passes the vertical-line test, showing that for each x-value in the domain there is only one y-value in the range. This is the graph of a function.

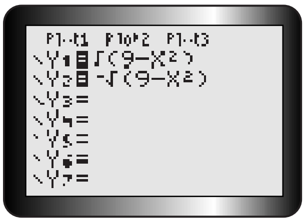


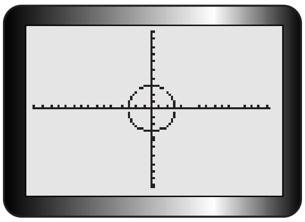
$y = 2x - 5$ is a function.

b) This equation defines the graph of a circle centred at (0, 0) with a radius of 3.

I used my graphing calculator and entered the upper half of the circle in Y1 and the lower half in Y2. Then I applied the vertical-line test to check.

This graph fails the vertical-line test, showing that there are x-values in the domain of this relation that correspond to two y-values in the range. This is not the graph of a function.

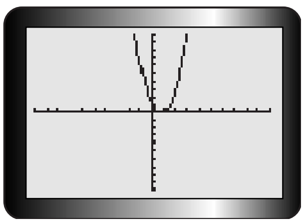




$x^2 + y^2 = 9$ is not a function.

c) This equation defines the graph of a parabola that opens upward.

I used my graphing calculator to enter $y = 2x^2 - 3x + 1$ and applied the vertical-line test to check.



This graph passes the vertical-line test, showing that for each x-value in the domain there is only one y-value in the range. This is the graph of a function.

$y = 2x^2 - 3x + 1$ is a function.

Mayda's Solution: Substituting Values

a) For any value of x, the equation $y = 2x - 5$ produces only one value of y. For example,

$y = 2(1) - 5 = -3$

This equation defines a function.

I substituted numbers for x in the equation.

No matter what number I substituted for x, I got only one answer for y when I doubled the number for x and then subtracted 5.

b) Substitute 0 for x in the equation

$$x^2 + y^2 = 9.$$
$$(0)^2 + y^2 = 9$$
$$y = 3 \text{ or } -3$$

There are two values for y when $x = 0$, so the equation defines a relation, but not a function.

I substituted 0 for x in the equation and solved for y. I used 0 because it's an easy value to calculate with. I got two values for y with $x = 0$.

c) Every value of x gives only one value of y in the equation $y = 2x^2 - 3x + 1$.
This equation represents a function.

No matter what number I choose for x, I get only one number for y that satisfies the equation.

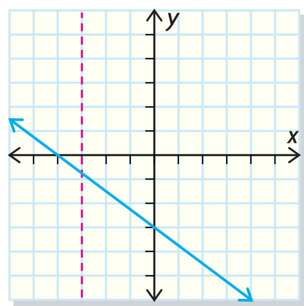
In Summary

Key Ideas

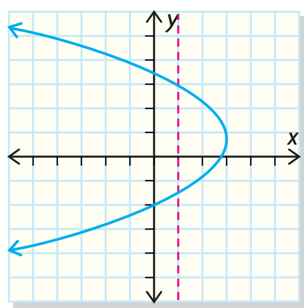
- A function is a relation in which each value of the independent variable corresponds with only one value of the dependent variable.
- Functions can be represented in various ways: in words, a table of values, a set of ordered pairs, a mapping diagram, a graph, or an equation.

Need To Know

- The domain of a relation or function is the set of all values of the independent variable. This is usually represented by the x-values on a coordinate grid.
- The range of a relation or function is the set of all values of the dependent variable. This is usually represented by the y-values on a coordinate grid.
- You can use the vertical-line test to check whether a graph represents a function. A graph represents a function if every vertical line intersects the graph in at most one point. This shows that there is only one element in the range for each element of the domain.
- You can recognize whether a relation is a function from its equation. If you can find even one value of x that gives more than one value of y when you substitute x into the equation, the relation is *not* a function. Linear relations, which have the general forms $y = mx + b$ or $Ax + By = C$ and whose graphs are straight lines, are all functions. Vertical lines are not functions but horizontal lines are. Quadratic relations, which have the general forms $y = ax^2 + bx + c$ or $y = a(x - h)^2 + k$ and whose graphs are parabolas, are also functions.



A relation that is a function



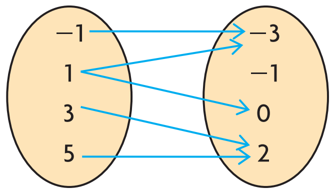
A relation that is not a function

CHECK Your Understanding

1. State which relations are functions. Explain.

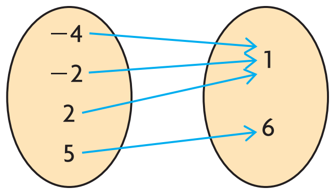
a) $\{(-5, 1), (-3, 2), (-1, 3), (1, 2)\}$

b)



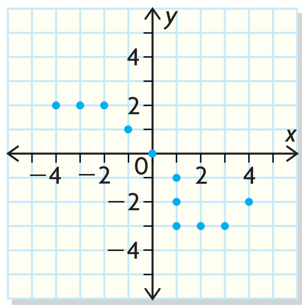
c) $\{(0, 4), (3, 5), (5, -2), (0, 1)\}$

d)

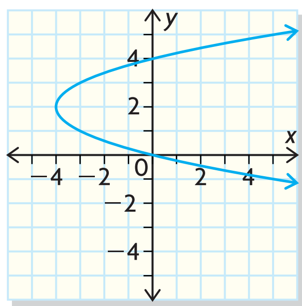


2. Use a ruler and the vertical-line test to determine which graphs are functions.

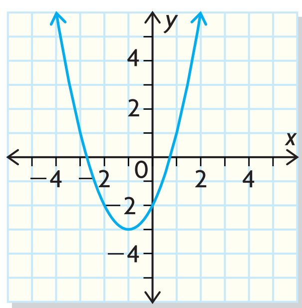
a)



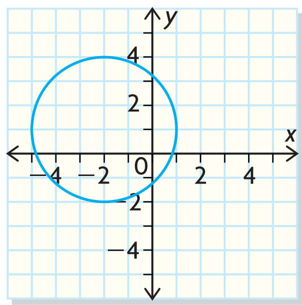
b)



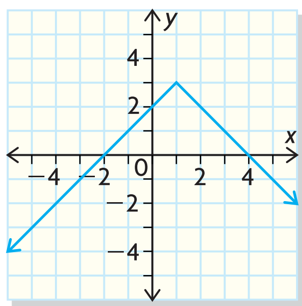
c)



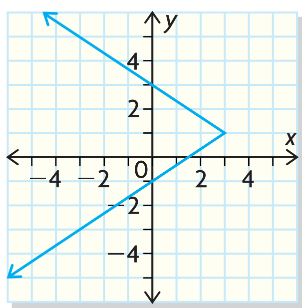
d)



e)



f)



3. Substitute -6 for x in each equation and solve for y. Use your results to explain why $y = x^2 - 5x$ is a function but $x = y^2 - 5y$ is not.

PRACTISING

4. The grades and numbers of credits for students are listed.
K

Student	Grade	Number of Credits
Barbara	10	8
Pierre	12	25
Kateri	11	15
Mandeep	11	18
Elly	10	16

a) Write a list of ordered pairs and create a mapping diagram for the relation between
- students and grades
- grades and numbers of credits
- students and numbers of credits

b) State the domain and range of each relation in part (a).
c) Which relations in part (a) are functions? Explain.

5. Graph the relations in question 4. Then use the vertical-line test to confirm your answers to part (c).

6. Describe the graphs of the relations $y = 3$ and $x = 3$. Are these relations functions? Explain.

7. Identify each type of relation and predict whether it is a function. Then graph each function and use the vertical-line test to determine whether your prediction was correct.

a) $y = 5 - 2x$ **c)** $y = -\frac{3}{4}(x + 3)^2 + 1$
b) $y = 2x^2 - 3$ **d)** $x^2 + y^2 = 25$

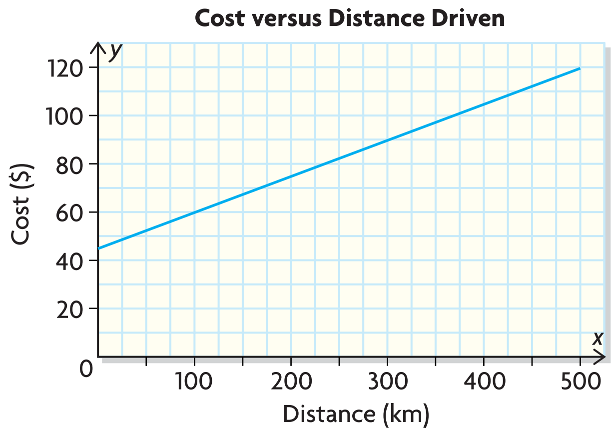
8. a) Substitute $x = 0$ into each equation and solve for y. Repeat for $x = -2$.
i) $3x + 4y = 5$ **iii)** $x^2 + y = 2$
ii) $x^2 + y^2 = 4$ **iv)** $x + y^2 = 0$
b) Which relations in part (a) appear to be functions?
c) How could you verify your answer to part (b)?

9. Determine which relations are functions.
a) $y = \sqrt{x + 2}$ **c)** $3x^2 - 4y^2 = 12$
b) $y = 2 - x$ **d)** $y = -3(x + 2)^2 - 4$

10. Use numeric and graphical representations to investigate whether the relation $x - y^2 = 2$ is a function. Explain your reasoning.

11. Determine which of the following relations are functions.
A
a) The relation between earnings and sales if Olwen earns \$400 per week plus 5% commission on sales
b) The relation between distance and time if Bran walks at 5 km/h
c) The relation between students' ages and the number of credits earned

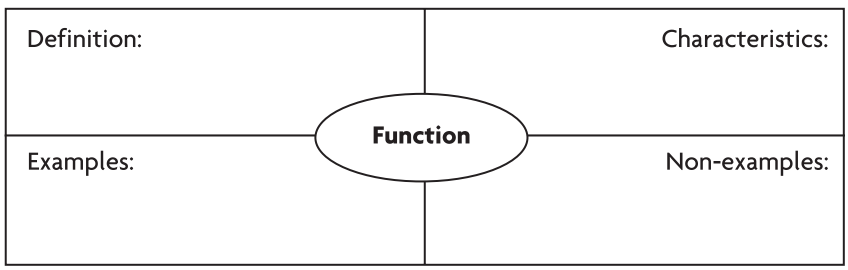
12. The cost of renting a car depends on the daily rental charge and the number of kilometres driven. A graph of cost versus the distance driven over a one-day period is shown.



a) What are the domain and range of this relation?
b) Explain why the domain and range have a lower limit.
c) Is the relation a function? Explain.

13. T **a)** Sketch a graph of a function that has the set of integers as its domain and all integers less than 5 as its range.
b) Sketch a graph of a relation that is not a function and that has the set of real numbers less than or equal to 10 as its domain and all real numbers greater than -5 as its range.

14. C Use a chart like the following to summarize what you have learned about functions.



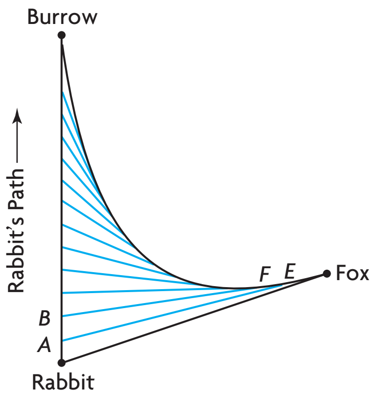
Extending

15. A freight delivery company charges $4/kg for any order less than 100 kg and $3.50/kg for any order of at least 100 kg.
a) Why must this relation be a function?
b) What is the domain of this function? What is its range?
c) Graph the function.
d) What suggestions can you offer to the company for a better pricing structure? Support your answer.

Curious | *Math*

Curves of Pursuit

A fox sees a rabbit sitting in the middle of a field and begins to run toward the rabbit. The rabbit sees the fox and runs in a straight line to its burrow. The fox continuously adjusts its direction so that it is always running directly toward the rabbit.



If the fox and rabbit are running at the same speed, the fox reaches point E when the rabbit reaches point A. The fox then changes direction to run along line EA. When the fox reaches point F, the rabbit is at B, so the fox begins to run along FB, and so on. The resulting curve is called a *curve of pursuit*.

1. If the original position of the rabbit represents the origin and the rabbit's path is along the positive y-axis, is the fox's path the graph of a function? Explain.

2. **a)** Investigate what happens by drawing a curve of pursuit if
  i) the burrow is farther away from the rabbit than it is in the first example
  ii) the burrow is closer to the rabbit than it is in the first example
b) Where does the fox finish in each case? How does the location of the burrow relative to the rabbit affect the fox's path?
c) Will the path of the fox always be a function, regardless of where the rabbit is relative to its burrow? Explain.

3. **a)** Draw a curve of pursuit in which
  i) the rabbit runs faster than the fox
  ii) the fox runs faster than the rabbit
b) Are these relations also functions? How do they differ from the one in question 1?

1.2 Function Notation

YOU WILL NEED

- graphing calculator

GOAL

Use function notation to represent linear and quadratic functions.

LEARN ABOUT the Math

The deepest mine in the world, East Rand mine in South Africa, reaches 3585 m into Earth's crust. Another South African mine, Western Deep, is being deepened to 4100 m. Suppose the temperature at the top of the mine shaft is 11 °C and that it increases at a rate of 0.015 °C/m as you descend.

? What is the temperature at the bottom of each mine?

function notation

notation, such as $f(x)$, used to represent the value of the dependent variable—the output—for a given value of the independent variable, x—the input

EXAMPLE 1 Representing a situation with a function and using it to solve a problem

a) Represent the temperature in a mine shaft with a function. Explain why your representation is a function, and write it in function notation.

b) Use your function to determine the temperature at the bottom of East Rand and Western Deep mines.

Lucy's Solution: Using an Equation

a) An equation for temperature is $T = 11 + 0.015d$, where T represents the temperature in degrees Celsius at a depth of d metres.

← I wrote a linear equation for the problem.

I used the fact that T starts at 11 °C and increases at a steady rate of 0.015 °C/m.

The equation represents a function. Temperature is a function of depth.

← Since this equation represents a linear relationship between temperature and depth, it is a function.

In function notation, $T(d) = 11 + 0.015d$.

← I wrote the equation again. $T(d)$ makes it clearer that T is a function of d.

Communication *Tip*

The notations y and $f(x)$ are interchangeable in the equation or graph of a function, so y is equal to $f(x)$. The notation $f(x)$ is read "f at x" or "f of x." The symbols $f(x)$, $g(x)$, and $h(x)$ are often used to name the outputs of functions, but other letters are also used, such as $v(t)$ for velocity as a function of time.

b) $T(3585) = 11 + 0.015(3585)$
$= 11 + 53.775$
$= 64.775$

I found the temperature at the bottom of East Rand mine by calculating the temperature at a depth of 3585 m. I substituted 3585 for *d* in the equation.

$T(4100) = 11 + 0.015(4100)$
$= 11 + 61.5$
$= 72.5$

For the new mine, I wanted the temperature when $d = 4100$, so I calculated $T(4100)$.

The temperatures at the bottom of East Rand mine and Western Deep mine are about 65 °C and 73 °C, respectively.

Stuart's Solution: Using a Graph

a) $T(d) = 11 + 0.015d$

This is a function because it is a linear relationship.

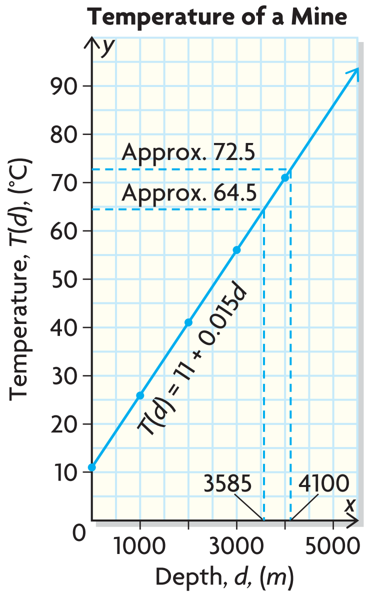
I wrote an equation to show how the temperature changes as you go down the mine.
I knew that the relationship was linear because the temperature increases at a steady rate. I used *d* for depth and called the function $T(d)$ for temperature.

b)

d **(m)**	***T*(*d*)(°C)**
0	$T(0) = 11 + 0.015(0) = 11$
1000	26
2000	41
3000	56
4000	71

I made a table of values for the function.

I substituted the *d*-values into the function equation to get the $T(d)$-values.



I plotted the points (0, 11), (1000, 26), (2000, 41), (3000, 56), and (4000, 71). Then I joined them with a straight line.

East Rand mine is 3585 m deep. The temperature at the bottom is $T(3585)$.

I interpolated to read $T(3585)$ from the graph. It was approximately 65.

The other mine is 4100 m deep.

By extrapolating, I found that $T(4100)$ was about 73.

The temperature at the bottom of East Rand mine is about 65 °C. The temperature at the bottom of Western Deep mine is about 73 °C.

Tech Support

For help using a graphing calculator to graph and evaluate functions, see Technical Appendix, B-2 and B-3.

Eli's Solution: Using a Graphing Calculator

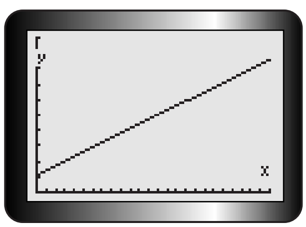
a) Let $T(d)$ represent the temperature in degrees Celsius at a depth of d metres.

I used function notation to write the equation.

$T(d) = 11 + 0.015d$

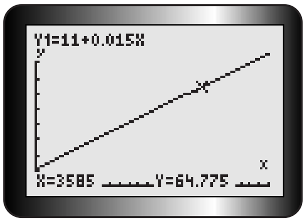
Temperature increases at a steady rate, so it is a function of depth.

b)

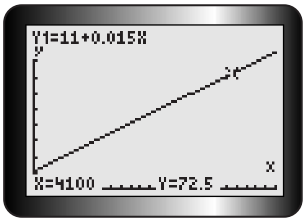


I graphed the function by entering Y1 = 11 + 0.015X into the equation editor.

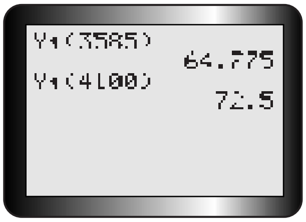
I used **WINDOW** settings of $0 \le X \le 5000$, Xscl 200, and $0 \le Y \le 100$, Yscl 10.



I used the value operation to find the temperature at the bottom of East Rand mine. This told me that $T(3585) = 64.775$.



Then I used the value operation again to find the temperature at the bottom of the other mine. I found that $T(4100) = 72.5$.



As a check, I called up the function on my calculator home screen, using **VARS** and function notation to display both answers.

The temperature at the bottom of the East Rand mine is about 65 °C. The temperature at the bottom of Western Deep mine is about 73 °C.

Reflecting

A. How did Lucy, Stuart, and Eli know that the relationship between temperature and depth is a function?

B. How did Lucy use the function equation to determine the two temperatures?

C. What does $T(3585)$ mean? How did Stuart use the graph to determine the value of $T(3585)$?

APPLY the Math

EXAMPLE 2 Representing a situation with a function model

A family played a game to decide who got to eat the last piece of pizza. Each person had to think of a number, double it, and subtract the result from 12. Finally, they each multiplied the resulting difference by the number they first thought of. The person with the highest final number won the pizza slice.

a) Use function notation to express the final answer in terms of the original number.

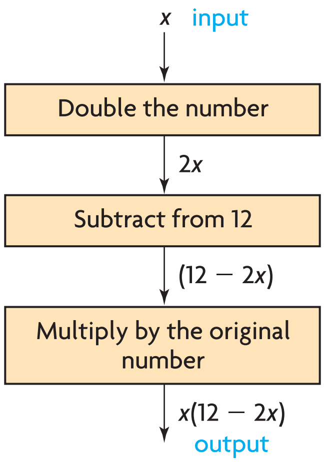
b) The original numbers chosen by the family members are shown. Who won the pizza slice?

c) What would be the best number to choose? Why?

Tim	5
Rhea	−2
Sara	7
Andy	10

Barbara's Solution

a)



I used a flow chart to show what happens to the original number in the game.

I chose x to be the original number, or input.

$$f(x) = x(12 - 2x)$$
$$= 12x - 2x^2$$

The expression for the final answer is quadratic, so the final result must be a function of the original number.

I chose $f(x)$ as the name for the final answer, or output.

b) Tim: $f(5) = 12(5) - 2(5)^2$

$= 60 - 2(25)$

$= 60 - 50$

$= 10$

I found the values of $f(5)$, $f(-2)$, $f(7)$, and $f(10)$ to see who had the highest answer.

Tim's answer was 10.

Rhea: $f(-2) = 12(-2) - 2(-2)^2$

$= -24 - 2(4)$

$= -24 - 8$

$= -32$

Rhea's answer was -32.

Sara: $f(7) = 12(7) - 2(7)^2$

$= 84 - 2(49)$

$= 84 - 98 = -14$

Sara's answer was -14.

Andy: $f(10) = 12(10) - 2(10)^2$

$= 120 - 2(100)$

$= 120 - 200 = -80$

Andy's answer was -80.

Tim won the pizza slice.

Tim's answer was the highest.

c) $f(x) = 12x - 2x^2$

I recognized that the equation was quadratic and that its graph would be a parabola that opened down, since the coefficient of x^2 was negative.

This meant that this quadratic function had a maximum value at its vertex.

$f(x) = -2x(x - 6)$

The x-intercepts are $x = 0$ and $x = 6$.

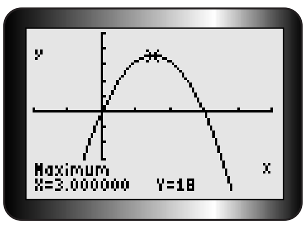
Vertex: $x = (0 + 6) \div 2$

$x = 3$

The best number to choose is 3.

I put the equation back in factored form by dividing out the common factor, $-2x$.

I remembered that the x-coordinate of the vertex is halfway between the two x-intercepts.



I checked my answer by graphing.

EXAMPLE 3 Connecting function notation to a graph

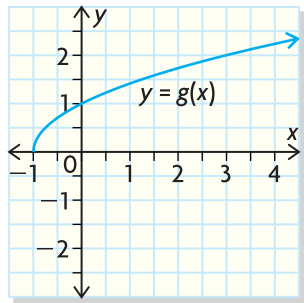
For the function shown in the graph, determine each value.

a) $g(3)$

b) $g(-1)$

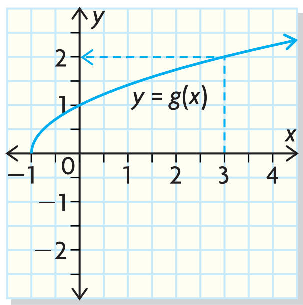
c) x if $g(x) = 1$

d) the domain and range of $g(x)$



Ernesto's Solution

a)



I looked at the graph to find the y-coordinate when $x = 3$.

I drew a line up to the graph from the x-axis at $x = 3$ and then a line across from that point of intersection to the y-axis.

When $x = 3, y = 2.$

$g(3) = 2$

The y-value was 2, so, in function notation, $g(3) = 2$.

b) When $x = -1, y = 0.$

$g(-1) = 0$

I saw that $y = 0$ when $x = -1$, so -1 is the x-intercept and $g(-1) = 0$.

c) $g(x) = 1$ when $x = 0$

I saw that the graph crosses the y-axis at $y = 1$. The x-value is 0 at this point.

d) The graph begins at the point $(-1, 0)$ and continues upward. The graph exists only for $x \geq -1$ and $y \geq 0$.

The domain is all real numbers greater than or equal to -1.

The range is all real numbers greater than or equal to 0.

I saw that there was no graph to the left of the point $(-1, 0)$ or below that point.

So the only possible x-values are $x \geq -1$, and the only possible y-values are $y \geq 0$.

EXAMPLE 4 Using algebraic expressions in functions

Consider the functions $f(x) = x^2 - 3x$ and $g(x) = 1 - 2x$.

a) Show that $f(2) > g(2)$, and explain what that means about their graphs.

b) Determine $g(3b)$.

c) Determine $f(c + 2) - g(c + 2)$.

Jamilla's Solution

a) $f(x) = x^2 - 3x$

$f(2) = (2)^2 - 3(2)$

$= 4 - 6$

$= -2$

$g(x) = 1 - 2x$

$g(2) = 1 - 2(2)$

$= 1 - 4$

$= -3$

I substituted 2 for x in both functions.

$-2 > -3$, so $f(2) > g(2)$

That means that the point on the graph of $f(x)$ is above the point on the graph of $g(x)$ when $x = 2$.

b) $g(3b) = 1 - 2(3b)$

$= 1 - 6b$

I substituted $3b$ for x.

I simplified the equation.

c) $f(c + 2) - g(c + 2) = [(c + 2)^2 - 3(c + 2)] - [1 - 2(c + 2)]$

$= [(c^2 + 4c + 4 - 3c - 6)] - [1 - 2c - 4]$

$= [c^2 + c - 2] - [-3 - 2c]$

$= c^2 + c - 2 + 3 + 2c$

$= c^2 + 3c + 1$

I substituted $c + 2$ for x in both functions.

I used square brackets to keep the functions separate until I had simplified each one.

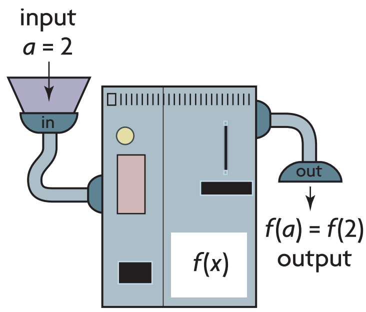
In Summary

Key Idea

- Symbols such as $f(x)$ are called function notation, which is used to represent the value of the dependent variable y for a given value of the independent variable x. For this reason, y and $f(x)$ are interchangeable in the equation of a function, so $y = f(x)$.

Need to Know

- $f(x)$ is read "*f* at *x*" or "*f* of *x*."
- $f(a)$ represents the value or output of the function when the input is $x = a$. The output depends on the equation of the function. To evaluate $f(a)$, substitute a for x in the equation for $f(x)$.
- $f(a)$ is the y-coordinate of the point on the graph of f with x-coordinate a. For example, if $f(x)$ takes the value 3 at $x = 2$, then $f(2) = 3$ and the point (2, 3) lies on the graph of f.

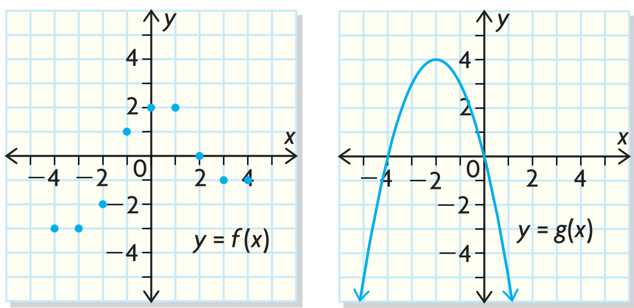


CHECK Your Understanding

1. Evaluate, where $f(x) = 2 - 3x$.

a) $f(2)$ **b)** $f(0)$ **c)** $f(-4)$ **d)** $f\left(\frac{1}{2}\right)$ **e)** $f(a)$ **f)** $f(3b)$

2. The graphs of $y = f(x)$ and $y = g(x)$ are shown.



Using the graphs, evaluate

a) $f(1)$ **b)** $g(-2)$ **c)** $f(4) - g(-2)$ **d)** x when $f(x) = -3$

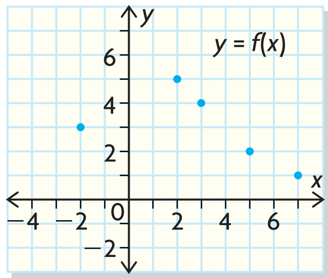
3. Milk is leaking from a carton at a rate of 3 mL/min. There is 1.2 L of milk in the carton at 11:00 a.m.

a) Use function notation to write an equation for this situation.
b) How much will be left in the carton at 1:00 p.m.?
c) At what time will 450 mL of milk be left in the carton?

PRACTISING

4. Evaluate $f(-1)$, $f(3)$, and $f(1.5)$ for
a) $f(x) = (x - 2)^2 - 1$ **b)** $f(x) = 2 + 3x - 4x^2$

5. For $f(x) = \frac{1}{2x}$, determine
a) $f(-3)$ **b)** $f(0)$ **c)** $f(1) - f(3)$ **d)** $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right)$

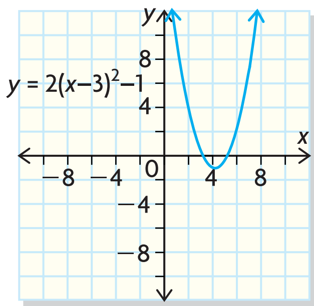


6. The graph of $y = f(x)$ is shown at the right.
a) State the domain and range of f.
b) Evaluate.
i) $f(3)$ **iii)** $f(5 - 3)$
ii) $f(5)$ **iv)** $f(5) - f(3)$

7. For $h(x) = 2x - 5$, determine
a) $h(a)$ **c)** $h(3c - 1)$
b) $h(b + 1)$ **d)** $h(2 - 5x)$

8. Consider the function $g(t) = 3t + 5$.
a) Create a table of values and graph the function.
b) Determine each value.
i) $g(0)$ **iv)** $g(2) - g(1)$
ii) $g(3)$ **v)** $g(1001) - g(1000)$
iii) $g(1) - g(0)$ **vi)** $g(a + 1) - g(a)$

9. Consider the function $f(s) = s^2 - 6s + 9$.
a) Create a table of values for the function.
b) Determine each value.
i) $f(0)$ **iv)** $f(3)$
ii) $f(1)$ **v)** $[f(2) - f(1)] - [f(1) - f(0)]$
iii) $f(2)$ **vi)** $[f(3) - f(2)] - [f(2) - f(1)]$
c) In part (b), what do you notice about the answers to parts (v) and (vi)? Explain why this happens.



10. **K** The graph at the right shows $f(x) = 2(x - 3)^2 - 1$.
a) Evaluate $f(-2)$.
b) What does $f(-2)$ represent on the graph of f?
c) State the domain and range of the relation.
d) How do you know that f is a function from its graph?

11. For $g(x) = 4 - 5x$, determine the input for x when the output of $g(x)$ is
a) -6 **b)** 2 **c)** 0 **d)** $\frac{3}{5}$

12. A company rents cars for \$50 per day plus \$0.15/km.

a) Express the daily rental cost as a function of the number of kilometres travelled.

b) Determine the rental cost if you drive 472 km in one day.

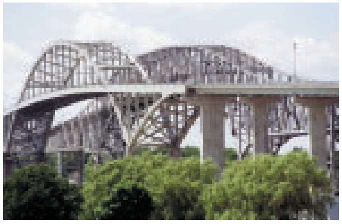
c) Determine how far you can drive in a day for \$80.

13. **A** As a mental arithmetic exercise, a teacher asked her students to think of a number, triple it, and subtract the resulting number from 24. Finally, they were asked to multiply the resulting difference by the number they first thought of.

a) Use function notation to express the final answer in terms of the original number.

b) Determine the result of choosing numbers 3, -5, and 10.

c) Determine the maximum result possible.



14. **T** The second span of the Bluewater Bridge in Sarnia, Ontario, is supported by two parabolic arches. Each arch is set in concrete foundations that are on opposite sides of the St. Clair River. The arches are 281 m apart. The top of each arch rises 71 m above the river. Write a function to model the arch.

15. **a)** Graph the function $f(x) = 3(x - 1)^2 - 4$.

b) What does $f(-1)$ represent on the graph? Indicate on the graph how you would find $f(-1)$.

c) Use the equation to determine

i) $f(2) - f(1)$ **ii)** $2f(3) - 7$ **iii)** $f(1 - x)$

16. Let $f(x) = x^2 + 2x - 15$. Determine the values of x for which

a) $f(x) = 0$ **b)** $f(x) = -12$ **c)** $f(x) = -16$

17. Let $f(x) = 3x + 1$ and $g(x) = 2 - x$. Determine values for a such that

a) $f(a) = g(a)$ **b)** $f(a^2) = g(2a)$

18. **C** Explain, with examples, what function notation is and how it relates to the graph of a function. Include a discussion of the advantages of using function notation.

Extending

19. The highest and lowest marks awarded on an examination were 285 and 75. All the marks must be reduced so that the highest and lowest marks become 200 and 60.

a) Determine a linear function that will convert 285 to 200 and 75 to 60.

b) Use the function to determine the new marks that correspond to original marks of 95, 175, 215, and 255.

20. A function $f(x)$ has these properties:

- The domain of f is the set of natural numbers.
- $f(1) = 1$
- $f(x + 1) = f(x) + 3x(x + 1) + 1$

a) Determine $f(2)$, $f(3)$, $f(4)$, $f(5)$, and $f(6)$.

b) Describe the function.

1.3 Exploring Properties of Parent Functions

GOAL

Explore and compare the graphs and equations of five basic functions.

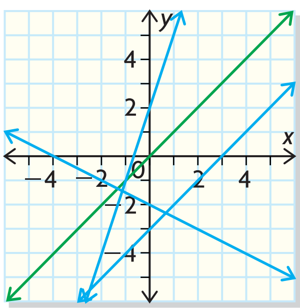
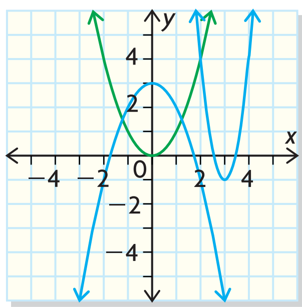
YOU WILL NEED

- graphing calculator or graphing technology
- graph paper

EXPLORE the Math

As a child, you learned how to recognize different animal families.

In mathematics, every function can be classified as a member of a **family**. Each member of a family of functions is related to the simplest, or most basic, function sharing the same characteristics. This function is called the **parent function**. Here are some members of the linear and quadratic families. The parent functions are in green.

Linear Functions	Quadratic Functions
 Parent function: $f(x) = x$	 Parent function: $f(x) = x^2$
Family members: $f(x) = mx + b$	Family members: $f(x) = a(x - h)^2 + k$
Examples: $f(x) = 3x + 2$, $f(x) = -\frac{1}{2}x - 2$	Examples: $f(x) = 5(x - 3)^2 - 1$, $f(x) = -x^2 + 3$

Communication *Tip*

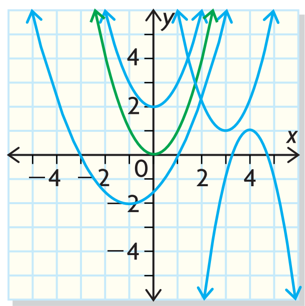
$\sqrt{x}$ always means the positive square root of x.

family

a collection of functions (or lines or curves) sharing common characteristics

parent function

the simplest, or base, function in a family



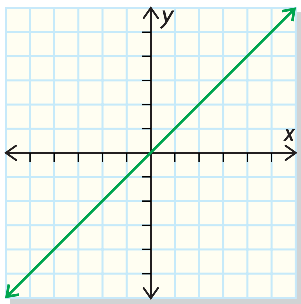
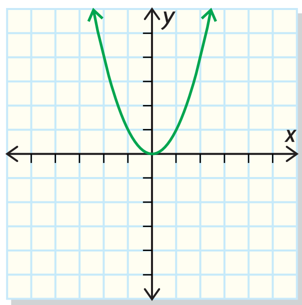
absolute value

written as |x|; describes the distance of x from 0; equals x when $x \geq 0$ or $-x$ when $x < 0$; for example, $|3| = 3$ and $|-3| = -(-3) = 3$

Three more parent functions are the square root function $f(x) = \sqrt{x}$, the reciprocal function $f(x) = \frac{1}{x}$, and the **absolute value** function $f(x) = |x|$.

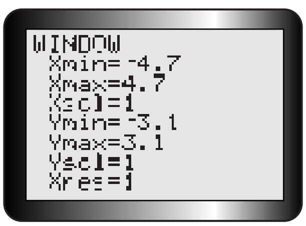
What are the characteristics of these parent functions that distinguish them from each other?

A. Make a table like the one shown.

Equation of Function	Name of Function	Sketch of Graph	Special Features/ Symmetry	Domain	Range
$f(x) = x$	linear function		• straight line that goes through the origin • slope is 1 • divides the plane exactly in half diagonally • graph only in quadrants 1 and 3		
$f(x) = x^2$	quadratic function		• parabola that opens up • vertex at the origin • y has a minimum value • y-axis is axis of symmetry • graph only in quadrants 1 and 2		
$f(x) = \sqrt{x}$	square root function				
$f(x) = \frac{1}{x}$	reciprocal function				
$f(x) = \|x\|$	absolute value function				

Tech *Support*

Use the following **WINDOW** settings to graph the functions:



You can change to these settings by pressing ZOOM 4 .

B. Use your graphing calculator to check the sketches shown for $f(x) = x$ and $f(x) = x^2$ and add anything you think is missing from the descriptions. Explain how you know that these equations are both functions.

C. In your table, record the domain and range of each of $f(x) = x$ and $f(x) = x^2$.

D. Clear all equations from the equation editor. Graph the square root function, $f(x) = \sqrt{x}$. In your table, sketch the graph and describe its shape. Is it a function? Explain. How is it different from the graphs of linear and quadratic functions?

E. Go to the table of values and scroll up and down the table. Does ERR: appear in the Y column? Explain why this happens.

F. Using the table of values and the graph, determine and record the domain and range of the function.

G. Repeat parts D through F for the reciprocal function $f(x) = \frac{1}{x}$. Use the table of values to see what happens to y when x is close to 0 and when x is far from 0. Explain why the graph is in two parts with a break in the middle.

H. Where are the asymptotes of this graph?

I. Repeat parts D through F for the absolute value function $f(x) = |x|$. Which of the other functions is the resulting graph most like? Explain. When you have finished, make sure that your table contains enough information for you to recognize each of the five parent functions.

Reflecting

J. Explain how each of the following helped you determine the domain and range.

a) the table of values
b) the graph
c) the function's equation

K. Which graphs lie in the listed quadrants?

a) the first and second quadrants
b) the first and third quadrants

L. Which graph has asymptotes? Why?

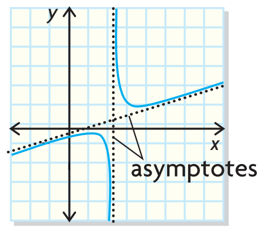
M. You have used the slope and y-intercept to sketch lines, vertices, and directions of opening to sketch parabolas. What characteristics of the new parent functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, and $f(x) = |x|$ could you use to sketch their graphs?

Tech Support

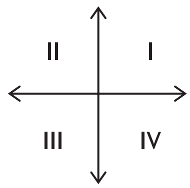
For help with the TABLE function of the graphing calculator, see Technical Appendix, B-6.
To graph $f(x) = |x|$, press MATH ▶ 1 X,T,Θ,n) .

asymptote

a line that the graph of a relation or function gets closer and closer to, but never meets, on some portion of its domain



Quadrants



In Summary

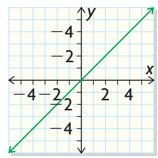
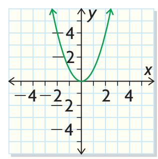
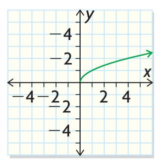
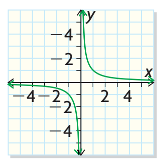
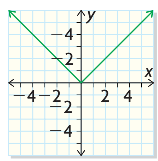
Key Idea

- Certain basic functions, called parent functions, form the building blocks for families of more complicated functions. Parent functions include, but are not limited to, $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, and $f(x) = |x|$.

(continued)

Need to Know

- Each of the parent functions $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, and $f(x) = |x|$ has unique characteristics that define the shape of its graph.

Equation of Function	Name of Function	Sketch of Graph
$f(x) = x$	linear function	
$f(x) = x^2$	quadratic function	
$f(x) = \sqrt{x}$	square root function	
$f(x) = \frac{1}{x}$	reciprocal function	
$f(x) = \|x\|$	absolute value function	

FURTHER Your Understanding

1. Sketch the graphs of $f(x) = x$ and $g(x) = \frac{1}{x}$ on the same axes. What do the graphs have in common? What is different about the graphs? Write equations of the asymptotes for the reciprocal function.

2. Sketch the graphs of $f(x) = x^2$ and $g(x) = |x|$ on the same axes. Describe how these graphs are alike and how they are different.

3. Sketch the graph of $f(x) = x^2$ for values of $x \geq 0$. On the same axes, sketch the graphs of $g(x) = \sqrt{x}$ and $h(x) = x$. Describe how the three graphs are related.

1.4 Determining the Domain and Range of a Function

GOAL

Use tables, graphs, and equations to find domains and ranges of functions.

YOU WILL NEED

- graph paper
- graphing calculator

LEARN ABOUT the Math

The CN Tower in Toronto has a lookout level that is 346 m above the ground.

A gull landing on the guardrail causes a pebble to fall off the edge.

The speed of the pebble as it falls to the ground is a **function** of how far it has fallen. The equation for this function is $v(d) = \sqrt{2gd}$, where

- d is the distance, in metres, the pebble has fallen
- $v(d)$ is the speed of the pebble, in metres per second (m/s)
- g is the acceleration due to gravity—about 9.8 metres per second squared (m/s^2)

? How can you determine the domain and range of the function $v(d)$?

EXAMPLE 1 Selecting a strategy to determine the domain and range

Determine the domain and range of $v(d)$, the pebble's speed.

Sally's Solution: Using a Graph

The pebble falls a total distance of 346 m. So the domain is $0 \le d \le 346$.

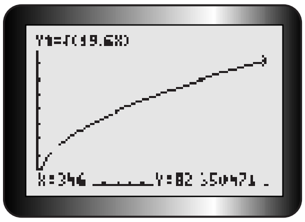
The distance d is 0 m when the pebble first falls off the edge and 346 m when the pebble lands on the ground. So d can take only values that lie in between 0 and 346. This gave me the domain for the function.

$2 \times 9.8 = 19.6$, so the function is

$v(d) = \sqrt{19.6d}$ for $0 \le d \le 346$

I entered Y= $\sqrt{19.6x}$ into my graphing calculator. I used $0 \le X \le 346$, Xscl 20 and $0 \le Y \le 100$, Yscl 10 for WINDOW settings.

Range:



$v(346) \doteq 82.4$

So the range is $0 \leq v(d) \leq 82.4$.

I saw that the graph started at the origin.

The pebble starts with no velocity. So 0 is the minimum value of the range.

The graph showed me that as the pebble's distance increases, so does its velocity. The pebble must be travelling the fastest when it hits the ground. This happens when $d = 346$. The maximum value of the range is $v(346)$. I evaluated this using the value operation.

Domain $= \{d \in \mathbf{R} \mid 0 \leq h \leq 346\}$

Range $= \{v(d) \in \mathbf{R} \mid 0 \leq v(d) \leq 82.4\}$

I used set notation to write the domain and range. I defined them as sets of **real numbers**.

real numbers

numbers that are either rational or irrational; these include positive and negative integers, zero, fractions, and irrational numbers such as $\sqrt{2}$ and π

Communication *Tip*

Set notation can be used to describe domains and ranges. For example, $\{x \in \mathbf{R} \mid 0 \leq x < 50\}$ is read "the set of all values x that belong to the set of real numbers, such that x is greater than or equal to 0 and less than 50." The symbol "|" stands for "such that."

David's Solution: Using the Function Equation

$d = 0$ when the pebble begins to fall, and $d = 346$ when it lands.

So the domain is $0 \leq d \leq 346$.

I found the domain by thinking about all the values that d could have. d is 0 m when the pebble first falls off the edge and 346 m when the pebble lands on the ground. So d must take values between 0 and 346.

The pebble starts with speed 0 m/s.

The pebble *fell* off the edge, so the speed was zero at the start.

$v(0) = \sqrt{19.6(0)} = 0$

I used the equation as a check.

As the pebble falls, its speed increases.

I knew that the pebble would gain speed until it hit the ground.

When the pebble lands, $d = 346$.

$$v(346) = \sqrt{19.6(346)}$$

$= 82.4$, to one decimal place

I used the function equation to find how fast the pebble was falling when it landed.

The domain is $\{d \in \mathbf{R} \mid 0 \leq d \leq 346\}$ and the range is $\{v(d) \in \mathbf{R} \mid 0 \leq v(d) \leq 82.4\}$.

I used set notation to write the domain and range.

Reflecting

A. Why did Sally need to think about the possible values for distance fallen before she graphed the function?

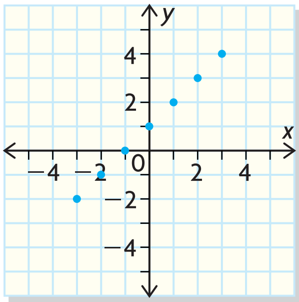
B. What properties of the square root function helped David use the given equation to find the domain and range?

APPLY the Math

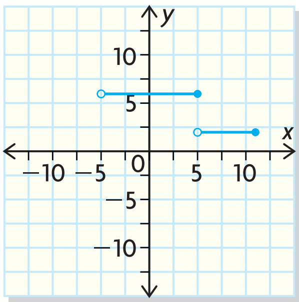
EXAMPLE 2 Determining domain and range from graphs

For each relation, state the domain and range and whether the relation is a function.

a)



b)



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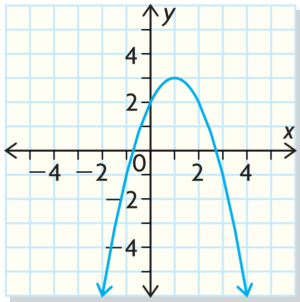
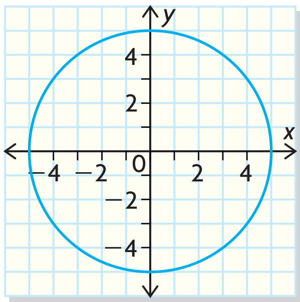


d)



Melanie's Solution

a) Domain $= \{x \in \mathbf{I} \mid -3 \leq x \leq 3\}$, or $\{-3, -2, -1, 0, 1, 2, 3\}$

Range $= \{y \in \mathbf{I} \mid -2 \leq y \leq 4\}$, or $\{-2, -1, 0, 1, 2, 3, 4\}$

I noticed that the x-coordinates were all the integers from -3 to 3 and the y-coordinates were all the integers from -2 to 4.

The graph is that of a function.

The graph passes the vertical-line test.

b) Domain $= \{x \in \mathbf{R} \mid -5 < x \leq 11\}$

An open circle on the graph shows that the endpoint of the line is not included in the graph. A closed circle means that the endpoint is included. So, x cannot be -5, but it can be 11.

Range $= \{2, 6\}$

There are only two y-values.

This is a function.

The graph passes the vertical-line test.

c) Domain $= \{x \in \mathbf{R}\}$

Range $= \{y \in \mathbf{R} \mid y \leq 3)\}$

The graph is a parabola with a maximum value at the vertex, which is the point (1, 3).

Therefore, x can be any real number, but y cannot be greater than 3.

This is a function.

The graph passes the vertical-line test.

d) Domain $= \{x \in \mathbf{R} \mid -5 \leq x \leq 5\}$

Range $= \{y \in \mathbf{R} \mid -5 \leq y \leq 5\}$

This is not a function.

The graph is a circle with centre (0, 0) and radius of 5. The graph fails the vertical-line test. There are many vertical lines that cross the graph in two places

EXAMPLE 3 Determining domain and range from the function equation

Determine the domain and range of each function.

a) $f(x) = 2x - 3$ **b)** $g(x) = -3(x + 1)^2 + 6$ **c)** $h(x) = \sqrt{2 - x}$

Jeff's Solution

a) $f(x) = 2x - 3$

This is a linear function, so x and y can be any value.

Domain $= \{x \in \mathbf{R}\}$

Range $= \{y \in \mathbf{R}\}$

This is the equation of a straight line that goes on forever in both directions.

x and $f(x)$ can be any numbers.

I used y instead of $f(x)$ to describe the range.

b) $g(x) = -3(x + 1)^2 + 6$

This is a quadratic equation in vertex form. The function has a maximum value at the vertex $(-1, 6)$. x can be any value.

Domain $= \{x \in \mathbf{R}\}$

Range $= \{y \in \mathbf{R} \mid y \leq 6\}$

This is the equation of a parabola that opens down, so y can never be more than its value at the vertex.

Any value of x will work in the equation, so x can be any number.

c) $h(x) = \sqrt{2 - x}$

$2 - x \geq 0$

$2 - x \geq 0$ as long as $x \leq 2$

Domain $= \{x \in \mathbf{R} \mid x \leq 2\}$

$\sqrt{2 - x}$ means the positive square root, so y is never negative.

Range $= \{y \in \mathbf{R} \mid y \geq 0\}$

You cannot take the square root of a negative number, so $2 - x$ must be positive or zero.

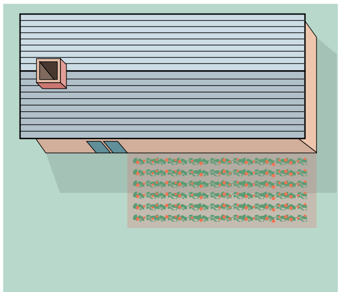
I thought about different possible values of x. 2 is okay, since $2 - 2 = 0$, but 4 is not, since $2 - 4$ is negative. I realized I had to use values less than or equal to 2.

EXAMPLE 4 Determining domain and range of an area function

Vitaly and Sherry have 24 m of fencing to enclose a rectangular garden at the back of their house.

a) Express the area of the garden as a function of its width.

b) Determine the domain and range of the area function.

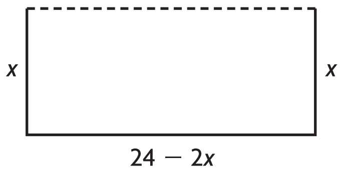


Jenny's Solution

a) Let the width of the garden be x m. Then the length is $(24 - 2x)$ m.

They need fencing on only three sides of the garden because the house forms the last side.

To find the length, I subtracted the two widths from 24.



Let the area be $A(x)$.

$$A(x) = x(24 - 2x)$$

$$A(x) = -2x(x - 12)$$

Area = width × length

I factored out -2 from $24 - 2x$ to write the function in factored form.

b) The smallest the width can approach is 0 m. The largest the width can approach is 12 m.

Domain $= \{x \in \mathbf{R} \mid 0 < x < 12\}$

This is a quadratic function that opens down. It has two zeros, at 0 and 12. The vertex lies halfway in between the zeros, above the x-axis, so the numbers in the domain have to be between 0 and 12. Any number ≤ 0 or ≥ 12 will result in a zero or negative area, which doesn't make sense.

$$x = (0 + 12) \div 2$$
$$x = 6$$

The vertex is halfway between $x = 0$ and $x = 12$. The x-coordinate of the vertex is 6.

$$A(6) = -2(6)(6 - 12)$$
$$= 72$$

The area ranges from 0 to 72 m^2.

Range $= \{A(x) \in \mathbf{R} \mid 0 < A(x) \leq 72\}$

I substituted $x = 6$ into the area function to find the y-coordinate of the vertex. Since area must be a positive quantity, all the output values of the function must lie between 0 and 72.

In Summary

Key Ideas

- The domain of a function is the set of values of the independent variable for which the function is defined. The range of a function depends on the equation of the function. The graph depends on the domain and range.
- The domain and range of a function can be determined from its graph, from a table of values, or from the function equation. They are usually easier to determine from a graph or a table of values.

Need to Know

- All linear functions include all the real numbers in their domains. Linear functions of the form $f(x) = mx + b$, where $m \neq 0$, have range $\{y \in \mathbf{R}\}$. Constant functions $f(x) = b$ have range $\{b\}$.
- All quadratic functions have domain $\{x \in \mathbf{R}\}$. The range of a quadratic function depends on the maximum or minimum value and the direction of opening.
- The domains of square root functions are restricted because the square root of a negative number is not a real number. The ranges are restricted because the square root sign refers to the positive square root. For example,
  - The function $f(x) = \sqrt{x}$ has domain $= \{x \in \mathbf{R} \mid x \geq 0\}$ and range $= \{y \in \mathbf{R} \mid y \geq 0\}$.
  - The function $g(x) = \sqrt{x - 3}$ has domain $= \{x \in \mathbf{R} \mid x \geq 3\}$ and range $= \{y \in \mathbf{R} \mid y \geq 0\}$.
- When working with functions that model real-world situations, consider whether there are any restrictions on the variables. For example, negative values often have no meaning in a real context, so the domain or range must be restricted to nonnegative values.

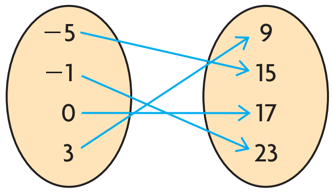
CHECK Your Understanding

1. State the domain and range of each relation.

a)

Year of Birth	Life Expectancy (years)
1900	47.3
1920	54.1
1940	62.9
1960	69.7
1980	73.7
2000	77.0

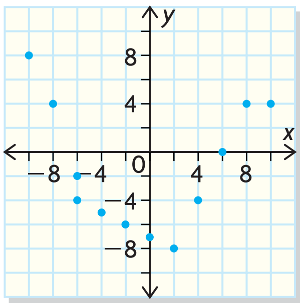
b)



c) $\{(-4, 7), (0, 5), (0, 3), (3, 0), (5, -1)\}$

2. State the domain and range of each relation.

a)



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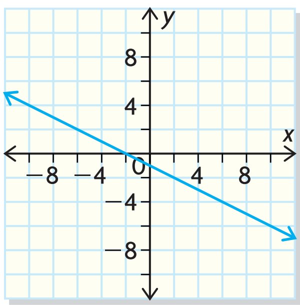

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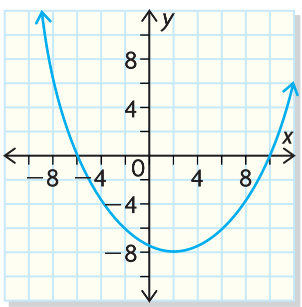

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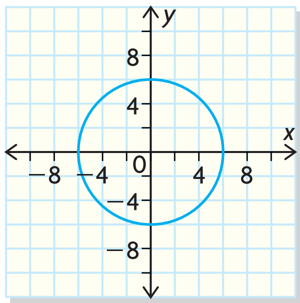
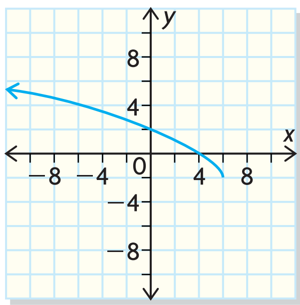
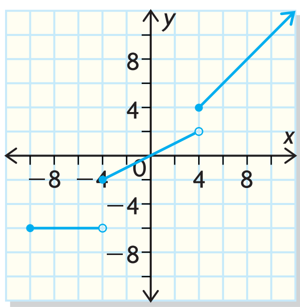


e)



f)

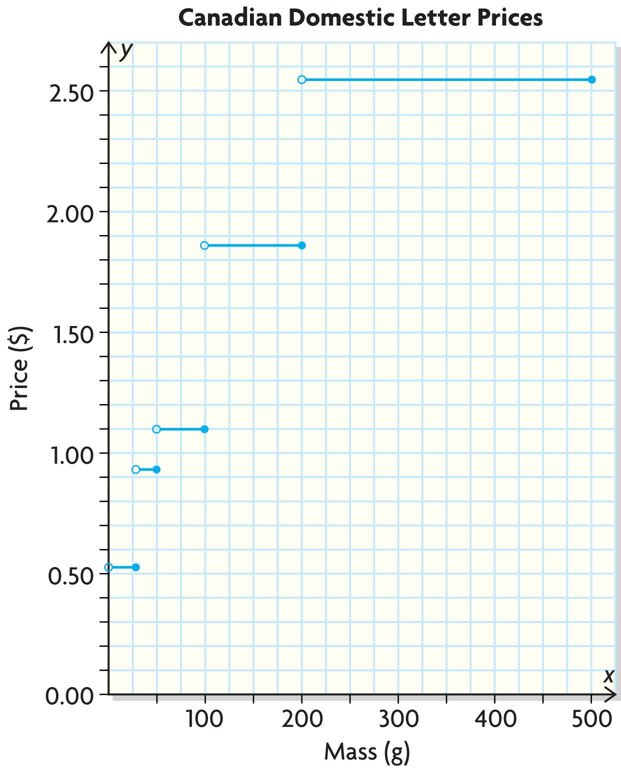


3. Identify which of the relations in questions 1 and 2 are functions.

4. Determine the domain and range of the function $f(x) = 2(x - 1)^2 - 3$ by sketching its graph.

PRACTISING

5. The graph shows how 2007 prices for mailing letters in Canada vary with mass.



a) Explain why this relation is a function. Why is it important for this to be so?

b) State the domain and range of the function.

6. The route for a marathon is 15 km long. Participants may walk, jog, run, or cycle. Copy and complete the table to show times for completing the marathon at different speeds.

Speed (km/h)	1	2	3	4	5	6	8	10	15	20
Time (h)	15.0	7.5								

Graph the relation in the table and explain how you know that it is a function. State the domain and range of the function.

7. A relation is defined by $x^2 + y^2 = 36$.

K **a)** Graph the relation.

b) State the domain and range of the relation.

c) Is the relation a function? Explain.

8. Write a function to describe coffee dripping into a 10-cup carafe at a rate of 1 mL/s. State the domain and range of the function (1 cup = 250 mL).

9. Determine the domain and range of each function.

a) $f(x) = -3x + 8$

b) $g(x) = -0.5(x + 3)^2 + 4$

c) $h(x) = \sqrt{x - 1}$

d) $p(x) = \frac{2}{3}(x - 2)^2 - 5$

e) $q(x) = 11 - \frac{5}{2}x$

f) $r(x) = \sqrt{5 - x}$

10. **A** A ball is thrown upward from the roof of a 25 m building. The ball reaches a height of 45 m above the ground after 2 s and hits the ground 5 s after being thrown.

a) Sketch a graph that shows the height of the ball as a function of time.
b) State the domain and range of the function.
c) Determine an equation for the function.

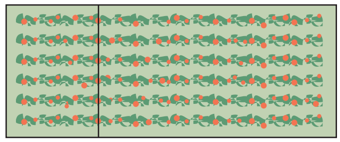
11. Write the domain and range of each function in set notation.

a) $f(x) = 4x + 1$
b) $f(x) = \sqrt{x - 2}$
c) $f(x) = 3(x + 1)^2 - 4$
d) $f(x) = -2x^2 - 5$

12. Use a graphing calculator to graph each function and determine the domain and range.

a) $f(x) = \sqrt{3 - x} + 2$
b) $g(x) = x^2 - 3x$
c) $h(x) = \frac{1}{x^2}$
d) $p(x) = \sqrt{x^2 - 5}$

13. **T** A farmer has 450 m of fencing to enclose a rectangular area and divide it into two sections as shown.



a) Write an equation to express the total area enclosed as a function of the width.
b) Determine the domain and range of this area function.
c) Determine the dimensions that give the maximum area.

14. Determine the range of each function if the domain is $\{-3, -1, 0, 2.5, 6\}$.

a) $f(x) = 4 - 3x$
b) $f(x) = 2x^2 - 3x + 1$

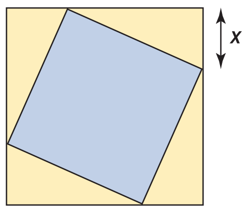
15. **C** Explain the terms "domain" and "range" as they apply to relations and functions. Describe, with examples, how the domain and range are determined from a table of values, a graph, and an equation.

Extending

16. **a)** Sketch the graph of a function whose domain is $\{x \in \mathbf{R}\}$ and range is $\{y \in \mathbf{R} \mid y \leq 2)\}$.

b) Sketch the graph of a relation that is not a function and whose domain is $\{x \in \mathbf{R} \mid x \geq -4\}$ and range is $\{y \in \mathbf{R}\}$.

17. You can draw a square inside another square by placing each vertex of the inner square on one side of the outer square. The large square in the diagram has side length 10 units.



a) Determine the area of the inscribed square as a function of x.
b) Determine the domain and range of this area function.
c) Determine the perimeter of the inscribed square as a function of x.
d) Determine the domain and range of this perimeter function.

1 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

- See Lesson 1.1, Examples 1, 2, and 3.
- Try Mid-Chapter Review Questions 1 and 2.

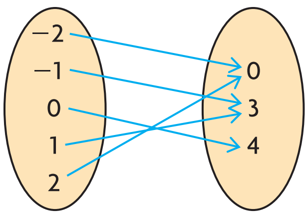
Q: How can you determine whether a relation is a function?

A1: For a relation to be a function, there must be only one value of the dependent variable for each value of the independent variable.

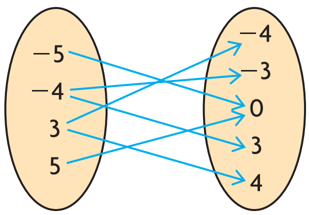
If the relation is described by a list of ordered pairs, you can see if any first elements appear more than once. If they do, the relation is not a function. For example, the relation $\{(-2, 0), (-1, 3), (0, 4), (1, 3), (2, 0)\}$ is a function; but the relation $\{(-5, 0), (-4, 3), (-4, -3), (3, -4), (3, 4), (5, 0)\}$ is not, because -4 and 3 each appear more than once as first elements.

A2: If the relation is shown in a mapping diagram, you can look at the arrows. If more than one arrow goes from an element of the domain (on the left) to an element of the range (on the right), then the relation is not a function. For example, diagram A shows a function but diagram B does not.

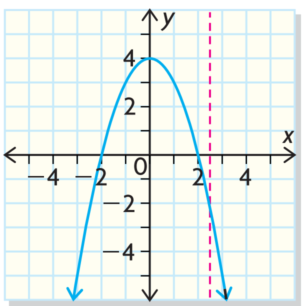
A



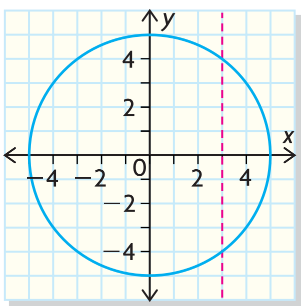
B



A



B



A3: If you have the graph of the relation, you can use the vertical-line test. If you can draw a vertical line that crosses the graph in more than one place, then an element in the domain corresponds to two elements in the range, so the relation is not a function. For example, graph A shows a function but graph B does not.

A4: If you have the equation of the relation, you can substitute numbers for x to see how many y-values correspond to each x-value. If a single x-value produces more than one corresponding y-value, the equation does not represent a function. For example, the equation $y = 4 - x^2$ is the equation of a function because you would get only one answer for y by putting a number in for x. The equation $x^2 + y^2 = 25$ does *not* represent a function because there are two values for y when x is any number between -5 and 5.

Study *Aid*

- See Lesson 1.2, Examples 1, 3, and 4.
- Try Mid-Chapter Review Questions 3 and 4.

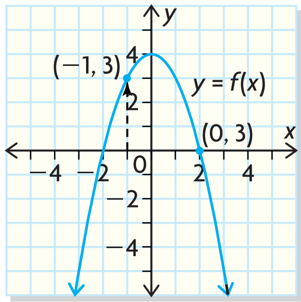
Q: What does function notation mean and why is it useful?

A: When a relation is a function, you can use function notation to write the equation. For example, you can write the equation $y = 4 - x^2$ in function notation as $f(x) = 4 - x^2$. f is a name for the function and $f(a)$ is the value of y or output when the input is $x = a$. The equation $f(-1) = 3$ means "When $x = -1$, $y = 3$;" in other words, the point $(-1, 3)$ belongs to the function.

To evaluate $f(-1)$, substitute -1 for x in the function equation:

$$\begin{aligned} f(-1) &= 4 - (-1)^2 \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

Or you can read the value from a graph.



Function notation is useful because writing $f(x) = 3$ gives more information about the function—you know that the independent variable is x—than writing $y = 3$. Also, you can work with more than one function at a time by giving each function a different name. You can choose meaningful names, such as $v(t)$ to describe velocity as a function of time, t, or $C(n)$ to describe the cost of producing n items.

Q: How can you determine the domain and range of a function?

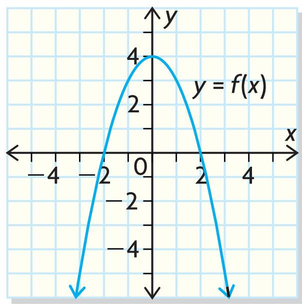
A: The domain of a function is the set of input values for which the function is defined. The range is the set of output values that correspond to the input values. Set notation can be used to describe the domain and range of a function.

Study *Aid*

- See Lesson 1.4, Examples 2 and 3.
- Try Mid-Chapter Review Questions 6, 7, and 8.

If you have the graph of a function, you can see the domain and range, as in the following examples:

A



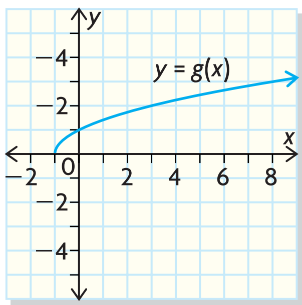
Because graph A goes on forever in both the positive and negative x direction, x can be any real number.

Because this function has a maximum value at the vertex, y cannot have a value greater than this maximum value.

You can express these facts in set notation:

$$\text{Domain} = \{x \in \mathbf{R}\};\ \text{Range} = \{y \in \mathbf{R} \mid y \leq 4\}$$

B



Graph B starts at the point $(-1, 0)$ and continues forever in the positive x direction and positive y direction. So x can be any real number greater than or equal to -1 and y can be any real number greater than or equal to 0.

$$\text{Domain} = \{x \in \mathbf{R} \mid x \geq -1\};\ \text{Range} = \{y \in \mathbf{R} \mid y \geq 0\}$$

You can also determine the domain and range from the equation of a function. For example, if $f(x) = 4 - x^2$, then any value of x will work in this equation, so $x \in \mathbf{R}$. Also, because x^2 is always positive or zero, $f(x)$ is always less than or equal to 4.

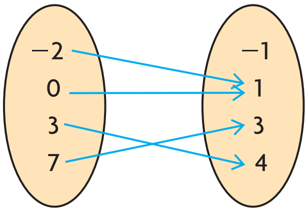
$$\text{Domain} = \{x \in \mathbf{R}\};\ \text{Range} = \{y \in \mathbf{R} \mid y \leq 4\}$$

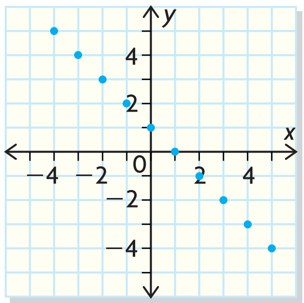
If $g(x) = \sqrt{x + 1}$, then x cannot be less than -1, or the number inside the square root sign would be negative. Also, the square root sign refers to the positive square root, so $g(x)$ is always positive or zero.

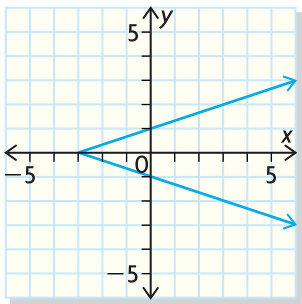
$$\text{Domain} = \{x \in \mathbf{R} \mid x \geq -1\};\ \text{Range} = \{y \in \mathbf{R} \mid y \geq 0\}$$

PRACTICE Questions

Lesson 1.1

1. Determine which relations are functions. For those which are, explain why.
   a) $\{(1, 2), (2, 3), (2, 4), (4, 5)\}$
   b)


   c)


   d)


   e) $y = -(x - 3)^2 + 5$
   f) $y = \sqrt{x - 4}$
2. Use numeric and graphical representations to show that $x^2 + y = 4$ is a function but $x^2 + y^2 = 4$ is not a function.

Lesson 1.2

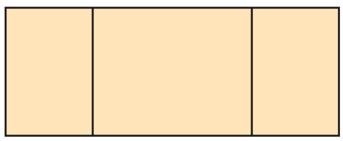
3. a) Graph the function $f(x) = -2(x + 1)^2 + 3$.
   b) Evaluate $f(-3)$.
   c) What does $f(-3)$ represent on the graph of f?
   d) Use the equation to determine **i)** $f(1) - f(0)$, **ii)** $3f(2) - 5$, and **iii)** $f(2 - x)$.
4. A teacher asked her students to think of a number, multiply it by 5, and subtract the product from 20. Then she asked them to multiply the resulting difference by the number they first thought of.
   a) Use function notation to express the final answer in terms of the original number.
   b) Determine the outputs for the input numbers 1, -1, and 7.
   c) Determine the maximum result possible.

Lesson 1.3

5. Graph each function and state its domain and range.
   a) $f(x) = x^2$
   b) $f(x) = \dfrac{1}{x}$
   c) $f(x) = \sqrt{x}$
   d) $f(x) = |x|$

Lesson 1.4

6. Determine the domain and range of each relation in question 1.
7. A farmer has 600 m of fencing to enclose a rectangular area and divide it into three sections as shown.



   a) Write an equation to express the total area enclosed as a function of the width.
   b) Determine the domain and range of this area function.
   c) Determine the dimensions that give the maximum area.
8. Determine the domain and range for each.
   a) A parabola has a vertex at $(-2, 5)$, and $y = 5$ is its maximum value.
   b) A parabola has a vertex at $(3, 4)$, and $y = 4$ is its minimum value.
   c) A circle has a centre at $(0, 0)$ and a radius of 7.
   d) A circle has a centre at $(2, 5)$ and a radius of 4.

1.5 The Inverse Function and Its Properties

YOU WILL NEED

- graph paper
- Mira™ (transparent mirror) (optional)

GOAL

Determine inverses of linear functions and investigate their properties.

INVESTIGATE the Math

The Backyard Paving Company charges $10/sq ft for installing interlocking paving stones, plus a $50 delivery fee. The company calculates the cost to the customer as a function of the area to be paved. Tom wants to express area in terms of cost to see how much of his yard he can pave for different budget amounts.

A

x Area (sq ft)	y Cost ($)
40	450
80	
120	
160	
200	

? What relation can Tom use, and how is it related to the function used by the company?

A. Copy and complete table A, using the company's prices. What is the independent variable in table A? the dependent variable?

B. Is the relation in table A a function? Explain.

C. Write the equation for $f(x)$ that describes the cost as a function of area.

D. Graph $f(x)$. Use the same scale of -100 to 2100 on each axis.

E. Tom needs to do the reverse of what the company's function does. Copy and complete table E for Tom. What is the independent variable? the dependent variable? How does this table compare with table A?

E

Cost ($)	Area (sq ft)
450	40
850	
1250	
1650	
2050	

F. The relationship in part E is the inverse of the cost function. Graph this inverse relation on the same axes as those in part D. Is this relation a function? Explain.

inverse of a function

the reverse of the original function; undoes what the original function has done

G. Draw the line $y = x$ on your graph. Place a Mira along the line $y = x$, or fold your graph paper along that line. What do you notice about the two graphs? Where do they intersect?

H. Compare the coordinates of points that lie on the graph of the cost function with those which lie on the graph of its inverse. What do you notice?

I. Write the slopes and y-intercepts of the two lines.
i) How are the slopes related?
ii) How are the y-intercepts related?
iii) Use the slope and y-intercept to write an equation for the inverse.

J. Use inverse operations on the cost function, f, to solve for x. Compare this equation with the equation of the inverse you found in part I.

K. Make a list of all the connections you have observed between the Backyard Paving Company's cost function and the one Tom will use.

Reflecting

L. How would a table of values for a linear function help you determine the inverse of that function?

M. **i)** How can you determine the coordinates of a point on the graph of the inverse function if you know a point on the graph of the original function?
ii) How could you use this relationship to graph the inverse?

N. How are the domain and range of the inverse related to the domain and range of a linear function?

O. How could you use inverse operations to determine the equation of the inverse of a linear function from the equation of the function?

APPLY the Math

EXAMPLE 1 Representing the equation of the inverse of a linear function

Find the inverse of the function defined by $f(x) = 2 - 5x$. Is the inverse a function? Explain.

Jamie's Solution: Reversing the Operations

In the equation $f(x) = 2 - 5x$, the operations on x are as follows: Multiply by -5 and then add 2.

I wrote down the operations on x in the order they were applied.

To reverse these operations, subtract 2 and then divide the result by -5.

Then I worked backward and wrote the inverse operations.

$$f^{-1}(x) = \frac{x-2}{-5} \quad \text{or}$$

$$f^{-1}(x) = -\frac{1}{5}x + \frac{2}{5}$$

I used these inverse operations to write the equation of the inverse.

The inverse is linear, so it must be a function, since all linear relations except vertical lines are functions.

I knew the inverse was a line.

Communication *Tip*

The function f^{-1} is the inverse of the function f. This use of -1 is different from raising values to the power -1.

Lynette's Solution: Interchanging the Variables

$$f(x) = 2 - 5x$$

I wrote the function in $y = mx + b$ form by putting y in place of $f(x)$.

$$y = -5x + 2$$

$$x = -5y + 2$$

I knew that if (x, y) is on the graph of $f(x)$, then (y, x) is on the inverse graph, so I switched x and y in the equation.

$$x - 2 = -5y + 2 - 2$$

$$x - 2 = -5y$$

$$\frac{x-2}{-5} = y$$

I solved for y by subtracting 2 from both sides and dividing both sides by -5.

$$f^{-1}(x) = \frac{x-2}{-5} \quad \text{or} \quad f^{-1}(x) = \frac{2-x}{5}$$

I wrote the equation in function notation.

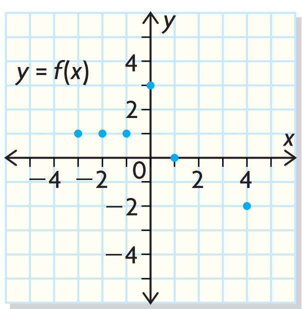
The inverse is a function.

The graph of $y = f^{-1}(x)$ is a straight line with slope $= -\frac{1}{5}$. The graph passes the vertical-line test.

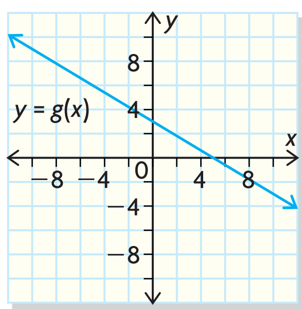
EXAMPLE 2 Relating the graphs of functions and their inverses

Use the graph of each function to obtain the graph of the inverse. Is the inverse a function? Explain.

a)



b)



Carlos's Solution

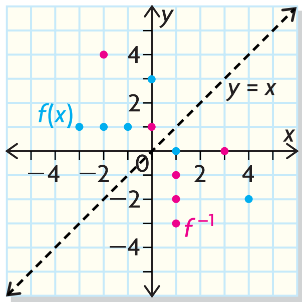
a) $f(x)$ is a function represented by the set of points $\{(-3, 1), (-2, 1), (-1, 1), (0, 3), (1, 0), (4, -2)\}$.

So $f^{-1}(x)$ is $\{(1, -3), (1, -2), (1, -1), (3, 0), (0, 1), (-2, 4)\}$.

← I wrote the coordinates of the points in the graph and then switched the *x*- and *y*-coordinates of each point. That gave me the inverse.

Plot the points for the inverse and draw the line $y = x$ to check for symmetry.

← I plotted the points in red.

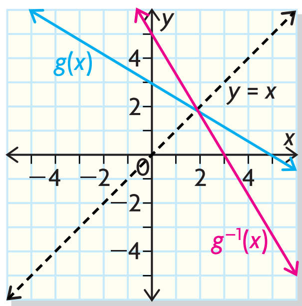


← I checked that the points on one side of the line $y = x$ were mirror images of the points on the other side.

The inverse is not a function: The graph fails the vertical-line test at $x = 1$.

← There are three red points for $x = 1$, so a vertical line drawn here would go through three points.

b)



← I wrote the coordinates of the *x*- and *y*-intercepts of $g(x)$: (5, 0) and (0, 3).

Then I switched the coordinates to find the two points (0, 5) and (3, 0) of $g^{-1}(x)$. I noticed that they were the intercepts.

I plotted the two points of $g^{-1}(x)$ and joined them with a straight line.

I drew the line $y = x$ and checked that the graphs of $g(x)$ and $g^{-1}(x)$ crossed on that line.

The inverse is a function.

← The inverse is a function because it passes the vertical-line test.

EXAMPLE 3 Using the inverse of a linear function to solve a problem

Recall from Lesson 1.2 that the temperature below Earth's surface is a function of depth and can be defined by $T(d) = 11 + 0.015d$.

a) State the domain and range of $T(d)$.
b) Determine the inverse of this function.
c) State the domain and range of $T^{-1}(d)$.
d) Explain what the inverse represents.

Erynn's Solution

a) Domain $= \{d \in \mathbf{R} \mid 0 \le d \le 5000\}$

I realized that *d* is 0 m on the surface. This is the beginning of the domain. The deeper mine has a depth of 4100 m, so I chose to end the domain at 5000.

Range $=$
$\{T(d) \in \mathbf{R} \mid 11 \le T(d) \le 86\}$

I calculated the beginning and end of the range by substituting $d = 0$ and $d = 5000$ into the equation for $T(d)$.

b)
$$T(d) = 11 + 0.015d$$

I wrote the temperature function with *y* and *x* instead of $T(d)$ and *d*.

$$y = 11 + 0.015x$$
$$x = 11 + 0.015y$$
$$x - 11 = 0.015y$$

I switched *x* and *y* and solved for *y* to get the inverse equation.

$$\frac{x - 11}{0.015} = y$$

$d(T) = \frac{T - 11}{0.015}$ is the inverse function.

Because I had switched the variables, I knew that *y* was now distance and *x* was temperature. I wrote the inverse in function notation.

c) Domain $\{T \in \mathbf{R} \mid 11 \le T \le 86\}$

Range $=$
$\{d(T) \in \mathbf{R} \mid 0 \le d(T) \le 5000\}$

The domain of the inverse is the same as the range of the original function, and the range of the inverse is the same as the domain of the original function.

d) The inverse shows the depth as a function of the temperature.

The inverse function is used to determine how far down a mine you would have to go to reach a temperature of, for example, 22 °C. I substituted 22 for *T* in the equation to get the answer.

$$d(22) = \frac{22 - 11}{0.015}$$
$$\doteq 733$$

When the temperature is 22 °C, the depth is about 733 m.

Someone planning a geothermal heating system would need this kind of information.

In Summary

Key Ideas

- The inverse of a linear function is the reverse of the original function. It undoes what the original has done and can be found using the inverse operations of the original function in reverse order. For example, to apply the function defined by $f(x) = 5x + 8$, multiply x by 5 and then add 8. To reverse this function, subtract 8 from x and then divide the result by 5: $f^{-1}(x) = \frac{x - 8}{5}$.
- The inverse of a function is not necessarily a function itself.

Need to Know

- A way to determine the inverse function is to switch the two variables and solve for the previously independent variable. For example, if $y = 5x + 8$, rewrite this equation as $x = 5y + 8$ and solve for y to get $y = \frac{x - 8}{5}$.
- If the original function is linear (with the exception of a horizontal line), the inverse is also a linear function.
- f^{-1} is the notation for the inverse function of f.
- If (a, b) is a point on the graph of $y = f(x)$, then (b, a) is a point on the graph of $y = f^{-1}(x)$. This implies that the domain of f is the range of f^{-1} and the range of f is the domain of f^{-1}.
- The graph of the inverse is the reflection of the graph of $y = f(x)$ in the line $y = x$.

CHECK Your Understanding

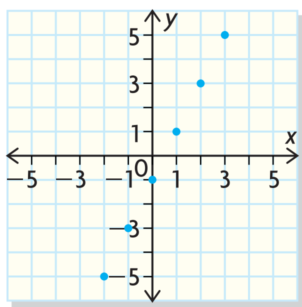
1. Determine the inverse relation for each set of ordered pairs. Graph each relation and its inverse. Which of the relations and inverse relations are functions?

a) $\{(-2, 3), (0, 4), (2, 5), (4, 6)\}$

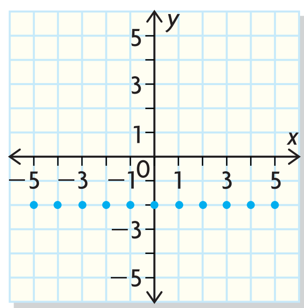
b) $\{(2, 5), (2, -1), (3, 1), (5, 1)\}$

2. Copy the graph of each function and graph its inverse. For each graph, identify the points that are common to the function and its inverse. Which inverse relations are functions?

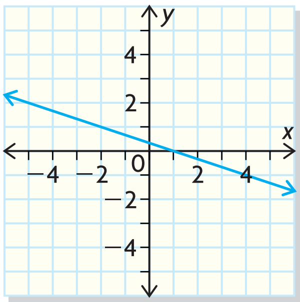
a)



b)



c)



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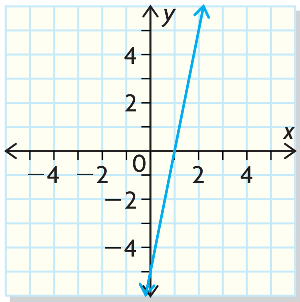
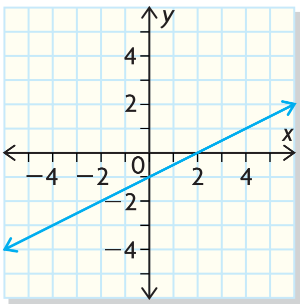
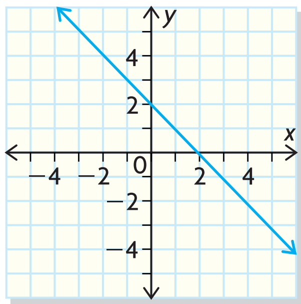


d)



f)



3. Determine whether each pair of functions described in words are inverses.

a) f: Multiply by 3, then add 1; g: Divide by 3, then subtract 1.

b) f: Multiply by 5, then subtract 2; g: Add 2, then divide by 5.

4. For each linear function, interchange x and y. Then solve for y to determine the inverse.

a) $y = 4x - 3$

b) $y = 2 - \frac{1}{2}x$

c) $3x + 4y = 6$

d) $2y - 10 = 5x$

PRACTISING

5. Determine the inverse of each linear function by reversing the operations.

a) $f(x) = x - 4$

b) $f(x) = 3x + 1$

c) $f(x) = 5x$

d) $f(x) = \frac{1}{2}x - 1$

e) $f(x) = 6 - 5x$

f) $f(x) = \frac{3}{4}x + 2$

6. Determine the inverse of each linear function by interchanging the variables.

a) $f(x) = x + 7$

b) $f(x) = 2 - x$

c) $f(x) = 5$

d) $f(x) = -\frac{1}{5}x - 2$

e) $f(x) = x$

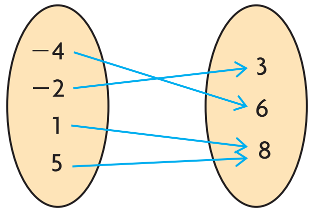
f) $f(x) = \frac{x - 3}{4}$

7. Sketch the graph of each function in questions 5 and 6, and sketch its inverse. Is each inverse linear? Is each inverse a function? Explain.

8. **K** For each function, determine the inverse, sketch the graphs of the function and its inverse, and state the domain and range of both the function and its inverse. In each case, how do the domain and range of the function compare with the domain and range of the inverse?

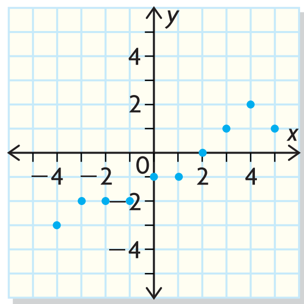
a) $\{(-1, 2), (1, 4), (2, 6), (3, 8)\}$

b)



c) $f(x) = 1 - 3x$

d)



9. **a)** Determine f^{-1} for the linear function $f(x) = 5x - 2$.

b) Graph f and f^{-1} on the same axes.

c) Explain how you can tell that f^{-1} is also a linear function.

d) State the coordinates of any points that are common to both f and f^{-1}.

e) Compare the slopes of the two lines.

f) Repeat parts (a) to (e) for $g(x) = -\frac{1}{2}x + 3$, $h(x) = 2x - 1$, $p(x) = 6 - x$, and $q(x) = 2$.

10. For $g(t) = 3t - 2$, determine each value.

a) $g(13)$

b) $g(7)$

c) $\dfrac{g(13) - g(7)}{13 - 7}$

d) $g^{-1}(13)$

e) $g^{-1}(7)$

f) $\dfrac{g^{-1}(13) - g^{-1}(7)}{13 - 7}$

11. Explain what parts (c) and (f) represent in question 10.

12. **A** The formula for converting a temperature in degrees Celsius into degrees Fahrenheit is $F = \frac{9}{5}C + 32$. Shirelle, an American visitor to Canada, uses a simpler rule to convert from Celsius to Fahrenheit: Double the Celsius temperature, then add 30.

a) Use function notation to write an equation for this rule. Call the function f and let x represent the temperature in degrees Celsius.

b) Write f^{-1} as a rule. Who might use this rule?

c) Determine $f^{-1}(x)$.

d) One day, the temperature was 14 °C. Use function notation to express this temperature in degrees Fahrenheit.

e) Another day, the temperature was 70 °F. Use function notation to express this temperature in degrees Celsius.

13. Ben, another American visitor to Canada, uses this rule to convert centimetres to inches: Multiply by 4 and then divide by 10. Let the function g be the method for converting centimetres to inches, according to Ben's rule.

a) Write g^{-1} as a rule.

b) Describe a situation in which the rule for g^{-1} might be useful.

c) Determine $g(x)$ and $g^{-1}(x)$.

d) One day, 15 cm of snow fell. Use function notation to represent this amount in inches.

e) Ben is 5 ft 10 in. tall. Use function notation to represent his height in centimetres.

14. Ali did his homework at school with a graphing calculator. He determined that the equation of the line of best fit for some data was $y = 2.63x - 1.29$. Once he got home, he realized he had mixed up the independent and dependent variables. Write the correct equation for the relation in the form $y = mx + b$.

15. Tiffany is paid \$8.05/h, plus 5% of her sales over \$1000, for a 40 h work week. For example, suppose Tiffany sold \$1800 worth of merchandise. Then she would earn \$8.05(40) + 0.05(\$800) = \$362.

a) Graph the relation between Tiffany's total pay for a 40 h work week and her sales for that week.

b) Write the relation in function notation.

c) Graph the inverse relation.

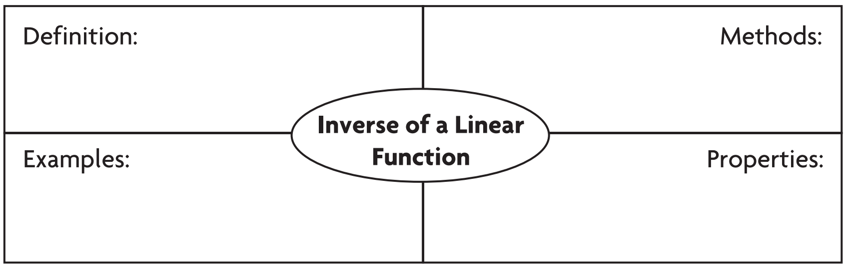
d) Write the inverse relation in function notation.

e) Write an expression in function notation that represents her sales if she earned \$420 one work week. Then evaluate.

16. The ordered pair (1, 5) belongs to a function f. Explain why the ordered pair (2, 1) cannot belong to f^{-1}.

17. Given $f(x) = k(2 + x)$, find the value of k if $f^{-1}(-2) = -3$.

18. Use a chart like the one shown to summarize what you have learned about the inverse of a linear function.



Extending

19. *Self-inverse* functions are their own inverses. Find three linear functions that are self-inverse.

20. Determine the inverse of the inverse of $f(x) = 3x + 4$.

1.6 Exploring Transformations of Parent Functions

YOU WILL NEED

- graphing calculator or graphing software

Communication *Tip*

The function defined by $g(x) = af(x - d) + c$ describes a transformation of the graph of f.

When $f(x) = x^2$,
$g(x) = a(x - d)^2 + c$.

When $f(x) = \sqrt{x}$,
$g(x) = a\sqrt{x - d} + c$.

When $f(x) = \frac{1}{x}$,
$g(x) = \frac{a}{x - d} + c$.

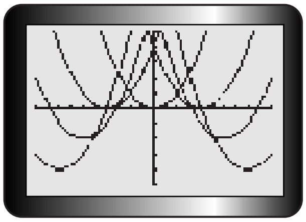
When $f(x) = |x|$,
$g(x) = a|x - d| + c$.

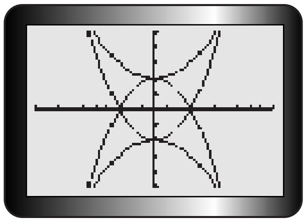
GOAL

Investigate transformations of parent functions.

EXPLORE the Math

Anastasia and Shelby made patterns with parabolas by applying **transformations** to the graph of the parent quadratic function $y = x^2$.





Anastasia thinks they could make more interesting patterns by applying transformations to other parent functions as well. Shelby wonders whether the transformations will have the same effect on the other functions as they do on quadratic functions.

? Do transformations of other parent functions behave in the same way as transformations of quadratic functions?

A. Graph the parent functions $f(x) = x^2$, $g(x) = \sqrt{x}$, $h(x) = \frac{1}{x}$, and $j(x) = |x|$. Sketch and label each graph.

B. Without using a calculator, use what you know about transformations of quadratic functions to sketch the graphs of $y = 3x^2$, $y = \frac{1}{2}x^2$, and $y = -2x^2$. Describe the transformations in words.

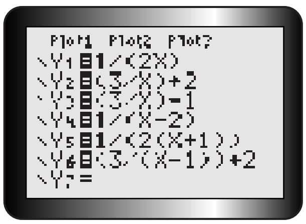
C. Predict what the graphs of $y = 3\sqrt{x}$, $y = \frac{1}{2}\sqrt{x}$, and $y = -2\sqrt{x}$ will look like. Use a graphing calculator to verify your predictions. Sketch and label each curve on the same axes, along with a sketch of the parent function. Compare the effect of these transformations with the effect of the same transformations on quadratic functions.

D. Repeat part C for $y = \frac{3}{x}$, $y = \frac{1}{2x}$, and $y = -\frac{2}{x}$, and for $y = 3|x|$, $y = \frac{1}{2}|x|$, and $y = -2|x|$.

E. Sketch $y = 3x^2 + 2$ and $y = 3x^2 - 1$ without a calculator. Describe the transformations in words. Predict what the graphs of $y = 3f(x) + 2$ and $y = 3f(x) - 1$ for each of the other parent functions will look like. Verify your predictions with a graphing calculator. Make labelled sketches and compare them with transformations on quadratic functions as before.

Tech *Support*

Use brackets when entering transformed versions of $y = \frac{1}{x}$:



To enter $f(x) = |x|$, press MATH ▶ 1 X,T,Θ,n.

F. Repeat part E for $y = f(x - 2)$, $y = \frac{1}{2}f(x + 1)$, and $y = 3f(x - 1) + 2$.

G. Examine your sketches for each type of transformation. Did the transformations have the same effect on the new parent functions as they had on quadratic functions? Explain.

Reflecting

H. How did the effect of transformations on parent functions compare with that on quadratic functions?

I. When you graphed $y = af(x - d) + c$, what were the effects of c and d?

J. How did the graphs with $a \geq 0$ compare with the graphs with $a \leq 0$?

K. How did the graphs for which $a > 1$ compare with the graph for which $0 < a < 1$?

In Summary

Key Idea

- In functions of the form $g(x) = af(x - d) + c$, the constants a, c, and d each change the location or shape of the graph of $f(x)$. The shape of the graph of $g(x)$ depends on the graph of the parent function $g(x)$ and on the value of a.

FURTHER Your Understanding

1. The graph of the equation $y = (x - 1)^2 + 2$ is the graph of a parabola that opens up and has its vertex at (1, 2). What do you know about the graphs of the following equations?

a) $y = \sqrt{x - 1} + 2$ b) $y = |x - 1| + 2$ c) $y = \dfrac{1}{x - 1} + 2$

2. The graph of $y = x^2$ opens up and the graph of $y = -x^2$ opens down. How would you compare the graphs of the following pairs of equations?

a) $y = \sqrt{x}$ and $y = -\sqrt{x}$

b) $y = |x|$ and $y = -|x|$

c) $y = \dfrac{1}{x}$ and $y = -\dfrac{1}{x}$

3. The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$. How do the following graphs compare?

a) $y = 2\sqrt{x}$ and $y = \sqrt{x}$

b) $y = 2|x|$ and $y = |x|$

c) $y = \dfrac{2}{x}$ and $y = \dfrac{1}{x}$

4. Experiment with each of the parent functions to create patterns on a graphing calculator screen.

1.7 Investigating Horizontal Stretches, Compressions, and Reflections

YOU WILL NEED

- graph paper (optional)
- graphing calculator

GOAL

Investigate and apply horizontal stretches, compressions, and reflections to parent functions

INVESTIGATE the Math

The function $p(L) = 2\pi\sqrt{\frac{1}{10}L}$ describes the time it takes a pendulum to complete one swing, from one side to the other and back, as a function of its length. In this formula,

- $p(L)$ represents the time in seconds
- L represents the pendulum's length in metres

Shannon wants to sketch the graph of this function. She knows that the parent function is $f(x) = \sqrt{x}$ and that the 2π causes a vertical stretch. She wonders what transformation is caused by multiplying x by $\frac{1}{10}$.

? What transformation must be applied to the graph of $y = f(x)$ to get the graph of $y = f(kx)$?

A. Copy and complete tables of values for $y = \sqrt{x}$ and $y = \sqrt{2x}$.

$y = \sqrt{x}$	
x	**y**
0	
1	
4	
9	
10	

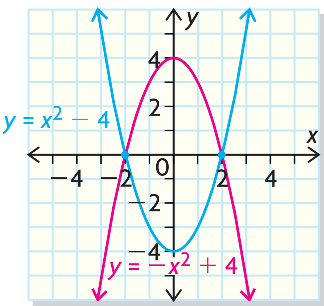
$y = \sqrt{2x}$	
x	**y**
0	
0.5	
2	
4.5	
8	

Graph both functions on the same set of axes. State the domain and range of each function.

B. Compare the position and shape of the two graphs. Are there any invariant points on the graphs? Explain.

invariant point

a point on a graph (or figure) that is unchanged by a transformation—for example, (−2, 0) and (2, 0) for this graph and transformation



C. How could you transform the graph of $y = \sqrt{x}$ to obtain the graph of $y = \sqrt{2x}$?

D. Compare the points in the tables of values. How could you use the first table to obtain the second? What happens to the point (x, y) under this transformation? This transformation is called a *horizontal compression of factor* $\frac{1}{2}$. Explain why this is a good description.

E. Repeat parts A through D for $y = \sqrt{x}$ and $y = \sqrt{\frac{1}{2}x}$. What happens to the point (x, y) under this transformation? Describe the transformation in words.

F. Repeat parts A through D for $y = \sqrt{x}$ and $y = \sqrt{-x}$.

G. Using a graphing calculator, investigate the effect of varying k in $y = f(kx)$ on the graphs of the given parent functions. In each case, try values of k that are **i)** between 0 and 1, **ii)** greater than 1, and **iii)** less than 0.

a) $f(x) = x^2$ **b)** $f(x) = \dfrac{1}{x}$ **c)** $f(x) = |x|$

H. Write a summary of the results of your investigations. Explain how you would use the graph of $y = f(x)$ to sketch the graph of $y = f(kx)$.

Communication *Tip*

In describing vertical stretches/compressions $af(x)$, the scale factor is a, but for horizontal stretches/compressions $f(kx)$, the scale factor is $\frac{1}{k}$. In both cases, for a scale factor greater than 1, a stretch occurs, and for a scale factor between 0 and 1, a compression occurs.

Reflecting

I. What transformation is caused by multiplying L by $\frac{1}{10}$ in the pendulum function $p(L) = 2\pi\sqrt{\frac{1}{10}L}$?

J. How is the graph of $y = f(2x)$ different from the graph of $y = 2f(x)$?

K. How is the graph of $y = f(-x)$ different from the graph of $y = -f(x)$?

L. What effect does k in $y = f(kx)$ have on the graph of $y = f(x)$ when
i) $|k| > 1$? **ii)** $0 < |k| < 1$? **iii)** $k < 0$?

APPLY the Math

EXAMPLE 1 Applying horizontal stretches, compressions, and reflections

For each pair of functions, identify the parent function, describe the transformations required to graph them from the parent function, and sketch all three graphs on the same set of axes.

a) $y = (4x)^2$; $y = \left(\frac{1}{5}x\right)^2$ **b)** $y = |0.25x|$; $y = |-x - 3|$

Ana's Solution

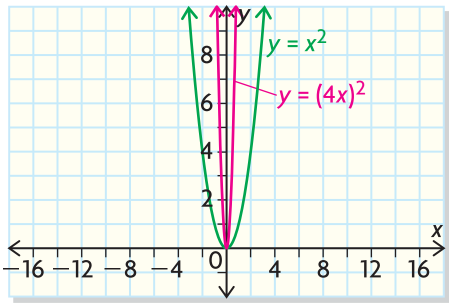
a) These functions are of the form $y = f(kx)^2$. The quadratic function $f(x) = x^2$ is the parent function.

I saw that these functions were $y = x^2$, with x multiplied by a number.

To graph $y = (4x)^2$, compress the graph of $y = (x)^2$ horizontally by the factor $\frac{1}{4}$.

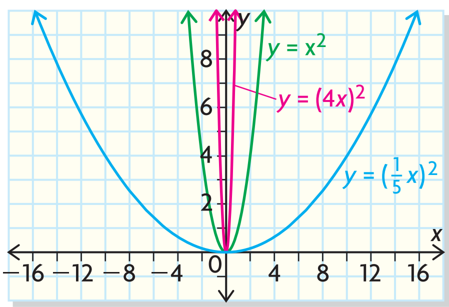
When x is multiplied by a number greater than 1, the graph is compressed horizontally. That makes sense, since the x-value required to make $y = 1$ is ± 1 for $y = x^2$, but is $\pm \frac{1}{4}$ for $y = (4x)^2$.

I multiplied the x-coordinates of the points (1, 1), (2, 4), and (3, 9) on $y = x^2$ by $\frac{1}{4}$ to find three points, $\left(\frac{1}{4}, 1\right)$, $\left(\frac{1}{2}, 4\right)$, and $\left(\frac{3}{4}, 9\right)$, on $y = (4x)^2$.



I plotted these points and joined them to the invariant point (0, 0) to graph one-half of the parabola. Then I used symmetry to complete the other half of the graph.

To graph $y = \left(\frac{1}{5}x^2\right)$, stretch the graph of $y = x^2$ horizontally by the factor 5.



This time, x is multiplied by a number between 0 and 1, so the graph is stretched horizontally. Instead of using an x-value of ± 1 to get a y-value of 1, I need an x-value of ± 5.

I used the same x-coordinates as before and multiplied by 5, which gave me points (5, 1), (10, 4), and (15, 9) to plot.

I used the invariant point (0, 0) and symmetry to complete the graph of $y = \left(\frac{1}{5}x\right)^2$.

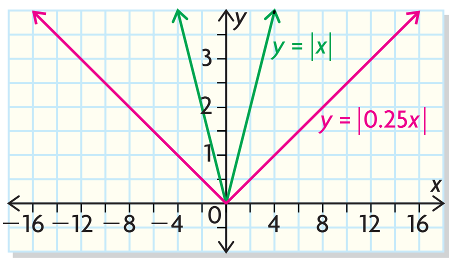
b) The parent function is $f(x) = |x|$.

To graph $y = |0.25x|$, stretch the graph of $y = |x|$ horizontally by the factor 4.

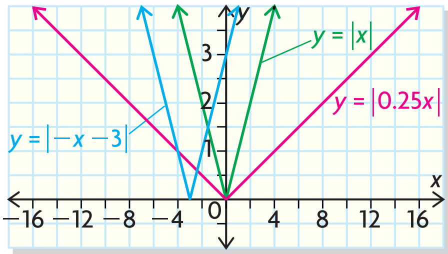
I knew from the absolute value signs that the parent function was the absolute value function.

I knew that the stretch factor was $\frac{1}{0.25} = 4$. The point that originally was (1, 1) corresponded to the new point (4, 1). So I multiplied the x-coordinates of the points (1, 1), (2, 2), and (3, 3) on $y = |x|$ by 4 to find the points (4, 1), (8, 2), and (12, 3) on the new graph.

I joined these points to the invariant point (0, 0) to graph one-half of the stretched absolute value function. I used symmetry to complete the graph.



To graph $y = |-x - 3|$, reflect the parent function graph in the y-axis, then translate it 3 units left.



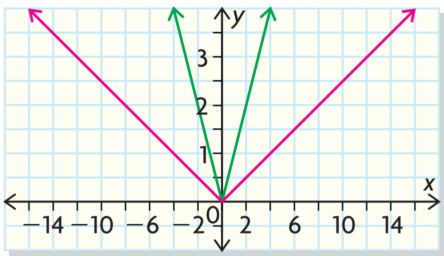
First I thought about the graph of $y = |-x|$. For this graph, I switched the values of x and $-x$, so I really reflected the graph of $y = |x|$ in the y-axis. The points that were originally $(-1, 1)$, $(0, 0)$, and $(1, 1)$ changed to $(1, 1)$, $(0, 0)$, and $(-1, 1)$; so the graph didn't change.

Next, I reasoned that since $|-x| = |x|$, $|-x - 3| = |x + 3|$. Therefore, I shifted the graph of $y = |x|$ 3 units left. The points that were originally $(-1, 1)$, $(0, 0)$, and $(1, 1)$ changed to $(-4, 1)$, $(-3, 0)$, and $(-2, 1)$.

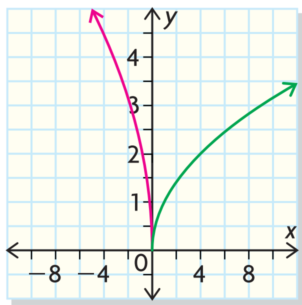
EXAMPLE 2 Using a graph to determine the equation of a transformed function

In the graphs shown, three parent functions have been graphed in green. The functions graphed in red have equations of the form $y = f(kx)$. Determine the equations. Explain your reasoning.

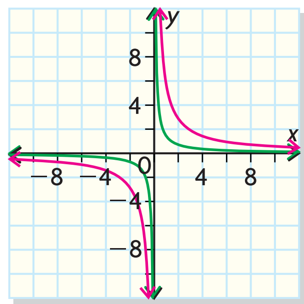
a)



b)



c)



Robert's Solution

a) The parent function is $f(x) = |x|$.

I recognized the V shape of the absolute value function.

Point $(1, 1)$ on $y = |x|$ corresponds to point $(4, 1)$ on the red graph.

I knew that $y = f(kx)$ represents a horizontal stretch or compression and/or a reflection in the y-axis.

Point (2, 2) corresponds to point (8, 2), and point (3, 3) corresponds to point (12, 3).

The red graph is a stretched-out version of the green graph, so k must be between 0 and 1.

The red graph is the green graph stretched horizontally by the factor 4.

The x-coordinates of points on the red graph are 4 times the ones on the green graph.

The equation is $y = |\frac{1}{4}x|$.

Since the stretch scale factor is 4, and $0 < k < 1$, it follows that $k = \frac{1}{4}$. So I could complete the equation.

b) The green graph is a graph of the square root function $f(x) = \sqrt{x}$.

The green graph is the square root function because it begins at (0, 0) and has the shape of a half parabola on its side.

The green graph has been compressed horizontally and reflected in the y-axis to produce the red graph.

The red graph is a compressed version of the green graph that had been flipped over the y-axis. Therefore, k is negative and less than -1.

(1, 1) corresponds to $(-0.25, 1)$.

(4, 2) corresponds to $(-1, 2)$.

(16, 4) corresponds to $(-4, 4)$.

Each x-coordinate has been divided by -4.

The equation is $y = \sqrt{-4x}$.

I divided the corresponding x-coordinates to find k:

$1 \div -0.25 = -4$

$4 \div -1 = -4$

$16 \div -4 = -4, \quad \text{so} \quad k = -4$

c) The parent function is $f(x) = \dfrac{1}{x}$.

I recognized the reciprocal function because the graph was in two parts and had asymptotes.

The graph has been stretched horizontally.

The red graph is further away from the asymptotes than the green graph, so it must have been stretched.

(1, 1) corresponds to (6, 1).

$(\frac{1}{2}, 2)$ corresponds to (3, 2).

$(-1, -1)$ corresponds to $(-6, -1)$.

$(-\frac{1}{2}, -2)$ corresponds to $(-3, -2)$.

Each x-coordinate has been multiplied by 6. The equation is $y = \dfrac{1}{(\frac{1}{6}x)}$.

Since the stretch scale factor is 6, and $0 < k < 1$, it follows that $k = \frac{1}{6}$.

EXAMPLE 3 Using transformations to sketch the graph for a real situation

Use transformations to sketch the graph of the pendulum function $p(L) = 2\pi\sqrt{\frac{1}{10}L}$, where $p(L)$ is the time, in seconds, that it takes for a pendulum to complete one swing and L is the length of the pendulum, in metres.

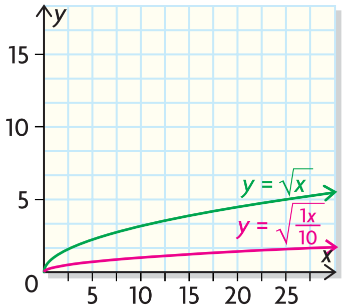
Shannon's Solution

The graph of $y = 2\pi\sqrt{\frac{1}{10}x}$ is the graph of the parent function $y = \sqrt{x}$ stretched horizontally by the factor 10 and vertically by the factor 2π.

The original equation was in the form $y = af(kx)$.

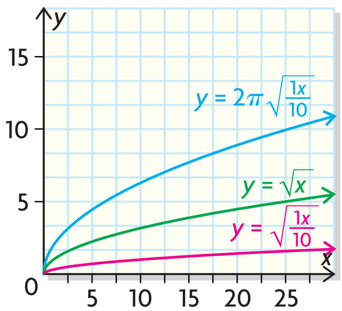
Since $0 < k < 1$, the graph is stretched horizontally by a scale factor of $\frac{1}{\frac{1}{10}} = 10$.

Because $a = 2\pi$ and $2\pi > 1$, the graph is stretched vertically by a scale factor of 2π.



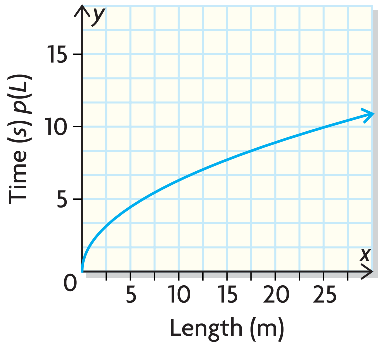
I applied the horizontal stretch.

I multiplied the x-coordinates by 10 to find points on the horizontally stretched graph:
(1, 1) moves to (10, 1).
(4, 2) moves to (40, 2).



Then I applied the vertical stretch to the red graph. I multiplied the y-coordinates by 2π, which is approximately 6.3 (to one decimal place):
(10, 1) moves to (10, 6.3).
(40, 2) moves to (40, 12.6) .

Period versus Length for a Pendulum



I drew a correctly labelled graph of the situation. I copied the sketch onto a graph with length L on the x-axis and time $p(L)$ on the y-axis.

In Summary

Key Idea

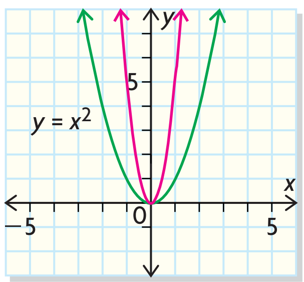
- Functions of the form $g(x) = f(kx)$ have graphs that are not congruent to the graph of $f(x)$. The differences in shape are a result of stretching or compressing in a horizontal direction.

Need to Know

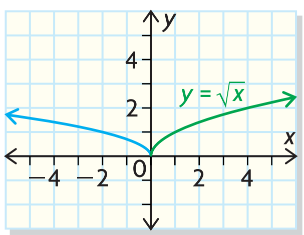
- The image of the point (x, y) on the graph of $f(x)$ is the point $\left(\frac{x}{k}, y\right)$ on the graph of $f(kx)$.
- If $g(x) = f(kx)$, then the value of k has the following effect on the graph of $f(x)$:
  - When $|k| > 1$, the graph is compressed horizontally by the factor $\frac{1}{|k|}$.
  - When $0 < |k| < 1$, the graph is stretched horizontally by the factor $\frac{1}{|k|}$.
  - When $k < 0$, the graph is also reflected in the y-axis.

CHECK Your Understanding

1. **a)** The red graph has been compressed horizontally by the factor $\frac{1}{3}$ relative to the graph of $y = x^2$. Write the equation of the red graph.



b) The blue graph has been stretched horizontally by the factor 2 relative to the graph of $y = \sqrt{x}$ and then reflected in the y-axis. Write the equation of the blue graph.



2. For each function, identify the parent function and describe how the graph of the function can be obtained from the graph of the parent function. Then sketch both graphs on the same set of axes.

a) $y = |0.5x|$ **b)** $y = \left(\frac{1}{4}x\right)^2$ **c)** $y = \sqrt{-2x}$ **d)** $y = \frac{1}{(5x)}$

PRACTISING

3. The point (3, 4) is on the graph of $y = f(x)$. State the coordinates of the image of this point on each graph.

   a) $y = f(2x)$ b) $y = f(0.5x)$ c) $y = f\left(\frac{1}{3}x\right)$ d) $y = f(-4x)$

4. Sketch graphs of each pair of transformed functions, along with the graph of the parent function, on the same set of axes. Describe the transformations in words and note any invariant points.

   a) $y = (2x)^2, y = (5x)^2$
   b) $y = \sqrt{3x}, y = \sqrt{4x}$
   c) $y = \frac{1}{(2x)}, y = \frac{1}{(3x)}$
   d) $y = |3x|, y = |5x|$

5. Repeat question 4 for each pair of transformed functions.

   a) $y = (-2x)^2, y = (-5x)^2$
   b) $y = \sqrt{-3x}, y = \sqrt{-4x}$
   c) $y = \frac{1}{(-2x)}, y = \frac{1}{(-3x)}$
   d) $y = |-3x|, y = |-5x|$

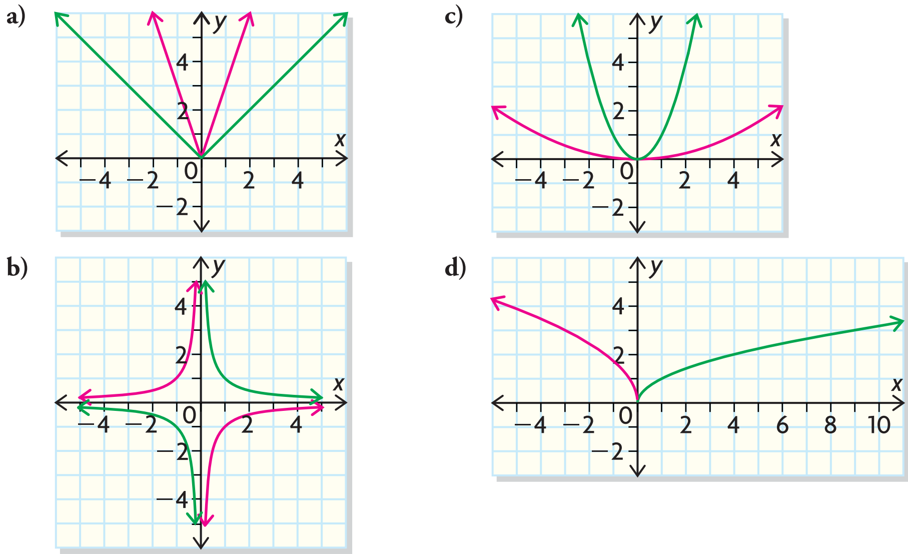
6. Repeat question 4 for each pair of transformed functions.

   a) $y = \left(\frac{1}{2}x\right)^2, y = \left(\frac{1}{3}x\right)^2$
   b) $y = \sqrt{\frac{1}{2}x}, y = \sqrt{\frac{1}{3}x}$
   c) $y = \frac{1}{\left(\frac{1}{2}x\right)}, y = \frac{1}{\left(\frac{1}{4}x\right)}$
   d) $y = \left|\frac{1}{3}x\right|, y = \left|\frac{1}{5}x\right|$

7. Repeat question 4 for each pair of transformed functions.

   a) $y = \left(-\frac{1}{2}x\right)^2, y = \left(-\frac{1}{3}x\right)^2$
   b) $y = \sqrt{-\frac{1}{2}x}, y = \sqrt{-\frac{1}{3}x}$
   c) $y = \frac{1}{\left(-\frac{1}{2}x\right)}, y = \frac{1}{\left(-\frac{1}{4}x\right)}$
   d) $y = \left|-\frac{1}{3}x\right|, y = \left|-\frac{1}{5}x\right|$

8. **K** In each graph, one of the parent functions $f(x) = x^2, f(x) = \sqrt{x}, f(x) = \frac{1}{x}$, and $f(x) = |x|$ has undergone a transformation of the form $f(kx)$. Determine the equations of the transformed functions graphed in red.



9. When an object is dropped from a height, the time it takes to reach the ground is a function of the height from which it was dropped. An equation for this function is $t(h) = \sqrt{\frac{h}{4.9}}$, where h is in metres and t is in seconds.

A

a) Describe the domain and range of the function.

b) Sketch the graph by applying a transformation to the graph of $t(h) = \sqrt{h}$.

10. For each set of functions, transform the graph of $f(x)$ to sketch $g(x)$ and $h(x)$, and state the domain and range of each function.

a) $f(x) = x^2, g(x) = \left(\frac{1}{4}x^2\right), h(x) = (-4x^2)$

b) $f(x) = \sqrt{x}, g(x) = \sqrt{\frac{1}{5}x}, h(x) = \sqrt{-5x}$

c) $f(x) = \frac{1}{x}, g(x) = \frac{1}{4x}, h(x) = \frac{1}{(-\frac{1}{3}x)}$

d) $f(x) = |x|, g(x) = |-2x|, h(x) = \left|\frac{1}{2}x\right|$

11. The function $y = f(x)$ has been transformed to $y = f(kx)$. Determine the value of k for each transformation.

a) a horizontal stretch by the factor 4

b) a horizontal compression by the factor $\frac{1}{2}$

c) a reflection in the y-axis

d) a horizontal compression by the factor $\frac{1}{5}$ and a reflection in the y-axis

12. A quadratic function has equation $f(x) = x^2 - x - 6$. Determine the x-intercepts for each function.

T

a) $y = f(2x)$ **b)** $y = f\left(\frac{1}{3}x\right)$ **c)** $y = f(-3x)$

13. **a)** Describe how the graph of $y = f(kx)$ can be obtained from the graph of $y = f(x)$. Include examples that show how the transformations vary with the value of k.

C

b) Compare the graph of $y = f(kx)$ with the graph of $y = kf(x)$ for different values of k and different functions $f(x)$. How are the transformations alike? How are they different?

Extending

14. **a)** Graph the function $y = \frac{1}{x}$.

b) Apply a horizontal stretch with factor 2.

c) Apply a vertical stretch with factor 2. What do you notice?

d) Write the equations of the functions that result from the transformations in parts (b) and (c). Explain why these equations are the same.

15. Suppose you are asked to graph $y = f(2x + 4)$. What two transformations are required? Does the order in which you apply these transformations make a difference? Choose one of the parent functions and investigate. If you get two different results, use a graphing calculator to verify which graph is correct.

1.8 Using Transformations to Graph Functions of the Form $y = af[k(x - d)] + c$

GOAL

Apply combinations of transformations, in a systematic order, to sketch graphs of functions.

YOU WILL NEED

- graph paper or graphing calculator

LEARN ABOUT the Math

Neil wants to sketch the graph of $f(x) = -3\sqrt{2(x + 4)} - 1$.

? How can Neil apply the transformations necessary to sketch the graph?

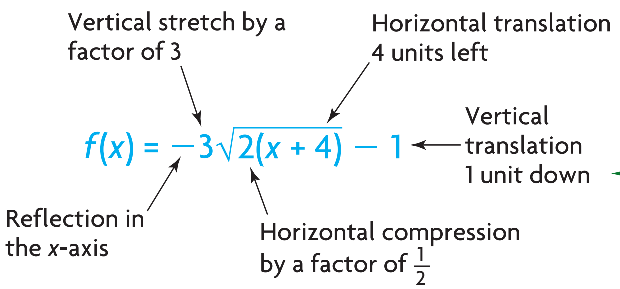
EXAMPLE 1 Applying a combination of transformations

Sketch the graph of $f(x) = -3\sqrt{2(x + 4)} - 1$. State the domain and range of the transformed function.

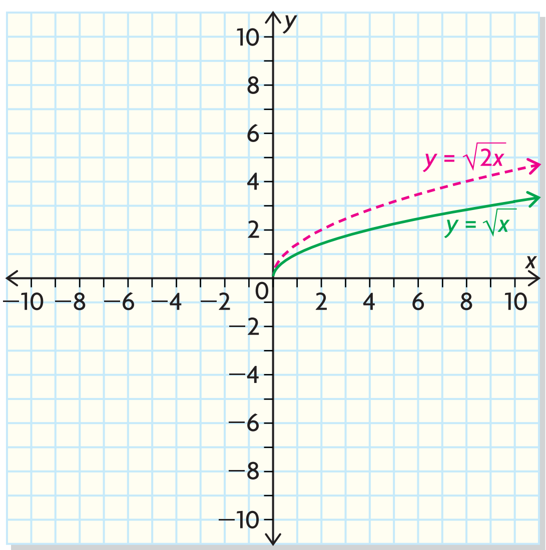
Neil's Solution

The parent function is $f(x) = \sqrt{x}$.

The function is a transformed square root function.

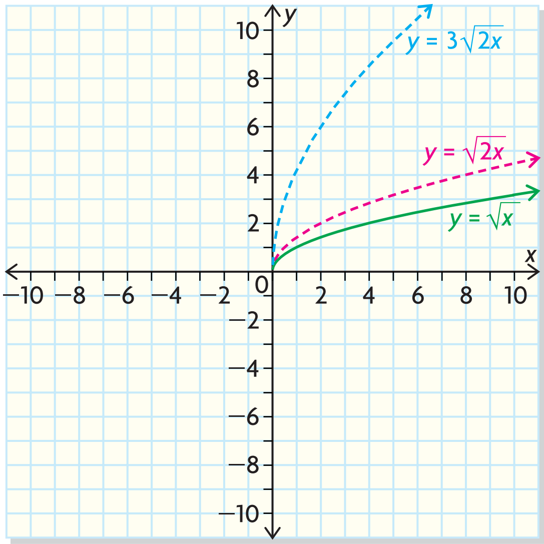


I looked at each part of the function and wrote down all the transformations I needed to apply.



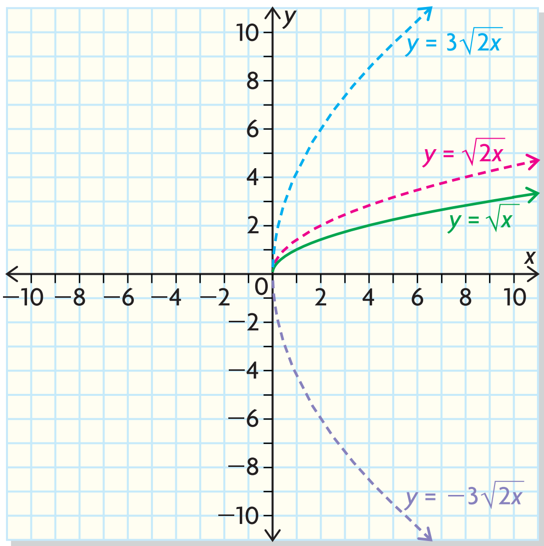
First I divided the x-coordinates of points on $y = \sqrt{x}$ by 2 to compress the graph horizontally by the factor $\frac{1}{2}$.

$f(x)$	$f(2x)$
(0, 0)	(0, 0)
(1, 1)	$\left(\frac{1}{2}, 1\right)$
(4, 2)	(2, 2)
(9, 3)	$\left(4\frac{1}{2}, 3\right)$



I multiplied the y-coordinates of $y = \sqrt{2x}$ by 3 to stretch the graph vertically by the factor 3.

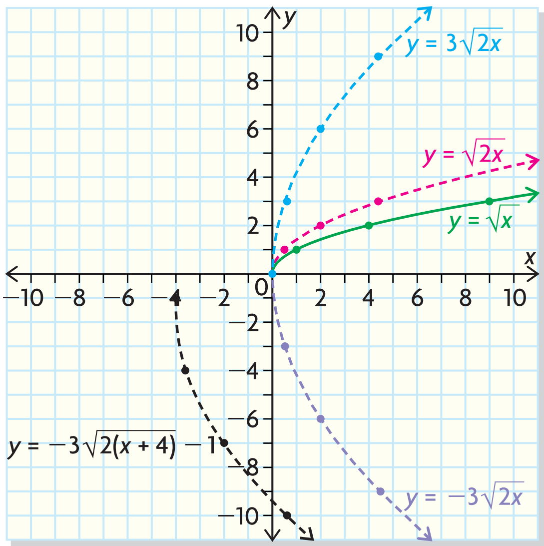
$f(x)$	$f(2x)$	$3f(2x)$
(0, 0)	(0, 0)	(0, 0)
(1, 1)	$\left(\frac{1}{2}, 1\right)$	$\left(\frac{1}{2}, 3\right)$
(4, 2)	(2, 2)	(2, 6)
(9, 3)	$\left(4\frac{1}{2}, 3\right)$	$\left(4\frac{1}{2}, 9\right)$



I flipped the graph of $y = 3\sqrt{2x}$ over the x-axis.

$f(x)$	$f(2x)$	$3f(2x)$	$-3f(2x)$
(0, 0)	(0, 0)	(0, 0)	(0, 0)
(1, 1)	$\left(\frac{1}{2}, 1\right)$	$\left(\frac{1}{2}, 3\right)$	$\left(\frac{1}{2}, -3\right)$
(4, 2)	(2, 2)	(2, 6)	(2, −6)
(9, 3)	$\left(4\frac{1}{2}, 3\right)$	$\left(4\frac{1}{2}, 9\right)$	$\left(4\frac{1}{2}, -9\right)$

Translate the graph 4 units left and 1 unit down.



I did both shifts together. I subtracted 4 from each of the x-coordinates and subtracted 1 from each of the y-coordinates of the graph of $y = -3\sqrt{2x}$.

$f(x)$	$f(2x)$	$3f(2x)$	$-3f(2x)$	$-3f(2(x+4))-1$
$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(-4, -1)$
$(1, 1)$	$\left(\frac{1}{2}, 1\right)$	$\left(\frac{1}{2}, 3\right)$	$\left(\frac{1}{2}, -3\right)$	$\left(-3\frac{1}{2}, -4\right)$
$(4, 2)$	$(2, 2)$	$(2, 6)$	$(2, -6)$	$(-2, -7)$
$(9, 3)$	$\left(4\frac{1}{2}, 3\right)$	$\left(4\frac{1}{2}, 9\right)$	$\left(4\frac{1}{2}, -9\right)$	$\left(\frac{1}{2}, -10\right)$

Domain $= \{x \in \mathbf{R} \mid x \geq -4\}$

Range $= \{y \in \mathbf{R} \mid y \leq -1\}$

From the final graph, $x \geq -4$ and $y \leq -1$.

Reflecting

A. How do the numbers in the function $f(x) = -3\sqrt{2(x+4)} - 1$ affect the x- and y-coordinates of each point on the parent function ?

B. How did Neil determine the domain and range of the final function?

C. How does the order in which Neil applied the transformations compare with the order of operations for numerical expressions?

D. Sarit says that she can graph the function in two steps. She would do both stretches or compressions and any reflections to the parent function first and then both translations. Do you think this will work? Explain.

APPLY the Math

EXAMPLE 2 Applying transformations to the equation and the graph

Some transformations are applied, in order, to the reciprocal function $f(x) = \frac{1}{x}$:

- horizontal stretch by the factor 3
- vertical stretch by the factor 2
- reflection in the y-axis
- translation 5 units right and 4 units up

a) Write the equation for the final transformed function $g(x)$.
b) Sketch the graphs of $f(x)$ and $g(x)$.
c) State the domain and range of both functions.

Lynn's Solution

a)
$$\begin{aligned} g(x) &= af[k(x - d)] + c \\ &= 2f\left[-\frac{1}{3}(x - 5)\right] + 4 \\ &= \frac{2}{\left(-\frac{1}{3}(x - 5)\right)} + 4 \end{aligned}$$

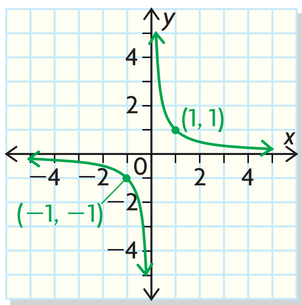
I built up the equation from the transformations.

A horizontal stretch by the factor 3 and a reflection in the y-axis means that $k = -\frac{1}{3}$.

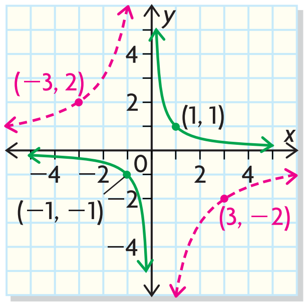
$a = 2$, because there is a vertical stretch by the factor 2.

$d = 5$ and $c = 4$, because the translation is 5 units right and 4 units up.

b) Graph of $f(x)$:



I sketched the graph of $f(x)$ and labelled the points (1, 1) and (−1, −1). The vertical asymptote is $x = 0$ and the horizontal asymptote is $y = 0$.

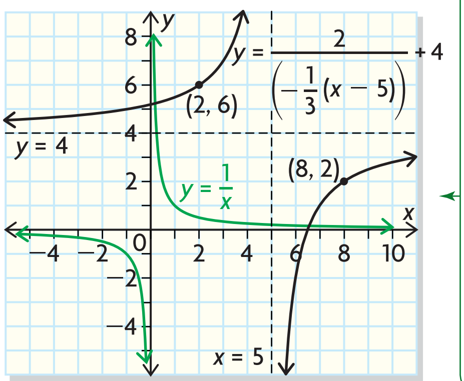


I applied the stretches and reflection to the labelled points by multiplying the x-coordinates by -3 and the y-coordinates by 2.

$(1, 1)$ became $(-3, 2)$ and $(-1, -1)$ became $(3, -2)$. The asymptotes did not change, since x and y still couldn't be 0.

I made a sketch of the stretched and reflected graph before applying the translation.

Graph of $g(x)$:



To apply the translations, 5 right and 4 up, I drew in the translated asymptotes first.

Since all the points moved 5 right, the new vertical asymptote is $x = 5$.

Since all the points moved up 4, the new horizontal asymptote is $y = 4$.

Then I drew the stretched and reflected graph in the new position after the translation.

I labelled the graphs and wrote the equations for the asymptotes.

c) For $f(x)$,

Domain $= \{x \in \mathbf{R} \mid x \neq 0\}$

Range $= \{y \in \mathbf{R} \mid y \neq 0\}$

For $g(x)$,

Domain $= \{x \in \mathbf{R} \mid x \neq 5\}$

Range $= \{y \in \mathbf{R} \mid y \neq 4\}$

I used the equations of the asymptotes to help determine the domain and range.

The graphs do not meet their asymptotes, so for $f(x)$, x cannot be 0 and y cannot be 0. Also, for $g(x)$, x cannot be 5 and y cannot be 4.

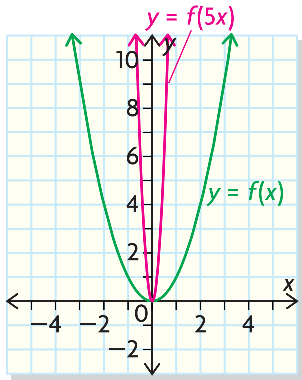
EXAMPLE 3 Factoring out k before applying transformations

For $f(x) = x^2$, sketch the graph of $g(x) = f(-5x + 10)$.

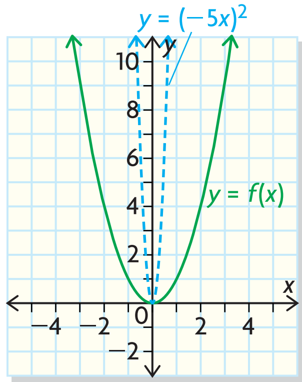
Stefan's Solution

$$g(x) = f(-5x + 10)$$
$$= f[-5(x - 2)]$$

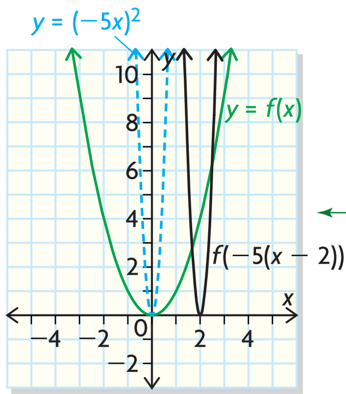
I wrote $g(x)$ in $af[k(x - d)] + c$ form by factoring out $k = -5$.



I graphed $y = f(x)$ and compressed the graph horizontally by the factor $\frac{1}{5}$. This gave me the graph of $y = f(5x)$.



I reflected $y = f(5x)$ in the y-axis. The graph of $y = f(-5x)$ looked the same because the y-axis is the axis of symmetry for $y = f(5x)$.



I translated the compressed and reflected graph 2 units right. This gave me the graph of $y = f[-5(x - 2)]$.

EXAMPLE 4 Identifying the equation of a transformed function from its graph

Match each equation to its graph. Explain your reasoning.

1. $y = \dfrac{1}{0.3(x + 1)} - 2$

2. $y = -4|x + 2| + 1$

3. $y = -\sqrt{3(x + 2)} - 1$

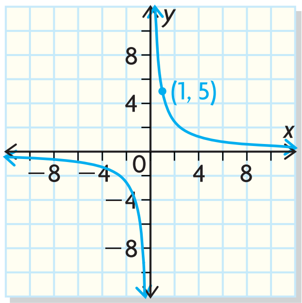
4. $y = \sqrt{-0.4(x - 4)} + 3$

5. $y = (0.5(x - 4))^2 + 2$

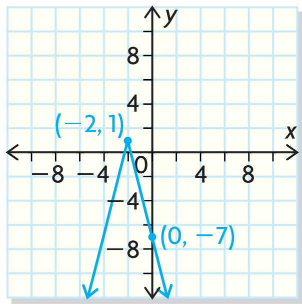
6. $y = \dfrac{5}{x}$

7. $y = -3(x + 1)^2 + 4$

A



D



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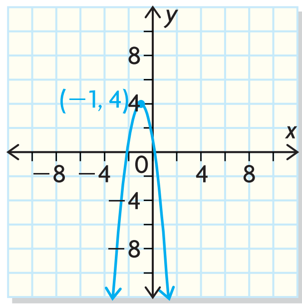

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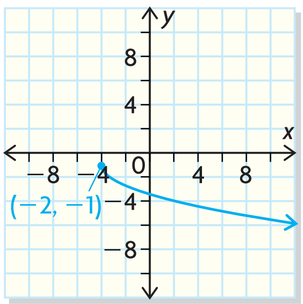

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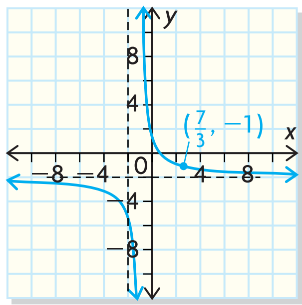

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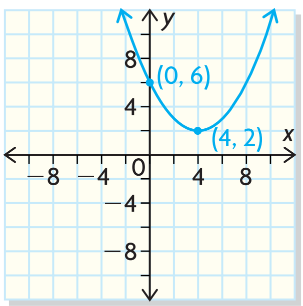

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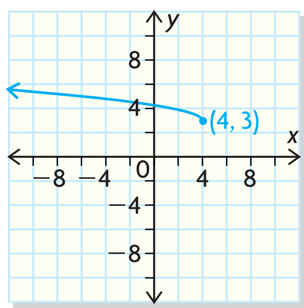


Donna's Solution

Graph A matches equation 6.

Graph A is like the graph of $y = \dfrac{1}{x}$, but it has been stretched vertically. The point (1, 1) has been stretched to (1, 5), so the scale factor is 5. The equation really is $y = \dfrac{5}{x}$.

Graph B matches equation 3.

This is the graph of a square root function that has been flipped over the x-axis, so, in the equation, a will be negative.

The parent square root graph has been compressed horizontally or stretched vertically. It starts at $(-2, -1)$ instead of $(0, 0)$, so it has been translated 2 units left and 1 unit down. So, $c = -2$ and $d = -1$.

Graph C matches equation 5.

Graph C is a parabola, so it has to match equation 5 or equation 7. Since $a > 0$, the parabola opens upward, so the answer can't be equation 7 and has to be equation 5.

I checked: The vertex is $(4, 2)$, so $d = 4$ and $c = 2$.

Graph C is wider than the parent function, so it has been stretched horizontally or compressed vertically. Equation 5 is the equation of a parabola with vertex $(4, 2)$, that opens up, and that has been stretched horizontally by the factor $\frac{1}{0.5} = 2$.

Graph D matches equation 2.

This is the graph of an absolute value function. The parent graph has been reflected in the x-axis, stretched vertically, and shifted 2 units left and 1 unit up. The equation must have $a < -1$, $d = -2$, and $c = 1$.

Graph E matches equation 1.

This is a transformation of the graph of $y = \frac{1}{x}$, so the answer has to be equation 1 or equation 6. The equations for the asymptotes are $x = -1$ and $y = -2$, so $d = -1$ and $c = -2$. This matches equation 1.

Also, in the equation, $k = 0.3$ means that the parent graph has been stretched horizontally by the factor $\frac{1}{0.3}$. The point $(1, 1)$ on the parent graph becomes $\left(\frac{10}{3}, 1\right)$, when you multiply the x-coordinate by $\frac{1}{0.3}$. Then, this point becomes $\left(\frac{7}{3}, -1\right)$, when you apply the translations by subtracting 1 from the x-coordinate and 2 from the y-coordinate.

Graph F matches equation 4.

This is another square root function. The parent function has been flipped over the y-axis, so $k < 0$. It has been stretched horizontally, so $-1 < k < 0$, and translated 4 units right and 3 units up, so $d = 4$ and $c = 3$.

Graph G matches equation 7.

Graph G is a parabola that opens down, so it has to match equation 7 because it is vertically stretched (narrow) and has vertex at $(-1, 4)$. Equation 7 has $a = -3$, which means that the parabola opens down and is vertically stretched by the factor 3. Also, $c = -1$ and $d = 4$, which means that the vertex is $(-1, 4)$, as in graph G.

In Summary

Key Ideas

- You can graph functions of the form $g(x) = af[k(x - d)] + c$ by applying the appropriate transformations to the key points of the parent function, one at a time, making sure to apply a and k before c and d. This order is like the order of operations for numerical expressions, since multiplications (stretches, compressions, and reflections) are done before additions and subtractions (translations).
- When using transformations to graph, you can apply a and k together, then c and d together, to get the desired graph in fewer steps.

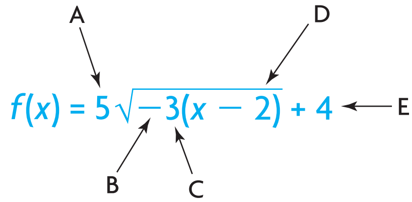
Need to Know

- The value of a determines the vertical stretch or compression and whether there is a reflection in the x-axis:
  - When $|a| > 1$, the graph of $y = f(x)$ is stretched vertically by the factor $|a|$.
  - For $0 < |a| < 1$, the graph is compressed vertically by the factor $|a|$.
  - When $a < 0$, the graph is also reflected in the x-axis.
- The value of k determines the horizontal stretch or compression and whether there is a reflection in the y-axis:
  - When $|k| > 1$, the graph is compressed horizontally by the factor $\frac{1}{|k|}$.
  - When $0 < |k| < 1$, the graph is stretched horizontally by the factor $\frac{1}{|k|}$.
  - When $k < 0$, the graph is also reflected in the y-axis.
- The value of d determines the horizontal translation:
  - For $d > 0$, the graph is translated d units right.
  - For $d < 0$, the graph is translated d units left.
- The value of c determines the vertical translation:
  - For $c > 0$, the graph is translated c units up.
  - For $c < 0$, the graph is translated c units down.

CHECK Your Understanding

1. Use words from the list to describe the transformations indicated by the arrows.

horizontal	*x*-axis
vertical	*y*-axis
stretch	factor
compression	up
reflection	down
translation	right
	left



2. Match each operation to one of the transformations from question 1.

Divide the *x*-coordinates by 3.	A
Multiply the *y*-coordinates by 5.	B
Multiply the *x*-coordinates by -1.	C
Add 4 to the *y*-coordinate.	D
Add 2 to the *x*-coordinate.	E

3. Complete the table for the point (1, 1).

$f(x)$	$f(3x)$	$f(-3x)$	$5f(-3x)$	$5f(-3(x-2))+4$
(1, 1)				

PRACTISING

4. **K** Explain what transformations you would need to apply to the graph of $y = f(x)$ to graph each function.

a) $y = 3f(x) - 1$ **c)** $y = f(2x) - 5$ **e)** $y = \frac{2}{3}f(x+3) + 1$

b) $y = f(x-2) + 3$ **d)** $y = -f\left(\frac{1}{2}x\right) - 2$ **f)** $y = 4f(-x) - 4$

5. Sketch each set of functions on the same set of axes.

a) $y = x^2, y = 3x^2, y = 3(x-2)^2 + 1$

b) $y = \sqrt{x}, y = \sqrt{3x}, y = \sqrt{-3x}, y = \sqrt{-3(x+1)} - 4$

c) $y = \frac{1}{x}, y = \frac{2}{x}, y = -\frac{2}{x}, y = -\frac{2}{x-1} + 3$

d) $y = |x|, y = \left|\frac{1}{2}x\right|, y = -\left|\frac{1}{2}x\right|, y = -\left|\frac{1}{2}(x+3)\right| - 2$

6. Explain what transformations you would need to apply to the graph of $y = f(x)$ to graph each function.

a) $y = f\left(\frac{1}{3}(x+4)\right)$ **c)** $y = -3f(2(x-1)) - 3$

b) $y = 2f(-(x-3)) + 1$

7. If $f(x) = x^2$, sketch the graph of each function and state the domain and range.

a) $y = f(x - 2) + 3$

b) $y = -f\left(\frac{1}{4}(x + 1)\right) + 2$

c) $y = 0.5f(3(x - 4)) - 1$

8. If $f(x) = \sqrt{x}$, sketch the graph of each function and state the domain and range.

a) $y = f(x - 1) + 4$

b) $y = f\left(-\frac{1}{2}(x + 4)\right) - 3$

c) $y = -2f(-(x - 2)) + 1$

9. If $f(x) = |x|$, sketch the graph of each function and state the domain and range.

a) $y = 2f(x - 3)$

b) $y = 4f(2(x - 1)) - 2$

c) $y = -\frac{1}{2}f(3(x + 2)) + 4$

10. Describe the transformations that you would apply to the graph of $f(x) = \frac{1}{x}$ to transform it into each of these graphs.

a) $y = \frac{1}{x - 2}$

b) $y = \frac{1}{x} + 2$

c) $y = 0.5\left(\frac{1}{x}\right)$

d) $y = \frac{2}{x}$

e) $y = \frac{1}{2x}$

f) $y = -\frac{1}{x}$

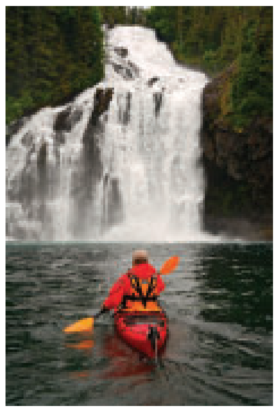
11. For $f(x) = x^2$, sketch the graph of $g(x) = f(2x + 6)$.

12. For $f(x) = \sqrt{x}$, sketch the graph of $h(x) = f(-3x - 12)$.

13. For $f(x) = |x|$, sketch the graph of $p(x) = f(4x + 8)$.

14. **A** Low and high blood pressure can both be dangerous. Doctors use a special index, P_d, to measure how far from normal someone's blood pressure is. In the equation $P_d = |P - \overline{P}|$, P is a person's systolic blood pressure and $\overline{P}$ is the normal systolic blood pressure. Sketch the graph of this index. Assume that normal systolic blood pressure is 120 mm(Hg).

15. Bhavesh uses the relationship Time $= \frac{\text{Distance}}{\text{Speed}}$ to plan his kayaking trips. Tomorrow Bhavesh plans to kayak 20 km across a calm lake. He wants to graph the relation $T(s) = \frac{20}{s}$ to see how the time, T, it will take varies with his kayaking speed, s. The next day, he will kayak 15 km up a river that flows at 3 km/h. He will need the graph of $T(s) = \frac{15}{s - 3}$ to plan this trip. Use transformations to sketch both graphs.



16. The graph of $g(x) = \sqrt{x}$ is reflected across the y-axis, stretched vertically by the factor 3, and then translated 5 units right and 2 units down. Draw the graph of the new function and write its equation.

17. The graph of $y = f(x)$ is reflected in the y-axis, stretched vertically by the factor 3, and then translated up 2 units and 1 unit left. Write the equation of the new function in terms of f.

18. Match each equation to its graph. Explain your reasoning.

a) $y = \dfrac{3}{-(x-2)} + 1$

b) $y = 2|x - 3| - 2$

c) $y = -2\sqrt{x+3} - 2$

d) $y = (0.25(x-2))^2 - 3$

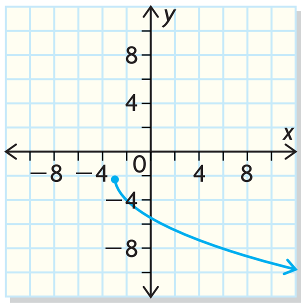
e) $y = -\dfrac{4}{x} - 3$

f) $y = -0.5|x + 4| + 2$

g) $y = -0.5\sqrt{1-x} + 1$

h) $y = -\dfrac{1}{2}(x+4)^2 + 1$

A



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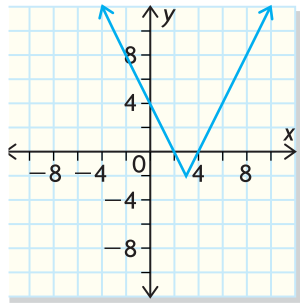

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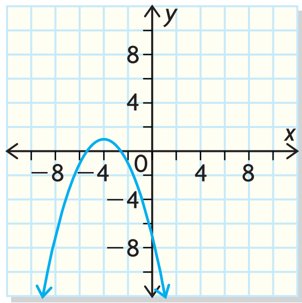

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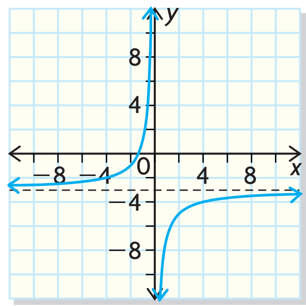

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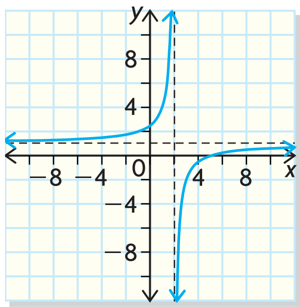

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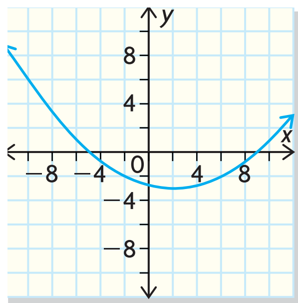

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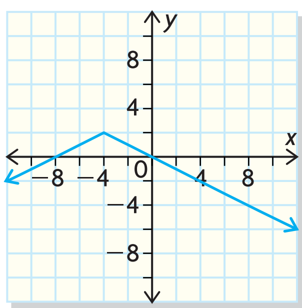

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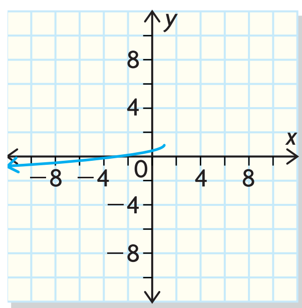


19. The function $y = f(x)$ has been transformed to $y = af[k(x - d)] + c$. Determine a, k, c, and d; sketch the graph; and state the domain and range for each transformation.

a) A vertical stretch by the factor 2, a reflection in the x-axis, and a translation 4 units right are applied to $y = \sqrt{x}$.

b) A vertical compression by the factor $\frac{1}{2}$, a reflection in the y-axis, a translation 3 units left, and a translation 4 units down are applied to $f(x) = \frac{1}{x}$.

c) A horizontal compression by the factor $\frac{1}{3}$, a vertical stretch by the factor 3, a translation 1 unit right, and a translation 6 units down are applied to $y = |x|$.

20. If $f(x) = (x - 2)(x + 5)$, determine the x-intercepts for each function.

a) $y = f(x)$

b) $y = -4f(x)$

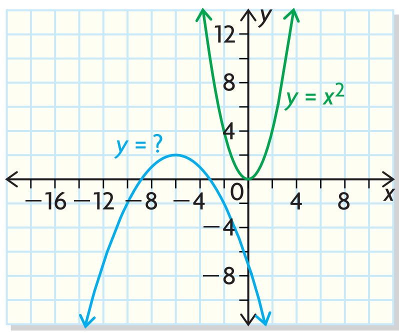
c) $y = f\left(-\frac{1}{3}x\right)$

d) $y = f(-(x + 2))$

21. **C** List the steps you would take to sketch the graph of a function of the form $y = af(k(x - d)) + c$ when $f(x)$ is one of the parent functions you have studied in this chapter. Discuss the roles of a, k, d, and c, and the order in which they would be applied.

Extending

22. The graphs of $y = x^2$ and another parabola are shown.



a) Determine a combination of transformations that would produce the second parabola from the first.

b) Determine a possible equation for the second parabola.

23. Compare the graphs and the domains and ranges of $f(x) = x^2$ and $g(x) = \sqrt{x}$. How are they alike? How are they different? Develop a procedure to obtain the graph of $g(x)$ from the graph of $f(x)$.

1 Chapter Review

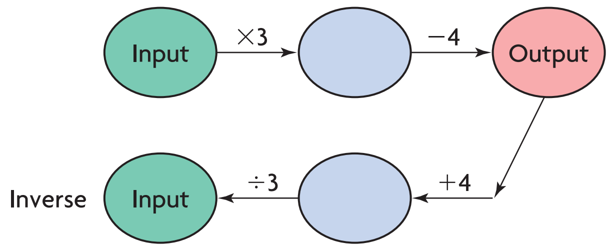
FREQUENTLY ASKED Questions

Study Aid

- See Lesson 1.5, Examples 1 and 2.
- Try Chapter Review Questions 10 and 11.

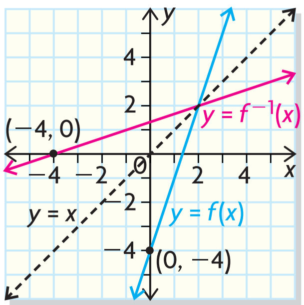
Q: How can you determine the inverse of a linear function?

A1: The inverse of a linear function is the reverse of the original function. It undoes what the original has done. This means that you can find the equation of the inverse by reversing the operations on x. For example, if $f(x) = 3x - 4$, the operations on x are as follows: Multiply by 3 and then subtract 4. To reverse these operations, you add 4 and divide by 3, so the inverse function is $f^{-1}(x) = \frac{x + 4}{3}$.



A2: If (x, y) is on the graph of $f(x)$, then (y, x) is on the inverse graph, so you can switch x and y in the equation to find the inverse equation. For example, if $f(x) = 3x - 4$, you can write this as $y = 3x - 4$. Then switch x and y and solve for y.

$$x = 3y - 4$$
$$x + 4 = 3y - 4 + 4$$
$$x + 4 = 3y$$
$$\frac{x + 4}{3} = y, \text{ so the inverse function is } f^{-1}(x) = \frac{x + 4}{3}.$$



A3: If you have the graph of a linear function, you can graph the inverse function by reflecting in the line $y = x$.

The inverse of a linear function is another linear function, unless the original function represents a horizontal line.

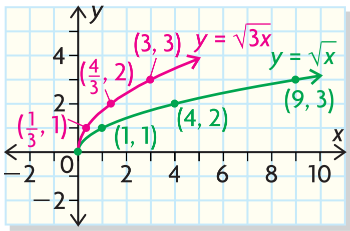
Study Aid

- See Lesson 1.7, Example 1.
- Try Chapter Review Questions 12 and 13.

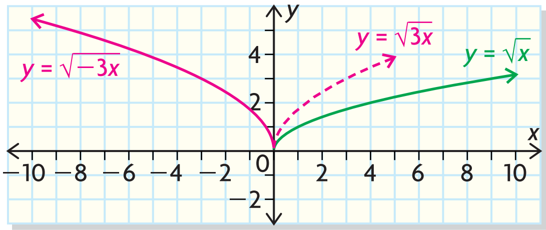
Q: How do you apply a horizontal stretch, compression, or reflection to the graph of a function?

A: The graph of $y = f(kx)$ is the graph of $y = f(x)$ after a horizontal stretch, compression, or reflection. When k is a number greater than 1 or less than -1, the graph is compressed horizontally by the factor $\frac{1}{k}$. When k is a number between -1 and 1, the graph is stretched horizontally by the factor $\frac{1}{k}$. Whenever k is negative, the graph is also reflected in the y-axis.

You apply a horizontal compression by dividing the x-coordinates of points on the original graph by k. For example, to graph $y = \sqrt{3x}$, graph $y = \sqrt{x}$ and then divide the x-coordinates of the points (0, 0), (1, 1), (4, 2), (9, 3) by 3 (or multiply by $\frac{1}{3}$) to get the points (0, 0), $(\frac{1}{3}, 1)$, $(\frac{4}{3}, 2)$, (3, 3) on the transformed graph.



When k is negative, you also reflect the graph in the y-axis. For example, $y = \sqrt{-3x}$.



Q: How do you sketch the graph of $y = af[k(x - d)] + c$ when you have the graph of $y = f(x)$?

A1: You can graph the parent function and then apply the transformations one by one, starting with the compressions, stretches, and reflections and leaving the translations until last. For example, to sketch the graph of $y = 3f(6 - 2x) - 5$ when $f(x) = \frac{1}{x}$, begin by putting the equation into the form $y = af[k(x - d)] + c$ by factoring. This gives $y = 3f[-2(x - 3)] - 5$. Then identify all the transformations you need to apply:

- $a = 3$ means a vertical stretch by the factor 3.
- $k = -2$ means a horizontal compression by the factor $\frac{1}{2}$ and a reflection in the y-axis.
- $d = 3$ means a horizontal translation 3 units right.
- $c = -5$ means a vertical translation 5 units down.

A2: You can graph the function in two steps: Apply both stretches or compressions and any reflections to the parent function first, and then both translations.

Study *Aid*

- See Lesson 1.8, Examples 1, 2, and 3.
- Try Chapter Review Questions 14 to 18.

PRACTICE Questions

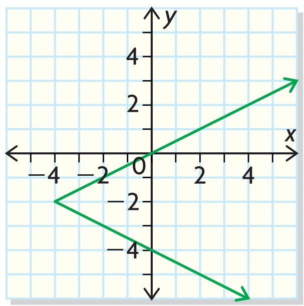
Lesson 1.1

1. For each relation, determine the domain and range and whether the relation is a function. Explain your reasoning.

a) $\{(-3, 0), (-1, 1), (0, 1), (4, 5), (0, 6)\}$

b) $y = 4 - x$

c)



d) $x^2 + y^2 = 16$

2. What rule can you use to determine, from the graph of a relation, whether the relation is a function? Graph each relation and determine which are functions.

a) $\{(-2. 1), (1, 1), (0, 0), (1, -1), (1, -2), (2, -2)\}$

b) $y = 4 - 3x$

c) $y = (x - 2)^2 + 4$

d) $x^2 + y^2 = 1$

e) $y = \frac{1}{x}$

f) $y = \sqrt{x}$

3. Sketch the graph of a function whose domain is the set of real numbers and whose range is the set of real numbers less than or equal to 3.

Lesson 1.2

4. If $f(x) = x^2 + 3x - 5$ and $g(x) = 2x - 3$, determine each.

a) $f(-1)$

b) $f(0)$

c) $g\left(\frac{1}{2}\right)$

d) $f(2b)$

e) $g(1 - 4a)$

f) x when $f(x) = g(x)$

5. a) Graph the function $f(x) = -2(x - 3)^2 + 4$, and state its domain and range.

b) What does $f(1)$ represent on the graph? Indicate, on the graph, how you would find $f(1)$.

c) Use the equation to determine each of the following.

i) $f(3) - f(2)$

ii) $2f(5) + 7$

iii) $f(1 - x)$

6. If $f(x) = x^2 - 4x + 3$, determine the input(s) for x whose output is $f(x) = 8$.

Lesson 1.4

7. A ball is thrown upward from the roof of a building 60 m tall. The ball reaches a height of 80 m above the ground after 2 s and hits the ground 6 s after being thrown.

a) Sketch a graph that shows the height of the ball as a function of time.

b) State the domain and range of the function.

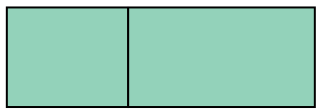
c) Determine an equation for the function.

8. State the domain and range of each function.

a) $f(x) = 2(x - 1)^2 + 3$

b) $f(x) = \sqrt{2x + 4}$

9. A farmer has 540 m of fencing to enclose a rectangular area and divide it into two sections as shown.



a) Write an equation to express the total area enclosed as a function of the width.

b) Determine the domain and range of this area function.

c) Determine the dimensions that give the maximum area.

Lesson 1.5

10. Using the functions listed as examples, describe three methods for determining the inverse of a linear function. Use a different method for each function.

a) $f(x) = 2x - 5$

b) $f(x) = \frac{x + 3}{7}$

c) $f(x) = 4 - \frac{1}{2}x$

11. For a fundraising event, a local charity organization expects to receive \$15 000 from corporate sponsorship, plus \$30 from each person who attends the event.

a) Use function notation to express the total income from the event as a function of the number of people who attend.

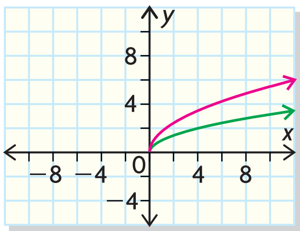
b) Suggest a reasonable domain and range for the function in part (a). Explain your reasoning.

c) The organizers want to know how many tickets they need to sell to reach their fundraising goal. Create a function to express the number of people as a function of expected income. State the domain of this new function.

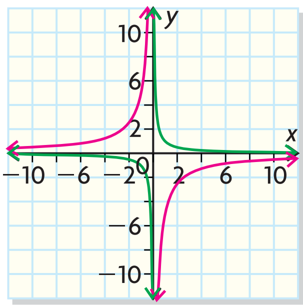
Lesson 1.7

12. In each graph, a parent function has undergone a transformation of the form $f(kx)$. Determine the equations of the transformed functions graphed in red. Explain your reasoning.

a)



b)



13. For each set of functions, transform the graph of $f(x)$ to sketch $g(x)$ and $h(x)$, and state the domain and range of each function.

a) $f(x) = x^2$, $g(x) = \left(\frac{1}{2}x\right)^2$, $h(x) = -(2x)^2$

b) $f(x) = |x|$, $g(x) = |-4x|$, $h(x) = \left|\frac{1}{4}x\right|$

Lesson 1.8

14. Three transformations are applied to $y = x^2$: a vertical stretch by the factor 2, a translation 3 units right, and a translation 4 units down.

a) Is the order of the transformations important?

b) Is there any other sequence of these transformations that could produce the same result?

15. The point $(1, 4)$ is on the graph of $y = f(x)$. Determine the coordinates of the image of this point on the graph of $y = 3f[-4(x + 1)] - 2$.

16. a) Explain what you would need to do to the graph of $y = f(x)$ to graph the function $y = -2f[(\frac{1}{3}x + 4)] - 1$.

b) Graph the function in part (a) for $f(x) = x^2$.

17. In each case, write the equation for the transformed function, sketch its graph, and state its domain and range.

a) The graph of $f(x) = \sqrt{x}$ is compressed horizontally by the factor $\frac{1}{2}$, reflected in the y-axis, and translated 3 units right and 2 units down.

b) The graph of $y = \frac{1}{x}$ is stretched vertically by the factor 3, reflected in the x-axis, and translated 4 units left and 1 unit up.

18. If $f(x) = (x - 4)(x + 3)$, determine the x-intercepts of each function.

a) $y = f(x)$

b) $y = -2f(x)$

c) $y = f\left(-\frac{1}{2}x\right)$

d) $y = f(-(x + 1))$

19. A function $f(x)$ has domain $\{x \in \mathbf{R} \mid x \geq -4\}$ and range $\{y \in \mathbf{R} \mid y < -1\}$. Determine the domain and range of each function.

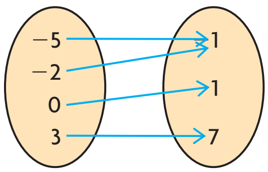
a) $y = 2f(x)$

b) $y = f(-x)$

c) $y = 3f(x + 1) + 4$

d) $y = -2f(-x + 5) + 1$

1 Chapter Self-Test



1. For each relation, determine the domain and range and whether the relation is a function. Explain your reasoning.

a) The function shown at the left.

b) $y = \sqrt{x + 2}$

2. An incandescent light bulb costs \$0.65 to buy and \$0.004/h for electricity to run. A fluorescent bulb costs \$3.50 to buy and \$0.001/h to run.

a) Use function notation to write a cost equation for each type of bulb.

b) State the domain and range of each function.

c) After how long is the fluorescent bulb cheaper than the regular bulb?

d) Determine the difference in costs after one year. Assume the light is on for an average of 6 h/day.

3. Determine the domain and range of each function. Show your steps.

a) $f(x) = \dfrac{1}{x - 2}$

b) $f(x) = \sqrt{3 - x} - 4$

c) $f(x) = -|x + 1| + 3$

4. Explain what the term *inverse* means in relation to a linear function. How are the domain and range of a linear function related to the domain and range of its inverse?

5. For each function, determine the inverse, sketch the graphs of the function and its inverse, and state the domain and range of both the function and its inverse.

a) $\{(-2, 3), (0, 5), (2, 6), (4, 8)\}$

b) $f(x) = 3 - 4x$

6. At Phoenix Fashions, Rebecca is paid a monthly salary of \$1500, plus 4% commission on her sales over \$2500.

a) Graph the relation between monthly earnings and sales.

b) Use function notation to write an equation of the relation.

c) Graph the inverse relation.

d) Use function notation to write an equation of the inverse.

e) Use the equation in part (d) to express Rebecca's sales if she earned \$1740 one month. Then evaluate.

7. The function $y = f(x)$ has been transformed to $y = f(kx)$. Determine the value of k for each transformation.

a) a horizontal stretch by the factor 5

b) a horizontal compression by the factor $\frac{1}{3}$ and a reflection in the y-axis

8. The function $y = f(x)$ has been transformed to $y = af[k(x - d)] + c$. Determine a, k, d, and c; write the equation; sketch the graph; and state the domain and range of each transformed function.

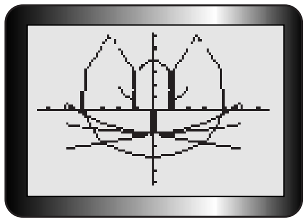
a) vertical compression by the factor $\frac{1}{2}$, reflection in the y-axis, and translation 2 units right, applied to $y = \sqrt{x}$

b) vertical stretch by the factor 4, reflection in the x-axis, translation 2 units left, and translation 3 units down, applied to $y = \dfrac{1}{x}$

c) horizontal compression by the factor $\frac{1}{4}$, vertical stretch by the factor $\frac{3}{2}$, reflection in the x-axis, translation 3 units right, and translation 2 units down, applied to $y = |x|$

1 Chapter Task

Functional Art



Parts of transformed parent functions were used to make this cat's face on a graphing calculator. The functions used are listed in the table that follows.

$$-6.58 \le X \le 6.58, \text{ Xscl } 1$$

$$-6.2 \le Y \le 6.2, \text{ Yscl } 1$$

Feature	Function	Domain
ears	$y = -\lvert 2x - 5\rvert + 6$ $y = -\lvert -2x - 5\rvert + 6$	$1 \le x \le 4$ $-4 \le x \le -1$
top of head	$y = -x^2 + 4$	$-1 \le x \le 1$
chin	$y = \dfrac{5}{x+5} - 5$ $y = \dfrac{5}{-x+5} - 5$	$-4 \le x \le 0$ $0 \le x \le 4$
eyes	$y = -\sqrt{4 - 2x} + 2$ $y = -\sqrt{4 - 2x} + 2$	$1 \le x \le 3$ $-3 \le x \le -1$
whiskers	$y = 0.1x^2 - 2$ $y = 0.03x^2 - 2$ $y = -0.25\lvert x\rvert - 2$	$-5 \le x \le 5$ $-5 \le x \le 5$ $-5 \le x \le 5$

? How can you use transformations of parent functions to create other pictures?

A. Re-create the cat's face on a graphing calculator. Begin by putting the calculator in DOT mode. Then enter each function listed in the table, along with its domain. The first three entries are shown.

B. Describe how transformations were used to create the cat's features. For each feature, describe
- which properties of the parent function were useful for that feature
- which transformations were used and why
- how symmetry was used and which transformations ensured symmetry

C. Create your own picture, using transformations of the parent functions $f(x) = x^2$, $f(x) = \sqrt{x}$, $y = \dfrac{1}{x}$, and $f(x) = \lvert x\rvert$. You must use each parent function at least once.

D. List the parts of your picture, the functions used, and the corresponding domains in a table. Explain why you chose each function and each transformation.

Task Checklist

- ✔ Did you use each parent function at least once?
- ✔ Did you list the transformed functions and the corresponding domains?
- ✔ Did you explain why you chose each function and each transformation?
- ✔ Did you describe the transformations in appropriate math language?